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Multi-Curve Valuation Approaches and their Application to Hedge Accounting according to IAS 39

by Dr. Dirk Schubert

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Foreword

The turmoil in financial markets in the course of the subprime crisis forced market participants to re-assess market risks inherent in financial instruments. The emergence of the re-assessment of market risks is mainly represented by the presence of "tenor and FX basis risks". Tenor dependence refers to repricing periods associated with the floating instruments, especially the floating side of interest rate swaps, while the FX basis refers to the relation in different currencies. Statistical evidence of financial market data reveals the persisting relevance of tenor and FX basis risks. It is highly unlikely that markets will return to previous conditions. Accordingly financial institutions are exposed to increased risk of higher volatility in profit & loss of their financial statements resulting from derivatives and other financial instruments.

The relevance of tenor and FX basis risks is accompanied by changes in market conventions with respect to the pricing of collateralized derivatives ("OIS discounting") and institutional changes in financial markets represented by the clearing houses ("central counterparties"). These facts from financial markets push financial institutions to accommodate to new valuation methodologies for the pricing and risk assessment of financial instruments, which is commonly summarized by the term "multi-curve valuation models".

Clearly, these developments also affect the financial accounting and especially the hedge accounting models applied under IAS 39. Hedge accounting is a very complex topic in financial accounting and its application is influenced by various components. This paper follows a step by step approach explaining the application of hedge accounting models according to IAS 39 in the course of the environmental changes in financial markets. Consequently the initial starting point of the analysis is a description of financial market conventions and the corresponding statistical facts. Based on this evidence simple and more complex multi-curve models which are applied in practice are described, and the impact on hedge accounting models according to IAS 39 is derived. As a result, a consistent approach of multi-curve and hedge accounting models according to IAS 39 can be achieved. This represents a contribution to the ongoing efforts in financial institutions to extend the new valuation models to financial accounting and aligning the financial reporting with economic risk assessment of hedging activities, which is advocated by IFRS 9.

This booklet was written by Dirk Schubert, Partner in Audit Financial Services of KPMG in Frankfurt/Main. It is an excellent example of how KPMG can contribute to cutting through complexity for the benefit of our clients.

KPMG AG Wirtschaftsprüfungsgesellschaft
The Management Board
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List of Abbreviations and Definitions

acc	Accrual
ACT	Actual
BFV	Bloomberg Fair Value
BGN	Bloomberg Generic (Price)
BIS	Bank for International Settlement
bp	Basis points
CCBS	Cross currency basis swap
CCP	Central counterparty
CCS	Cross currency swap
CDS	Credit default swap
CET	Central European Time
cf.	Confer – compare
CIP	Covered interest parity
CSA	Credit Support Annex to the ISDA Master Agreement (2002)
CVA	Counterparty valuation adjustment
d	Day
DVA	Debt valuation adjustment
ECB	European Central Bank
EMIR	European Market Infra-structure Regulation
EONIA	European Overnight Index Average
ESMA	European Securities and Markets Authority
EUR	Euro
EURIBOR	Euro Interbank Offered Rate
FAS	Financial Accounting Standard
FED Funds	Federal Funds
FFV	Full fair value
FV	Fair value
FRA	Forward rate agreement
FRN	Floating rate note
FOMC	Federal Open Market Committee
FX	Foreign exchange
GBP	Pound sterling
HFV	Hedge fair value
IAS	International Accounting Standard
IASB	International Accounting Standard Board
IFRS	International Financial Reporting Standards
IRS	Interest rate swap
ISDA	International Swaps and Derivatives Association
ISDA Master Agreement (2002)	Master Agreement for derivative contracts issued by ISDA (date: 2002)
ISDA Credit Derivatives Definitions (2003)	Definitions to the ISDA Master Agreement (2002) (date: 2003 in the form of the revision of the Big Bang Protocol 2009)

ISDA Confirmation	Document and other confirming evidence exchanged between the parties or otherwise effective for the purpose of confirming or evidencing transactions entered into under the ISDA Master Agreement (2002)
JPY	Japanese yen
LCH	London Clearing House
l.h.s.	Left-hand side
LIBOR	London Interbank Offered Rate
m	Month
MM	Money market
MtM	Mark-to-market
OCI	Other comprehensive income
OIS	Overnight index swap
ON	Overnight
OTC	Over-the-counter
PV	Present value
P&L	Profit and Loss
Repo	Repurchase agreement
resp.	Respectively
SCSA	Standard Credit Support Annex to the ISDA Master Agreement (2002)
SLI	Separate line item
SONIA	Sterling Overnight Interbank Average Rate
TARGET	Trans-European Automated Real-time Gross Settlement Express Transfer
TEUR	Thousand euro
TUSD	Thousand US dollar
USD	US dollar
VaR	Value at Risk
vs.	Versus
w	Week
w.r.t.	With respect to
y	Year
1M	1-month
3M	3-month
6M	6-month
12M	12-month

1 Introduction

Multi-curve approaches for pricing derivatives, especially interest rate derivatives are currently in the process of being implemented by banks in order to adapt to changes in market practice.[1] As a consequence of the financial crisis, market participants take into account the collateralization of over-the-counter (OTC) derivatives, e.g. based on the Credit Support Annex to the International Swaps and Derivatives Association Master Agreement (2002) (CSA), as well as additional risk factors like tenor basis spreads in order to derive market consistent prices for derivatives. This stems from the fact that e.g. interest rates quoted in the money market as well as prices in the derivative market revealed significant differences between different tenors.

Irrespective of the financial crisis money markets and derivative markets always represented separate markets, but before 2008 differences in pricings according to the tenor have been considered negligible. During the financial crisis these differences in pricings according to the tenor became more and more significant and have to be factored into pricing models accordingly. Money markets, derivative markets and cash markets for bonds/loans and foreign exchange (FX) rates are all different with respect to market participants, market conventions, pricings and quotations etc., which is referred to as "market segmentation".

"Market segmentation" is not a new phenomenon as e.g. cash markets for bonds/loans and derivatives have always been different in pricings, market conventions etc., but "market segmentation" is also a characteristic of derivative markets. Cross currency basis swaps (CCBS) are traded in the derivative market as separate financial instruments bridging the gap between derivative markets related to different currencies, and since the early 90s have been proven to be significant. As a consequence, cross currency swaps (CCS) cannot be replicated solely

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1 E.g. refer to *New valuation and pricing approaches for derivatives in the wake of financial crises,* KPMG International October 2011.

by plain vanilla interest rate swaps (IRS) denominated in different currencies traded as separate financial instrument in different markets. This revealed that consistent derivative pricing frameworks had to be constructed in order to price interest rate swaps and cross currency basis swaps consistently.[2] In these cases discounting future cash flows is performed on a different basis than forwarding, which is termed as a "multi-curve setup". The "interest rate" derivative market itself is also subdivided into different (sub-)markets, e.g. interest rate swap vs. 3-month Euro Interbank Offered Rate (EURIBOR) and interest rate swaps vs. 6-month EURIBOR[3], but before the financial crises the differences between interest rate swaps for different tenors were not considered significant. Multi-curve setups take into account these differences as well as collateralization and create an integrated market for all derivatives. In view of these facts and the evolving changes in market conventions like overnight index swap (OIS) discounting and the implementation of multi-curve setups for derivative pricing, it is obvious to ask about the consequences for hedge accounting models applied in practice.

Pricing different derivatives consistently is usually performed by applying the absence of arbitrage principle, which is a widely adopted theoretical framework for the pricing of derivatives.

Quotes of derivative pricing parameters like swap rates play a pivotal role in hedge accounting under International Accounting Standard (IAS) 39 as well as Financial Accounting Standard (FAS) 133, since these quotes are utilized to determine the benchmark interest rate used to specify the "hedged risk" and the fair value (FV) of the hedged item (e.g. cash instruments) with respect to the hedged risk in conjunction with the designated portion.[4] Economically hedge accounting models

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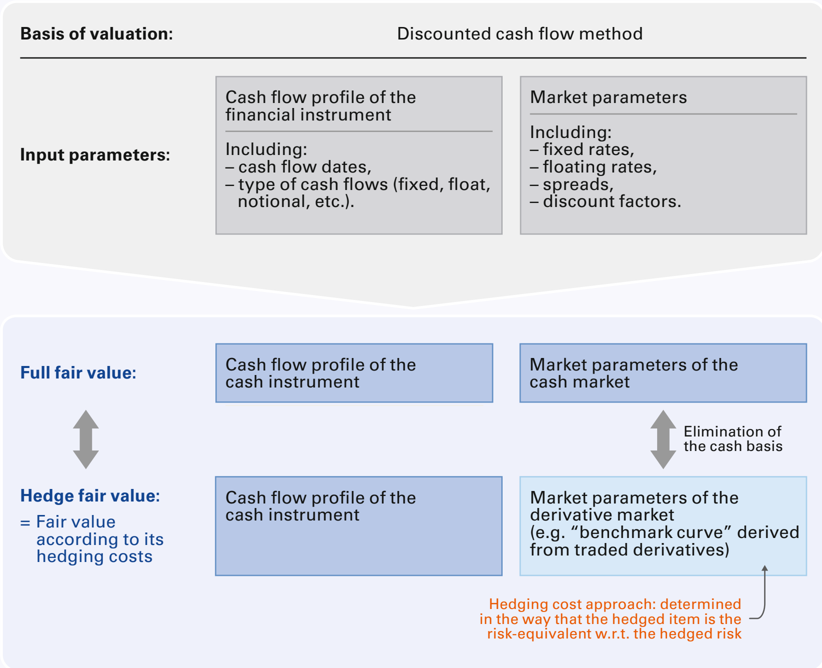
2 Fruchard, E., Zammouri, C. and Willems, E. (1995) or Tuckman, B. and Porfirio, P. (2003).

3 Also described in Bianchetti, M. and Carlicchi, M. (2011) and references therein.

4 For details and the underlying economics refer to Schubert, D. (2011).

FIGURE 1 **Illustration of the Hedging Cost Approach**



reflect an integrated market for derivatives (hedging instruments) and cash instruments (hedged item) using the absence of arbitrage principle. Accordingly for hedge accounting purposes the hedged items (cash instruments) are fair valued with respect to the hedged risk (benchmark curve) and the portion is determined according to the hedging cost approach independently of the funding model.[5]

In the following the economics of multi-curve setups are described as well as the impact on hedge accounting models – especially for interest rate hedging. Consequently the paper commences with statistical

5 For details and the underlying economics refer to Schubert, D. (2011) in particular section 3.2.1.

evidence as well as a depiction of the various markets and market conventions for derivative and cash instruments involved in hedge accounting models. As a result it will be demonstrated that the utilization of market quotations already includes economic modeling and proves the fact of market segmentation. This is followed by an introduction to the relation of multi-curve approaches and hedge accounting using a simple multi-curve setup. Since hedge accounting of FX risk already implies the utilization of a multi-curve setup, the paper continues with FX hedge accounting and the results are then extended to interest rate hedge accounting in a multi-curve setup including collateralization. In order to simplify the analysis, only hedges with deterministic cash flow profiles are treated in the present document. Hedges with stochastic cash flow profiles will be considered separately in a forthcoming paper. Multi-curve setups do not only affect derivative pricing but also the performance measurement of interest rate and credit treasury departments within banks (International Financial Reporting Standards (IFRS) 8 "Segment Reporting") as well as financial risk management techniques, but these aspects are beyond the scope of this paper.

SECTION TWO

2 Executive Summary and Conclusion

2.1 Relevance of Multi-Curve Models for Pricing and Valuation

2.1.1 Economic Background

Multi-curve models for pricing and valuation represent a general term for the description of circumstances in which discount curves and forward curves differ. The reason for its relevance is threefold:

- Price discovery in financial markets in the course of the financial crisis takes into account differences in tenors which result in different tenor-specific interest rates. The difference is also termed "tenor basis spread".
- Besides the relevance of tenor basis spreads for pricing within a single currency also (basis) spreads between currencies are of importance. Differences between currencies are termed cross currency basis spreads (FX basis spreads).

- Changes in market conventions and institutional changes within financial markets drive the implementation of multi-curve models e.g. “OIS-discounting” for collateralized trades or the intended increase in regulation of OTC markets.

During the financial crisis the spreads between interest rates of different tenors widened and have since that time remained on a significant level. Consequently the dependence of interest rates on tenors for the valuation of derivatives became significant. An illustrative example for “money market rates” is given in *Figure 2.* Before the financial crisis

FIGURE 2 **Money Market Rates before and after the Financial Crisis**

FIGURE 3 **Cross Currency Basis Spreads: 3M EURIBOR vs. 3M USD LIBOR (5-Year Maturity) in Basis Points (bp)**

the impact of tenors was regarded as negligible and discounting and forwarding on the same curve irrespective of the tenor for valuation purposes resulted only in insignificant differences from a more sophisticated tenor specific valuation.

For illustration: Before the financial crisis differences between 6-month EURIBOR and 3-month EURIBOR interest rate swap rates for the same maturity were regarded as low, so discounting a 6-month or 3-month EURIBOR interest rate swap by using the same curve for both resulted in only small valuation differences. After the financial crisis the tenor became significant, as is visible from the difference of the swap rates reflecting the differentiated markets' risk assessment of these products. Therefore both interest rate swaps have to be valued using different i.e. tenor specific market data.

In contrast the FX basis – regarded as difference in interest rates between two foreign currencies – has a longer tradition since it has been of relevance long before the financial crisis. The first "multi-curve" models have been created in order to take into account the FX basis for derivative pricing in foreign currencies.

Nevertheless, as can be seen from *Figure 3,* the financial crisis also affected cross currency basis spreads, which show a similar behaviour as tenor basis spreads of a single (foreign) currency: after the financial crisis these remained significant and volatile. Consequently the traditional foreign currency interest rate parity – a well known model in international economics – does not necessarily hold anymore, since the valuation model has to be augmented by incorporating the cross currency basis spread. Therefore cross currency swaps, FX forward contracts etc. have to take into account the cross currency basis spreads in order to appropriately reflect current market valuation practices.

Reasons of the occurrence of FX- and tenor basis spreads are currently analyzed by academic research. Possible explanations are: differences in credit risk and/or liquidity. During the financial crisis market participants realized that shorter tenors bear less credit risk than longer tenors and additionally considered the US dollar (USD) currency as more liquid than e.g. the euro (EUR) currency. All these possible explanations represent hypotheses and therefore involve financial modeling and econometric testing. But irrespective of such explanations, a re-assessment of risks by market participants involved in derivative transactions is a fact.

Changes in market conventions and institutional changes within financial markets accelerate the implementation of multi-curve models. Clearing houses[6] such as SwapClear (LCH.Clearnet)[7], Eurex Clearing AG (Deutsche Börse AG)[8] etc. require the utilization of an overnight index for valuation purposes, e.g. the European Overnight Index Average (EONIA). The increasing involvement of clearing houses and central counterparties in derivative transactions in order to eliminate counterparty risk pushes financial markets towards a standardization concerning discount curves derived from OIS rates.

This development is closely related to the treatment of collateralizations in derivative transactions, since clearing houses require daily collateral postings ("margins") and corresponding interest payments on cash collaterals. According to daily exchanges of collateral postings, an overnight index to determine the interest on the collateral postings is considered adequate.

These changes in the market environment are accompanied by modifications of the legal framework of the derivative business. In this context the International Swaps and Derivatives Association (ISDA) Master Agreement (2002)[9] represents the market standard for derivative

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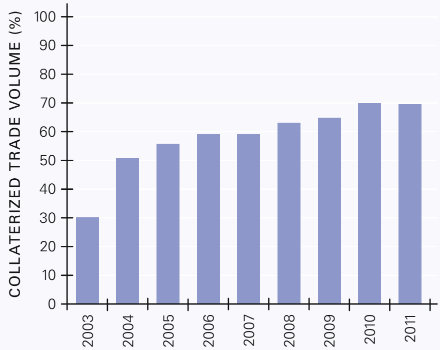
6 For a description of the economics of clearing houses refer to Pirrong, C. (2011). See also *Memo/12/232 – Regulation on Over-the-Counter Derivatives and Market Infrastructures – Frequently Asked Questions,* March 29, 2012.

7 cf. e.g. Whittall, C. (2010b) "LCH.Clearnet re-values $ 218 trillion swap portfolio using OIS", in: *Risk Magazine,* June 2010.

8 cf. Eurex Clearing (2012).

transactions between two counterparties supplemented by a credit support annex (CSA). Accordingly in such a CSA (to the ISDA Master Agreement (2002)) the evaluation of the interest associated with cash collateral postings of derivative transactions is changed so as to require the utilization of an overnight index e.g. EONIA. Currently cash collateral is commonly eligible and posted in selected reference currencies (e.g. USD, EUR, GBP, JPY). According to this specific feature commonly used under the terms and conditions in the relevant framework documents for derivatives issued by ISDA[10], the cross currency basis spread cannot be neglected in connection with collateral postings, since the cash collateral can be referenced to a different (foreign) currency than the derivative transaction.

FIGURE 4: OTC Derivatives: Percentage of Collateralized Trade Volume, 2003–2011 (Source: ISDA Margin Survey 2011 and earlier years)



This feature is currently under debate[11], since ISDA plans to change this market practice. But this plan does not facilitate the situation for cross currency products which by definition include at least two currencies or deals traded in minor currencies with collateral postings in a reference currency, and thus for the valuation the FX basis has to be taken into account. Additional legal changes under the ISDA Master Agreement (2002) are the rules concerning “disputes” and “close outs” of derivative transactions, since these also require the utilization of overnight indices in order to determine the close out amount.

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9 Master agreement for derivative contracts issued by ISDA (as of 2002).

10 E.g. ISDA Master Agreement (2002), ISDA Credit Derivative Definitions (2003) (Definitions to the ISDA Master Agreement (2002) (as of 2003 in the form of the revision of the Big Bang Protocol 2009) and CSA.

11 For a description refer to Sawyer, N. and Vaghela, V. (2012), “Standard CSA: the dollar dominance dispute”, in: *Risk Magazine,* January 2012.

The features described represent the market standard for derivative transactions only in the interbank market ("collateralized derivative transactions"). Corporates also use the ISDA documentation for derivatives as a standard, but not the CSA ("uncollateralized derivative transactions") due to liquidity requirements of collateral postings, which are considered unfavourable for corporates due to their liquidity constraints. Consequently overnight indices are not applied as discount rates for those derivative transactions and therefore e.g. the EURIBOR or London Interbank Offered Rate (LIBOR) rates are applied. As a result of changes in market conventions, discount rates for derivative transactions become counterparty specific and yield a segmentation of derivative markets. It is also important to note that the evaluation of interest on a cash collateral using an overnight index does not mean that a financial institution (bank) is able to (fully) fund on an overnight index basis!

With respect to these developments within financial markets, valuation models have to be modified in order to reflect the increased number of risk and counterparty specific factors. Additionally, discount curves cannot be derived from market data (e.g. swap rates) without taking into account different tenors. For example (in the interbank market) EURIBOR or LIBOR discount rates cannot be derived independently from overnight index rates, so a "pure" EURIBOR or LIBOR curve ceased to exist. Under these circumstances financial institutions started to implement "multi-curve" models for derivative pricing in order to take into account tenor dependence and collateralization. Within these valuation models forwarding and discounting is performed by means of different curves. The implementation of such models requires significant changes of pricing and risk management routines in banks:

- Consistent and arbitrage free setup of discount and forward curves involving several risk factors like tenor and cross currency basis spreads, which additionally distinguishes between collateralized and uncollateralized trades.

- In the course of modified curve setups counterparty valuation adjustment (CVA)/debt valuation adjustment (DVA) methods need to be adjusted.
- Based on consistent curve setups, loan and transfer pricing (treasury departments) need to be modified. For example, granting loans in foreign currency and neglecting the FX basis in loan pricing can result in an immediate and significant economic loss.
- Alignment of market risk management methods like Value at Risk (VaR) evaluations in order to cope with re-assessment of market risk. This issue is also addressed by banking regulators.[12]
- Alignment with or integration of collateral management processes.
- Changes in valuation practices affect the entire front-to-back office processes within financial institutions including financial accounting.

2.1.2 Implications for Hedge Accounting

Interest rate hedge accounting is applied in order to avoid Profit and Loss (P&L) volatility resulting from accounting mismatch. According to IAS 39, derivatives have to be measured at fair value through P&L, while e.g. loans are measured at amortized cost. Only if the requirements concerning fair value hedge accounting under IAS 39 are met, loans can be measured at fair value related to interest rate risk so that the fair value changes effectively offset[13] the fair value changes of the hedging interest rate derivatives in P&L.

Changes in valuation methods for derivatives naturally affect hedge accounting models. IAS 39 mainly distinguishes between two types of models: cash flow and fair value hedge accounting models. The impact on both hedge accounting models is different: Cash flow hedge

12 Refer e.g. *Recommendations European Systemic Risk Board,* December 22th, 2011 on US dollar denominated funding of credit institutions (ESRB/2011/2) (2012/C 72/01).

13 Ineffectiveness may arise from e.g. counterparty risk in respect of the hedging instrument.

accounting directly accommodates to the new market conventions according to the changes in derivative pricing due to the application of the hypothetical derivative method, whereas the adaption to the fair value model entails several issues. As will be elaborated, the economic rationale is very similar in both hedge accounting (valuation) models, although the impact is different since fair value hedge accounting requires the valuation of the hedged item with respect to the "hedged risk". Due to the changes in financial markets the question arises: What is the hedged risk and how can several risk factors be incorporated into the valuation of the hedged item?

Before the financial crisis, the application of fair value hedge accounting according to IAS 39 (single and portfolio hedges) was simplified because, e.g. for hedges comprising of 6-month EURIBOR and 3-month EURIBOR interest rate swaps, only one curve for discounting and forwarding had to be applied. Since the crisis for every hedge accounting relationship the tenor dependence has to be taken into account. As described above, market discount rates cannot be derived independently from each other. Therefore applying single-curve hedge accounting individually for each set of hedging instruments, e.g. 3-month EURIBOR interest rate swap and 6-month EURIBOR interest rate swap with a separate set of discount curves, does not seem appropriate. Now the issue arises how to deal with multi-curve models in hedge accounting, since financial institutions intend to use multi-curve models for hedge accounting purposes and – for operational reasons – do not want to assign individual discount curves for every hedging relationship or apply different valuation methods for derivatives in the front office and financial accounting. Although IAS 39 does not explicitly prescribe the use of the same benchmark curve for discounting the hedged item and the hedging instrument, the definition of the hedged risk and the application of a consistent multi-curve setup imply the usage of the identical discount curve for the hedged item and the hedging instrument. By applying multi-curve models without modifications in the valuation of the hedged item, a significant increase of ineffectiveness of hedging relationships is expected due to tenor or cross currency basis spreads (FX basis spreads) although there have been no changes in contractual cash flows or the funding position (economic hedge relationship).

A possible solution to this hedge accounting issue can be achieved by designating a different portion of cash flows (covering several risk factors) associated with the hedged item and regular adjustments including re-designation of the hedging relationship.

2.2 Structure and Impact of Multi-Curve Valuation Models

2.2.1 Markets' Assessment of Risk and Its Impact on Valuation Models

Irrespectively of whether a single or multiple risk factors in financial markets are considered, all risk factors have one property in common: they are unobservable (the only exception being FX spot rates). For example interest rate risk is unobservable; one cannot go into the market and buy or observe "interest rate risk". In order to determine the interest rate risk a "yard stick" or "benchmark" is required. Interest rates are always tied to traded financial instruments and cannot be "observed" or "determined" independently. Deciding on a "yard stick" or "benchmark" firstly requires a decision for a set of traded financial instruments in order to derive prices and corresponding risk factors (= changes in prices of the benchmark). But as an immediate consequence the determination of risk factors itself is a model and represents an approximation of reality. With respect to "interest rate risk" there are a number of possible financial instruments to be considered: interest rate swaps, government bonds, corporate bonds, repos etc. Market participants prevalently consider the derivative market as the most liquid, reliable source of prices and as a means of deriving risk factors. This inevitably implies that the "benchmark" is tied to market conditions/conventions (e.g. credit and counterparty risk), price discovery (e.g. supply and demand) and the legal framework of the set of financial instruments utilized to derive the benchmark and cannot be separated. In the case of derivatives, which are described in *Section 3,* e.g. the legal framework is illustrated on the basis of the ISDA documentation for derivatives including collateralization according to CSA etc.[14] In *Section 4* the connection between the economic rationale and hedge accounting according to IAS 39 in a single-curve model is shown.

Figure 5 compares the markets' assessment of risk and valuation factors before and after the financial market crisis. In the strict sense FX basis and tenor risk existed before the financial market crises but were considered of minor importance. To simplify matters, the model for collateralized interest rate derivatives is portrayed. For uncollateralized derivatives the set of "traded financial instruments" differs but the economic rationale is similar.

Following the fact of unobservability of each risk factor mentioned in *Figure 5* – except the USD/EUR spot exchange rate – market participants have to assign a set of liquidly traded financial instruments in order to measure the market risk (approximation of reality – an economic model!). Independently from single or multiple risk factors modeling FX risk forms an exception. The USD/EUR spot exchange rate is a cash price and observable, but it includes the exchange of cash (converting EUR cash into USD cash or vice versa). But as soon as the re-exchange into cash takes place at some future point in time interest rate risk is present. This is the reason why there is an interrelationship between three types of risk, FX, exchange in cash (liquidity) and interest rate risk. Therefore FX risk is in most cases also unobservable,

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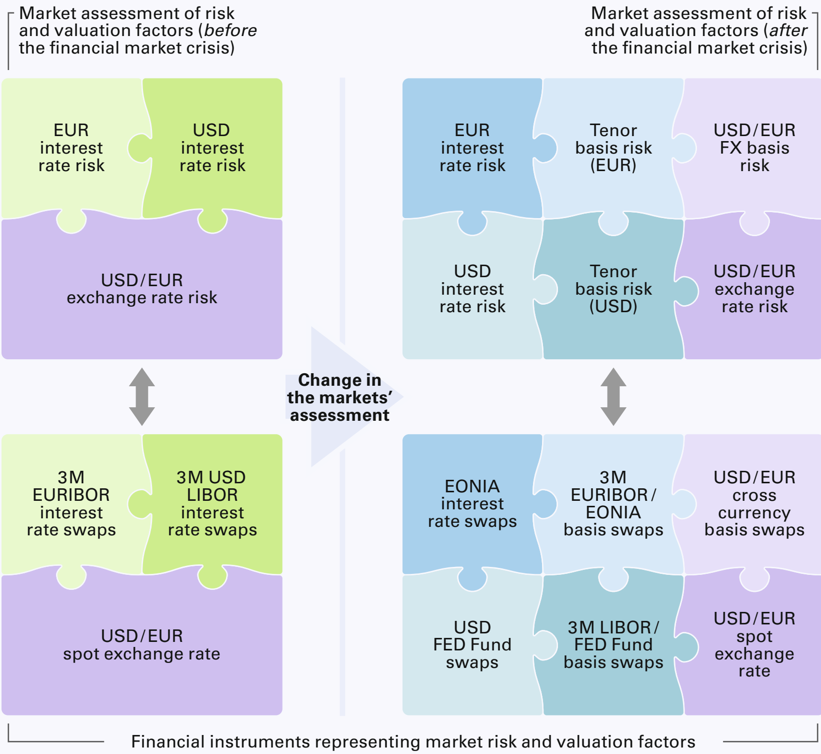
14 Further frameworks are e.g. the German master agreement for financial transactions *(Rahmenvertrag für Finanztermingeschäfte)* with its annexes and supplements issued by the Association of German Banks *(Bundesverband Deutscher Banken)* and the European Master Agreement for financial transactions with its annexes and supplements issued by the Banking Federation of the European Union. The legal mechanism of these master agreements is similar to the ISDA Master Agreement (2002).

15 For further reading on these related topics, refer to Krugman, P. R., Obstfeld, M. (2011), chapter 13 and chapter 14; Shapiro, A. C. (2006), chapter 10 and Krugman, P. R., Wells, R., Graddy, K. (2008), chapter 34.

FIGURE 5: The Markets' Assessment of Risk and Valuation Factors before and after the Financial Market Crisis



because it is tied to several risk factors (like e.g. interest rate risk), and a model is needed to separate all types of risks. This feature is also termed "overlay risk" and is present in both: single or multiple factor models.

In the context of FX risk and hedge accounting according to IAS 39 there are three concepts, which need to be distinguished:[15]

- **Economic exposure:** economic exposure relates to the impact of future changes in exchange rates/FX basis to future operating cash flows of a bank/firm. This type of exposure also includes long term changes in foreign currency/FX basis, which affects the competitive advantage of a bank/firm. E.g. a local based bank/firm can be exposed to economic exposure even if it is not operating in foreign countries.

- **Transaction exposure:** Transaction exposure is a subset of economic exposure and represents future gains or losses of existing foreign currency denominated contractual obligations. Hedge accounting according to IAS 39 mainly relates to transaction exposures.
- **Translation exposure:** Translation exposure relates to the conversion of recognized assets and liabilities into the functional currency of the balance sheet preparer, which does not necessarily coincide with the transaction exposure.

Keeping the described exception of FX risk in mind and resuming the approach outlined above to measure market risk originating from mainly unobservable risk factors as presented in *Figure 5,* each type of risk factor is assigned to a corresponding set of traded financial instruments which is summarized in *Table 1.*

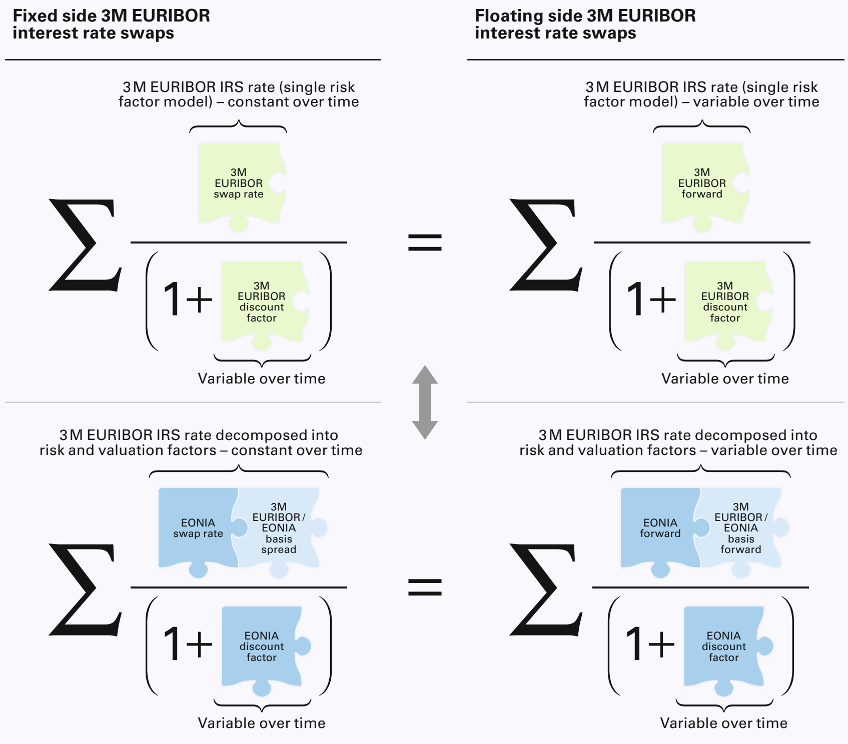
TABLE 1: Assignment of Types of Financial Market Risk to Corresponding Sets of Traded Financial Instruments (Collateralized)

	Types of financial market risk	Sets of traded financial instruments
Before the financial market crisis – single-curve models	**EUR interest rate risk**	3M EURIBOR interest rate swaps
	USD interest rate risk	3M USD LIBOR swaps
	USD/EUR exchange rate risk	USD/EUR spot exchange rate
After the financial market crises – multi-curve models	**EUR interest rate risk**	EONIA interest rate swaps
	Tenor basis risk (EUR)	3M EURIBOR/EONIA basis swaps
	USD/EUR FX basis risk	USD/EUR cross currency basis swaps
	USD interest rate risk	FED Funds interest rate swaps
	Tenor basis risk (USD)	3M USD LIBOR/FED Funds basis swaps
	USD/EUR exchange rate risk	USD/EUR spot exchange rate

The re-assessment of market risk does not imply that e.g. 3-month EURIBOR interest rate swaps are not traded anymore. These instruments are still liquid derivative instruments but the assessment of the inherent market risks in the 3-month EURIBOR interest rate swaps has changed.

Figure 6 reveals that the re-assessment of market risk factors affects every traded financial instrument. In this example a 3-month EURIBOR interest rate swap is decomposed into two risk factors: EONIA risk and the 3-month EURIBOR/EONIA basis risk. According to this decomposition the assessment of risk changes, but neither the contractual

FIGURE 6: Comparison of Single- and Multi-Curve Valuation Models for an Interest Rate Swap (at t_0)



cash flows of the derivative nor the fair value of zero at inception (at t_0) do. In a single-curve model (single risk factor model applied before the financial market crises) the 3-month EURIBOR interest rate swap was exposed to only one risk factor: 3-month EURIBOR risk.

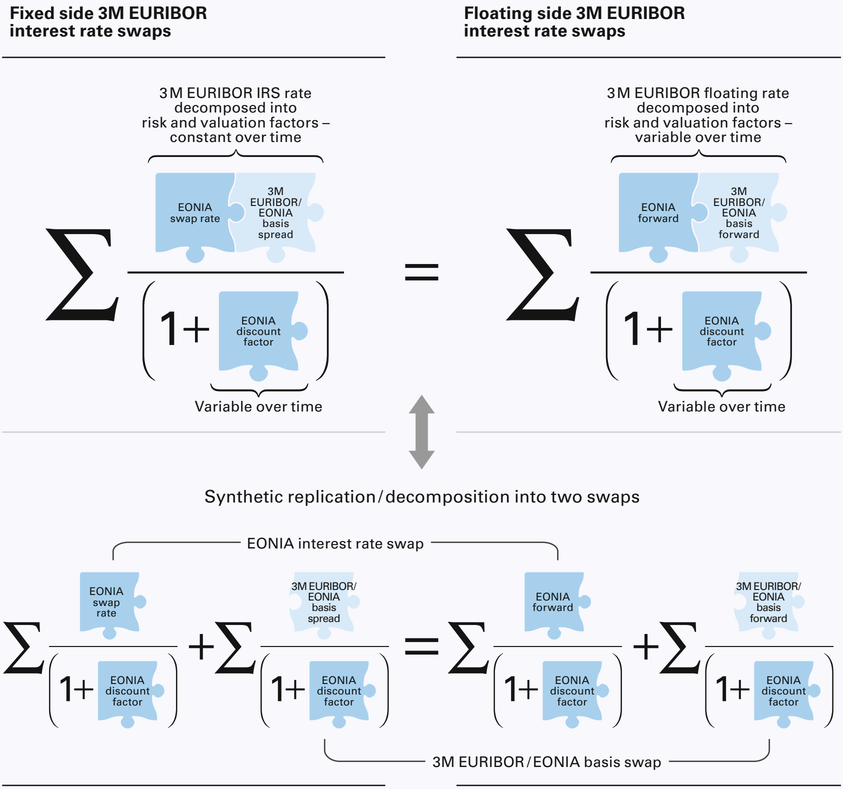
The described decomposition of the 3-month EURIBOR interest rate swap into market risk factors can also be stated as a synthetic decomposition into derivatives. The assignment of types of financial market risk to corresponding sets of traded financial instruments also implies that the assigned traded financial instruments (in this case interest rate derivatives) form the set of "basis" instruments which can be utilized to replicate any other derivative traded in the market such as the 3-month EURIBOR interest rate swap. This is illustrated in *Figure 7.*

As illustrated in *Figure 7,* a 3-month EURIBOR interest rate swap can be synthetically decomposed into two derivatives: EONIA interest rate swap and a 3-month EURIBOR/EONIA basis swap (at t_0).

There are some important properties and features in context with *Figure 7:*

- It can be observed that market participants consider traded derivatives as the best estimate of risk and prices in financial markets. The choice of derivatives represents an assumption of the economic behavior of market participants. Please keep in mind that any economic model requires a behavioral assumption of market participants.
- In a multiple risk factor model economy the risk factors cannot be modeled independently. Simultaneous modeling of risk factors is a model itself.
- The selection of risk factors, like in the charts above, depends not only on the assessment by market participants but also on individual preferences of market participants towards risk (balance sheet preparers). For example: the 6-month EURIBOR/EONIA basis swaps are not included in *Table 1,* because this type of risk

FIGURE 7: **Synthetic Replication / Decomposition of a 3M EURIBOR Interest Rate Swap by an EONIA Swap and a 3M EURIBOR / EONIA Basis Swap (at t_0)**



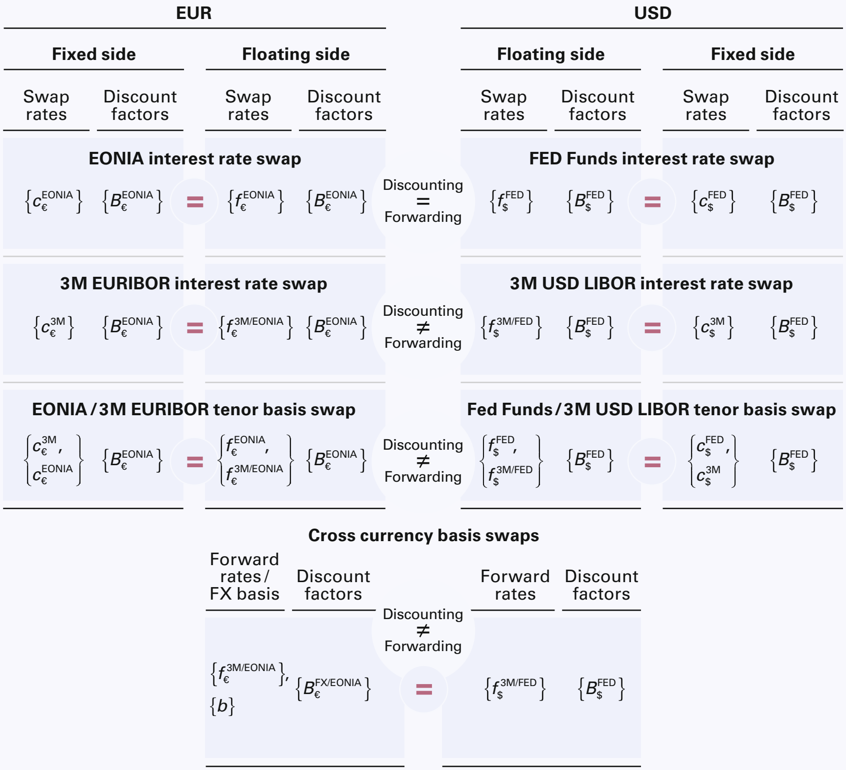
is not relevant for the specific market participant using this model. Otherwise the list above would have to be augmented by this supplementary risk factor, e.g. 6-month EURIBOR/EONIA basis swap.

- The list of market risk factors as mentioned above also represents valuation factors.
- Like in the single risk factor model, since derivative prices are considered the best estimate for pricing all financial instruments (bonds, loans etc.), these will be priced ("fair valued") according to their hedging costs. As will be described in *Section 4,* this can be reasoned by the absence of arbitrage principle.

2.2.2 The Structure of Multi-Curve Valuation Models

The list in *Table 1* of the market risk factors and its corresponding traded "basis" financial instruments (derivatives) form the starting point for multi-curve models. The idea is straightforward: after a set of interest rate derivatives is defined to measure market risk and valuation factors, this set of derivatives constitutes a system of equations which defines the multi-curve model. This is illustrated in *Figure 8.* The "equations" (marked in red) represent equilibrium conditions for each set of derivatives and not "algebraic" equations. The system of equations consists of "known" and "unknown" variables. Swap rates (e.g. $\left\{c_{€}^{EONIA}\right\}$) taken from market quotes represent "known" variables, while forward rates, e.g. $\left\{f_{€}^{EONIA}, f_{€}^{3M/EONIA}\right\}$ and discount factors e.g. $\left\{B_{\$}^{FED}\right\}$ are unknown variables and need to be evaluated by solving the system of equations simultaneously using "bootstrapping" algorithms (for examples please refer to *Section 4*). Please observe that the system of equations essentially comprises only two discount curves: the EONIA and the FED Funds discount curve (according to the two currencies EUR and USD). These represent the risk-free discount curves for collateralized derivatives (in EUR and USD respectively) according to market convention. An entire description of this model is provided in *Section 6.* All derivatives are priced against these two discount curves ("numeraire") and therefore e.g. no LIBOR or EURIBOR discount curve exists.

FIGURE 8: **Model Structure of a Multi-Curve Model for Collateralized Derivatives**

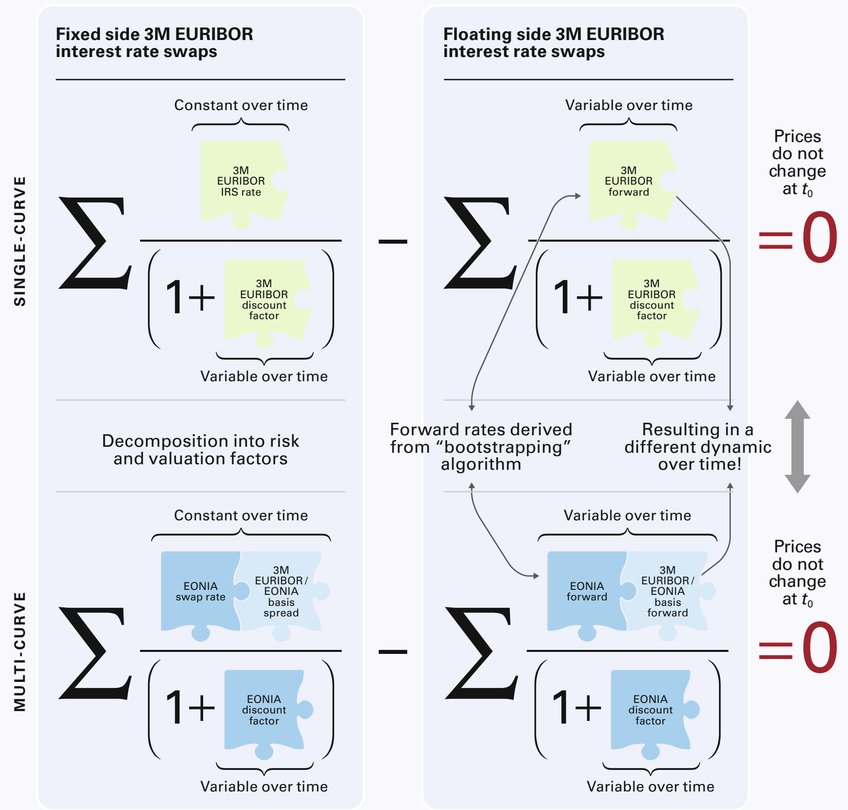


Three main results are achieved by solving the system of equations:

- Swap rates are input parameters and are not changed by the application of the multi-curve model. EONIA or 3-month EURIBOR interest rate swap rates ($\{c_{€}^{EONIA}\}$, $\{c_{€}^{3M}\}$) remain unaffected. That is also termed "calibration" of the multi-curve model. In multi-curve setups the prices of derivatives ("fair value") at inception t_0 do not change. At t_0 all derivatives have a price of zero – according to the market convention and the equilibrium condition. Using this equilibrium condition, e.g. the forward rates for the 3-month EURIBOR interest rate swaps $\{f_{€}^{3M/EONIA}\}$ are derived. These forward rates comprise two risk factors: 3-month EURIBOR tenor basis risk and EONIA risk. In this case "forwarding" is not equal to "discounting", since the EONIA discount curve is used. In case of EONIA and FED Funds interest rate swaps (with cash collateral posting in EUR and USD respectively) forwarding and discounting coincide.
- Each derivative in the multi-curve model is decomposed into risk factors considered in the multi-curve model.
- The multi-curve model changes the dynamic of prices ("fair values") at every time $t > t_0$. Fair value changes of derivatives recognized in P&L depend on the chosen multi-curve model. This is very similar to single-curve models, and a description of additional modeling assumptions and a comparison to single-curve models is given in *Section 6.* The change in dynamic – in comparison to single-curve models – mainly results from the floating side of the derivatives (set of forward rates), since by market convention the changes due to the alteration of discount curves is incorporated in forward rates, which are evaluated from the system of equations above.

In order to illustrate these properties *Figure 6* is augmented and modified, the "=" signs are marked in red to indicate equilibrium conditions (see *Figure 9*).

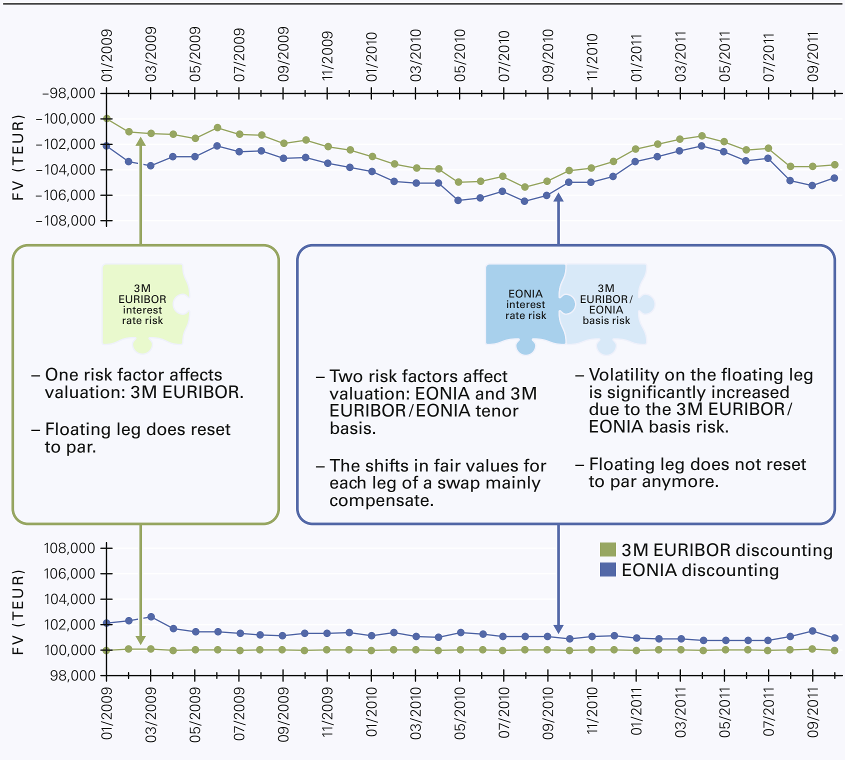
FIGURE 9: **Results from Solutions of Multi- and Single-Curve Valuation Models for an Interest Rate Swap (at t_0)**



In *Figure 10* an example of a 3-month EURIBOR interest rate swap is provided, for which the fixed leg and the floating leg are considered separately. As shown in *Figure 10,* the incorporation of an additional risk factor, which results from the change in discounting from EURIBOR to EONIA, leads to increased volatility in fair value of the floating leg (including repayment); forward rates $\left\{f_{€}^{3M/EONIA}\right\}$ are discounted with EONIA. As becomes apparent from the *Figure 10,* the floating leg (including repayment) does not reset to par anymore.

FIGURE 10: Comparison of a Fixed and Floating Leg of a 3M EURIBOR Interest Rate Swap (IRS) Discounted with 3M EURIBOR and EONIA

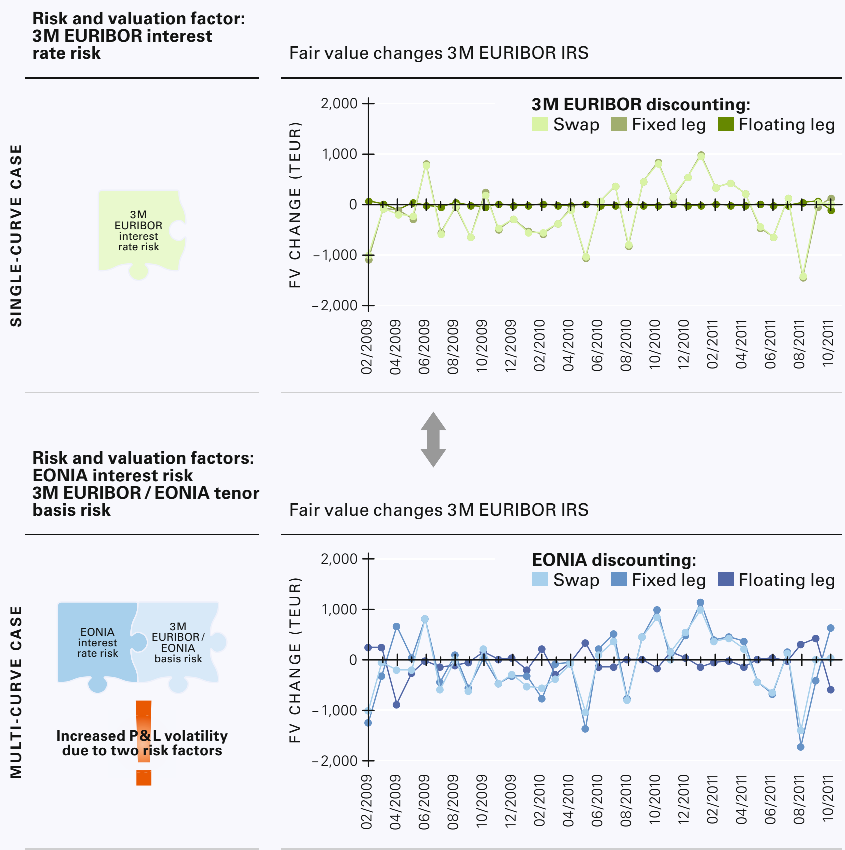
Fair value 3M EURIBOR IRS fixed leg incl. repayment



Fair value 3M EURIBOR IRS floating leg incl. repayment

In order to compare fair value changes in a single-curve and a multi-curve setup, the fair value changes of a 3-month EURIBOR interest rate swap separately for each leg (including repayment) as well as for the entire swap are portrayed in *Figure 11* for each setup. The increased volatility of fair value changes in the floating leg leads to an increased overall volatility in fair value changes of the 3-month EURIBOR interest rate swap.

FIGURE 11: Comparison of Fair Value (FV) Changes of a 3M EURIBOR Interest Rate Swap Discounted with 3M EURIBOR and EONIA



Similar examples and results can be derived for 3-month USD LIBOR interest rate swaps including foreign currency and the FX basis tenor risk. The impact on multi-curve modeling setups including FX is analyzed in *Section 5* and *Section 6*. Market conventions of cash instruments and derivative instruments can be found in *Section 4*.

There are many more features and properties on multi-curve modeling than those mentioned in the short description and the results above. In the following enumeration the relevant topics for this paper are briefly listed and a reference to the corresponding section is provided:

- Multi-curve models create an integrated market for all derivatives included in the model setup. These include market conventions on traded derivatives as well as collateralization (refer to *Section 3* and *Section 6*).
- Like single-curve models, multi-curve models represent relative valuation models. All derivatives are valued relatively to the given discount curve. This property of derivative pricing is carried over to hedge accounting according to IAS 39.
- Contractual cash flows as defined by the terms and conditions of the legal contract remain unchanged by the application of multi-curve models.
- Market prices of derivatives remain unchanged at inception but the chosen multi-curve setup affects P&L changes and volatility. The results of multi-curve models as well as single-curve models are – for many reasons – specific to each financial institution (balance sheet preparer) and cannot be easily compared. Similarities exist in methodologies like bootstrapping algorithms and the application of the absence of arbitrage principle (refer to *Section 4* and *Section 6*).
- The consideration of multiple risk factors in a multi-curve setup increases the likelihood of higher P&L volatility resulting from FV movements of derivatives.

- Due to changes in market conventions EURIBOR and LIBOR discount curves (single-curve model setups) depend on EONIA or FED Funds discount curves, derived from collateralized derivatives. This is especially the case for the short term (< 1–2y), since (collateralized) forward rate agreements (FRA) or futures are utilized to model the short term for which OIS discount curves are used (refer to *Section 6*).
- A distinction must be made between the application of the multi-curve model setup on existing and that on new derivative transactions. For new derivative transactions the fair values (prices) do not change, but those of existing derivative transactions alter. This is analyzed in *Section 6.* The change in fair value of existing derivatives can be regarded as the present value (PV) of differences in future cash collateral interest payments arising from the change of 3-month EURIBOR to EONIA. Therefore the change in discount curves results in a "real" economic gain or loss.
- Due to changes in discount curves the overall funding position of a financial institution changes (refer to *Section 6*).
- Funding of a financial institution requires economic modeling and must be distinguished from discounting, which is also subject to economic modeling. It is only in special cases that both coincide (refer to *Section 6*).
- Single-curve models as well as multi-curve models determine the "roll out of cash flows" or "projection of cash flows" in connection with the application of the discounted cash flow method. In single-curve models as well as in some cases in multi-curve models closed form solutions for forward rates can be proven (refer to *Section 6*). Accordingly rolling out cash flows requires an economic model, and in case of the multi-curve setup different results are achieved.

2.3 Hedge Accounting in Multi-Curve Model Setups according to IAS 39

2.3.1 The Hedge Accounting Puzzle

The hedge accounting puzzle is established by three major and interrelated questions:

- **Market segmentation:** How can cash flow and fair value hedge accounting (IAS 39) approaches be justified in presence of market segmentation for cash (hedged item) and derivative instruments (hedging instruments)? For example, why can the hypothetical derivative method be applied in a cash flow hedging relationship (floating rate loan and interest rate swap) despite the fact that empirical evidence shows that fair value changes of the floating side of an interest rate swap (derivative price) and "full" fair value change of a floating rate note (cash price) have nothing in common (see *Figure 12*)? Effectiveness testing in a cash flow hedging relationship would fail if carried out using "full" fair value changes of the 3-month floating rate note (FRN).
  Similar questions arise in the case of fair value hedge accounting according to IAS 39 (see *Figure 13*): Fair value changes of interest rate swaps (derivative) cannot "explain" "full" fair value changes (cash price) of a fixed rate bond. How can it be justified that the interest rate risk, measured by interest rate derivatives, is incorporated in the fixed rate bond?
- **Impact of multiple risk factors:** What is the impact and the interrelationship of the previous question and the recent re-assessment of risk and valuation factors in financial markets? As the aforementioned *Figures 12* and *13* reveal, the question remains the same also in a multi-curve environment if the discount curve is changed according to changes in market conventions.
- **Financial accounting of economic hedges:** The answer to these questions is of immediate practical relevance since economic hedging relationships incorporate multiple risk factors which should qualify for hedge accounting under IAS 39. Currently

FIGURE 12: **Comparison of Full Fair Value (FFV) Changes of the 3M Floating Rate Note (FRN) (Rating Comparable to AA⁻) and the 3M EURIBOR Interest Rate Swap with Different Discounting Factors**[16]

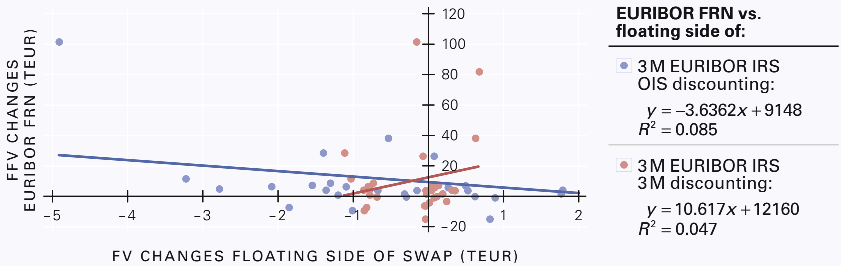
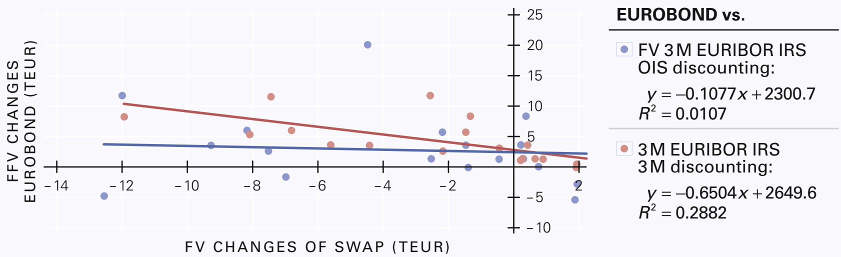


FIGURE 13: **Comparison of FFV Changes of a Fixed Rate Bond (Rating Comparable to AA⁻) and the 3M EURIBOR Interest Rate Swap with Different Discounting Factors**



market participants (balance sheet preparers) are exposed to unreasonable P&L volatility if the multiple risk factors are not taken into account in the valuation of the hedged item despite sound hedging relationships, which are represented by offsetting cash flow profiles (for examples refer to *Section 2.3.3*).

There are neither explanations nor any explicit guidance in IAS 39/ED 2010/13 with respect to the questions raised above. In the following a brief answer and solution are provided. A detailed description of market conventions in derivative and cash markets – including statistical evidence – can be found in *Section 3*. References with respect to multi-curve models and IAS 39 requirements are given below.

16 The rating of the floating rate note is comparable to AA⁻, which is the rating of discount curves (market convention) derived from traded collateralized derivatives. Given the same rating in *Figure 12* financial instruments with similar credit risk are compared.

2.3.2 Cash Flow Hedge Accounting in a Multi-Curve Model Setup

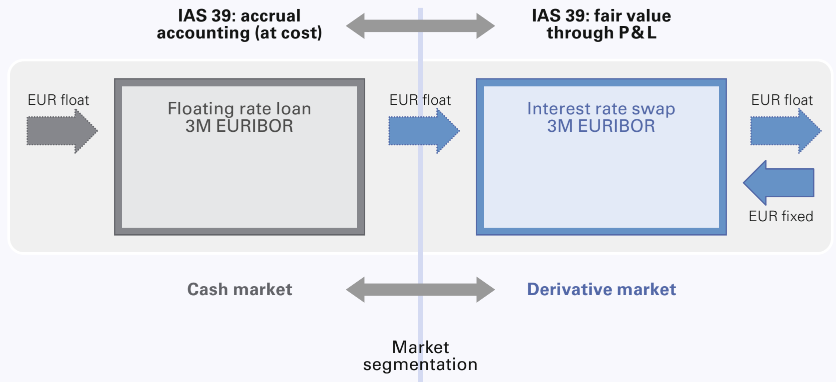
Let's consider a 3-month EURIBOR floating rate note (cash market) and 3-month EURIBOR interest rate swap. According to the "Mixed Model" Approach applied by IAS 39 the 3-month EURIBOR floating rate note (cash market) is carried at cost, while the derivative is carried at fair value through P&L. Since both financial instruments are tied to the 3-month EURIBOR money market rate and therefore the cash flow changes in both financial instruments correspond to changes in 3-month EURIBOR, it is assumed that the requirements according to IAS 39.88(b) are met ("variability of cash flow criteria") and cash flow hedge accounting can be applied. *Figure 14* illustrates the cash flow hedging relationship.

Summarizing the facts with respect to the cash flow hedging relationship:

- Derivative markets and cash markets are segmented markets with different pricings.
- Comparing (full) fair value changes of the 3-month EURIBOR floating rate note with fair value changes of the floating side of the 3-month EURIBOR shows low explanatory power. Therefore an empirical relationship between both prices and markets is not supported by analysis of real examples, as *Figure 12* shows.
- Equating cash flows such as 3-month EURIBOR payments of a floating rate note with 3-month EURIBOR payments of the floating side of the interest rate swap already represent an economic model (equilibrium condition), since both cash flows – and in particular their expectation – are tied to different financial instruments and markets.
- If effectiveness testing were performed by comparing (full) fair value changes of the 3-month EURIBOR floating rate note with fair value changes of the floating side of the 3-month EURIBOR, effectiveness would not be achieved and therefore cash flow hedge accounting could not be applied (see *Figure 12*).

FIGURE 14: **Example of Cash Flow Hedge Qualifying for Hedge Accounting according to IAS 39**

IAS 39: "Mixed Model" Approach



- 3-month EURIBOR rates represent money market offer rates and not necessarily represent transaction rates. Obviously the proof of the variability of cash flow criteria according to IAS 39 does not require that the amount of contractual cash flows is referenced to an index (e.g. as an average (OIS) or offer rates (EURIBOR, cf. outline in *Section 3.1*)), which is derived from real transactions. So even here modeling is involved on the level of the index.

The impact of multiple risk factors is illustrated in *Figure 15*. Three major features are of importance:

- Multiple factor models do not change the contractual cash flows of the financial instruments involved.
- Multiple risk factors affect all relevant cash flows in terms of their risk assessment. Therefore the cash flow perspective and the risk/valuation perspective must be distinguished.
- Derivative prices are the only relevant source of prices and representatives of risk factors!

Figure 15 portrays the impact of markets' assessment of risk and valuation factors on the cash flow profile of a cash flow hedge (a hedge accounting relationship consisting of a 3-month EURIBOR floating rate instrument and a 3-month EURIBOR interest rate swap discounted on EONIA) according to IAS 39. In this hedge accounting relationship EUR interest rate risk and tenor basis risk is present, since EONIA interest rate swaps and 3-month EURIBOR/EONIA basis swaps (derivative market) are considered as the relevant source for risk measurement.

Following the illustration in *Figure 15* the impact on markets' assessment on risk factors in the cash flow hedge accounting model also resolves the market segmentation of cash and derivative markets. Additionally due to the application of the hypothetical derivative method the impact of the decomposition of derivatives defined by the multi-curve model *(Figure 8)* into its risk factors does not affect the effectiveness testing since the fair value changes of the hypothetical and real derivative are compared. Consequently cash flow hedge accounting can be justified if an integrated market for cash and derivative instruments is assumed and the derivative prices are used to evaluate the "fair value" of the 3-month EURIBOR floating rate note (cash instrument).

Furthermore the decomposition into risk factors defined by the multi-curve model (cf. *Figure 8*) is also transferred to the 3-month EURIBOR floating rate note (cash instrument). In this respect the role of tenor basis swaps and the decomposition of derivatives according to the multi-curve model setup into its risk components are crucial. In the cash flow hedge accounting model this is covered by cf. IAS 39.86(b), KPMG Insights 7.7.630.30[17], 7.7.630.40[18], 7.7.630.50[19] and 7.7.640.10[20]

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17 Change of discount rate in the hypothetical derivative without de-designation/re-designation.

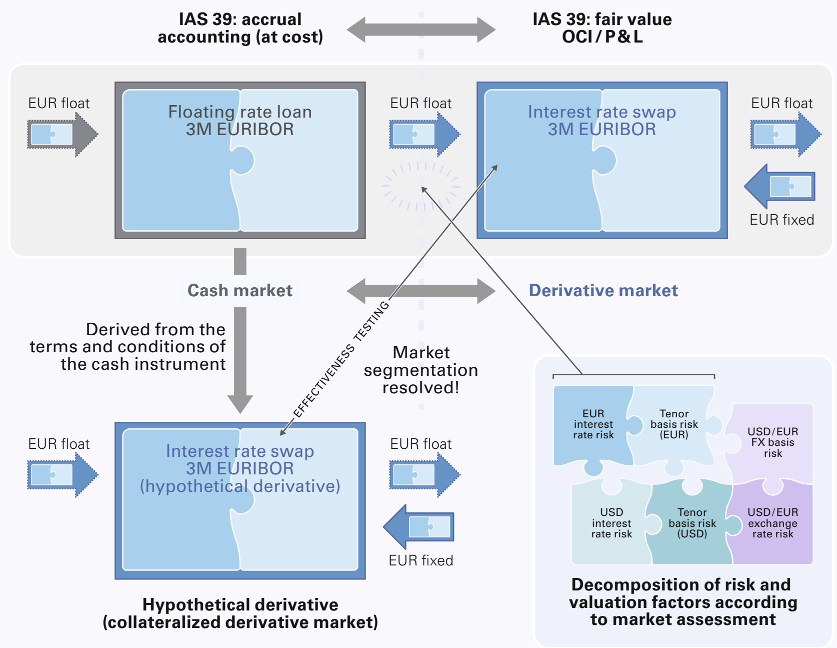
18 Adjustment of the discount rate of a hypothetical derivative.

19 Hypothetical derivative method not available in a fair value hedging relationship.

20 Consideration of only the changes in fair value of the floating leg of the swap for effectiveness testing purposes when using the hypothetical derivative.

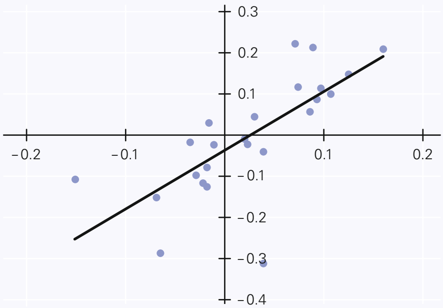
FIGURE 15: **Transition from Cash Flow to Risk Factor and Valuation Perspective in a Cash Flow Hedge Accounting Model**

IAS 39: cash flow hedge accounting



and does not affect the results of the effectiveness test. This approach followed by IAS 39 reveals that for financial accounting purposes the decomposition of the cash instrument remains synthetic. In *Figure 16* the 3-month EURIBOR money market rates are compared with the 3-month EONIA swap rates (3-month compounded EONIA money market rates). If the decomposition into risk factors were performed in real terms, which means decomposing 3-month EURIBOR cash flows into EONIA and 3-month EURIBOR / EONIA tenor, the "variability of cash flow criteria" would not be met. In case of a real decomposition the "variability of cash flow criteria" according to IAS 39 would have to be proven by comparing changes of 3-month EURIBOR rates corresponding to the 3-month EURIBOR floating rate note (cash

FIGURE 16: Scatter Plot and Regression Analysis of 3M EURIBOR Money Market (MM) Rates vs. 3M EONIA Swap Rates



instrument) with changes in EONIA rates resulting from the synthetic decomposition of the 3-month EURIBOR interest rate swap into an EONIA and 3-month EURIBOR/EONIA basis swap.

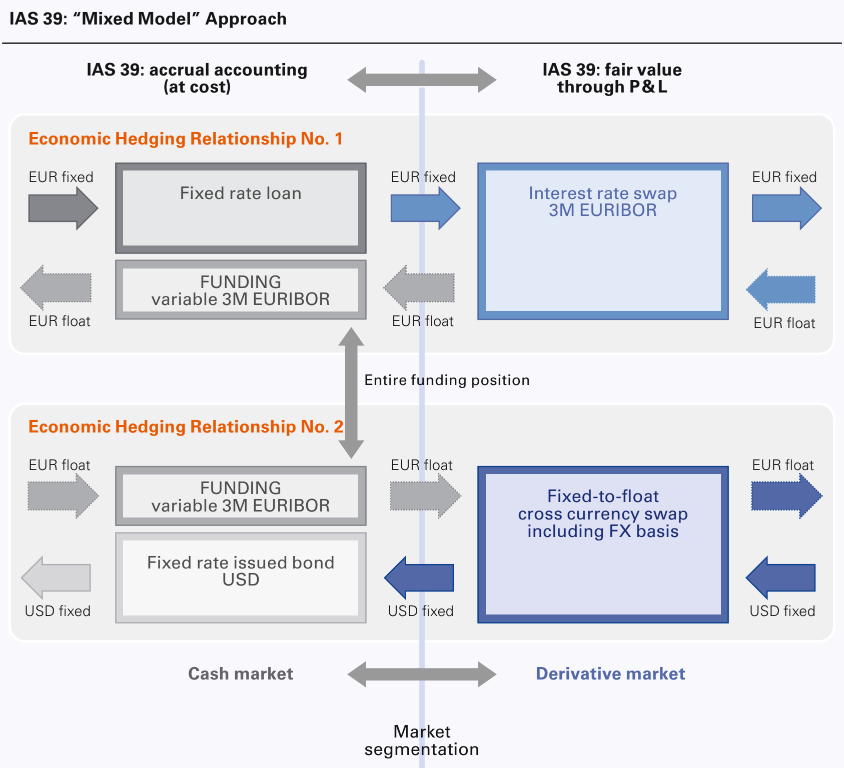
The cash flow hedge accounting model according to IAS 39 can be justified by the absence of arbitrage principle using the derivative prices as the only relevant price for cash and derivative markets as well as its corresponding risk decomposition. The impact of multi-curve models is limited: due to the application of the hypothetical derivative method no additional ineffectiveness is expected, provided that terms and conditions of the cash and derivative instrument (floating side) match to a sufficiently high degree. The economic underpinnings of cash flow and fair value hedge accounting are similar but, due to the structure of the fair value hedge accounting model, the impact of multi-curve models is different. Both hedge accounting models represent valuation models which can be applied if the requirements of IAS 39 are met.

2.3.3 Fair Value Hedge Accounting in a Multi-Curve Model Setup

Figure 17 shows two typical economic hedge accounting relationships applied by financial institutions. The first economic hedging relationship (Economic Hedging Relationship No. 1) refers to interest rate hedging; the interest rate risk of a fixed rate loan is "swapped" into a floating interest rate. On a cash flow basis this hedging relationship entirely matches, i.e. the cash flows (considering the cash flow profile with the internal coupon assumption for the fixed rate loan) exactly offset. But the hedging instrument (interest rate swap) and the hedged item (fixed rate loan) are financial instruments, which are traded in different markets (market segmentation). According to the mixed model approach hedging instrument and hedged item are carried at different values. This results in P&L volatility stemming from fair value changes of the derivative recognized at fair value in P&L.

The application of hedge accounting resolves this accounting mismatch by "fair valuing" the fixed rate loan with respect to interest rate risk ("hedged risk"). A similar situation arises in case of FX cross currency hedging (Economic Hedging Relationship No. 2). A USD denominated

FIGURE 17: Typical Economic Hedge Relationships

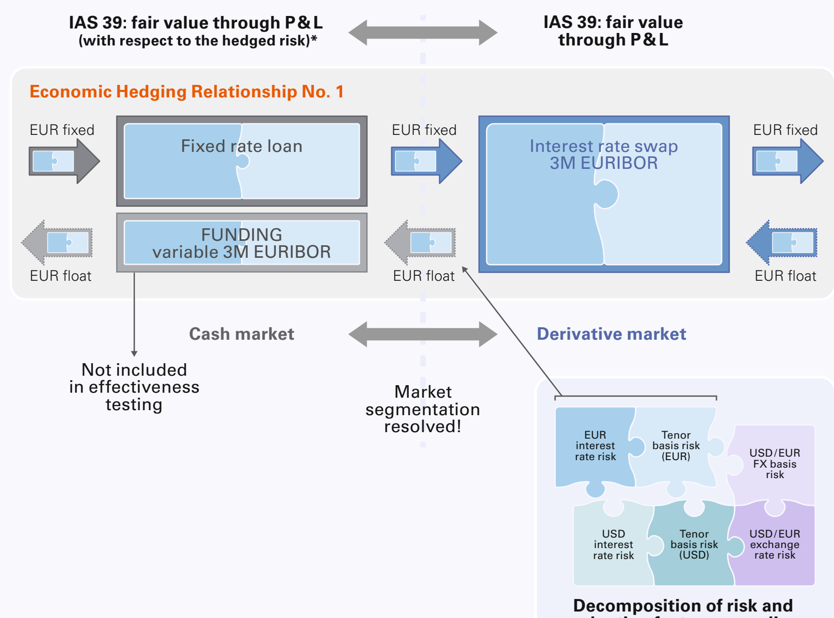


fixed rate bond is issued and hedged with a fixed-to-float cross currency swap including the FX basis. In this hedging relationship the FX risk and interest rate risk are hedged simultaneously. On a cash flow basis the net cash flow of the economic hedging relationship is zero. Again, both financial instruments involved in the economic hedging relationship belong to different markets.

Considering the Economic Hedging Relationship No. 1 it is assumed that the requirements of IAS 39 are met and fair value hedge accounting can be applied. Following the economic rationale outlined above (refer to *Section 2.2.1*), the relevant risk and valuation factors determined by assessment of markets' participants result in the following impact (see *Figure 18*):

FIGURE 18: Interest Rate Fair Value Hedge Accounting according to IAS 39

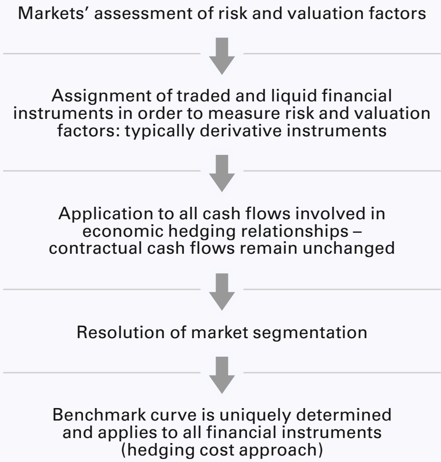
IAS 39: fair value hedge accounting



* The contractually determined cash flow profile is valued using parameters (benchmark curve) of the derivative market.

- The relevant risk and valuation factors with respect to the economic and hedge accounting relationship are EUR and the tenor basis risk. Both risk factors are represented by liquidly traded derivatives: EONIA interest rate swaps and 3-month EURIBOR/EONIA basis swaps (refer to *Table 1*).
- The risk and valuation factors are assigned to all cash flows involved in the economic hedging relationship. This is also of importance in connection with risk management techniques such as evaluation of VaR figures (refer to *Section 6*).
- The assignment of risk and valuation factors does not change contractual cash flows – a common feature of single- and multi-curve valuation models.
- Since the relevant risk factors are assigned to all cash flows – cash and derivative instruments alike – involved in the economic and hedge accounting relationship an integrated market for all financial instruments is created.
- Since the determined risk factors are measured by derivative prices, the relevant prices for all financial instruments in the integrated market are derivative prices. Consequently market segmentation is resolved.
- By the creation of an integrated market for cash and derivative instruments and the resulting resolution of market segmentation the cash basis between cash and derivative markets is eliminated.
- According to the multi-curve model setup (refer to *Figure 8*) there are only two relevant discount curves: EONIA and FED Funds discount curves. This results from the property that all derivatives in the multi-curve setup are measured relatively to these two discount curves. With respect to fair value hedge accounting these two curves serve as the “benchmark curve” according to IAS 39.
- Using derivative prices for the determination of the fair value resulting from the designated hedged risk does not represent a “short cut method”. The utilization of derivative prices to “fair value” cash instruments is equivalent to price cash instruments (“hedged items”) according to their hedging costs. The main steps are summarized in *Figure 19*. This also reveals that fair value

FIGURE 19: Financial Economics of Hedge Accounting



hedge accounting according to IAS 39 is a valuation model, which can be applied if the requirements are met. A brief description and references are given at the end of *Section 4*.

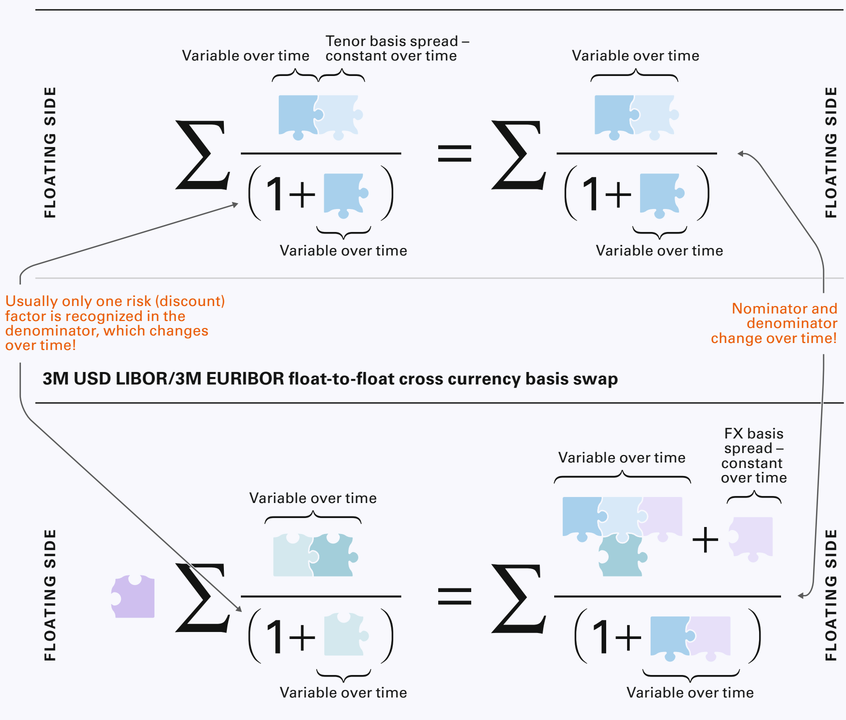
- The described economic features above also apply to fair value hedge accounting models in case of single-curve models. Therefore multi-curve models can be considered as a generalization of single-curve models.

With respect to fair value hedge accounting some special features arise. Firstly the measurement of market and risk factors outlined in *Table 1* also involves tenor basis swaps. Tenor basis swaps are liquidly traded instruments; their changes in fair value are measured with respect to the benchmark curve (denominator) and additionally relate to changes in cash flows ("variable cash flows") represented in the nominator of the floating sides. This is illustrated in *Figure 20*.[21]

Now the special feature is that tenor and cross currency basis spreads (FX basis spreads) meet the requirements of IAS 39.AG99F, since they measure "differences" in benchmark curves and are thus separately identifiable and reliably measurable; otherwise a logical inconsistency is created. For example: liquidly traded cross currency basis swaps can be considered as the difference between the EURIBOR and USD LIBOR benchmark curve. If the cross currency basis swap does not meet the above-mentioned conditions from IAS 39, this also holds true for the EURIBOR and USD LIBOR benchmark curve. This also

21 For illustration purposes the US convention for tenor basis swaps as will be described in section 3.2.1 is used.

FIGURE 20: **Schematic Representation of a Tenor Basis Swap and a Cross Currency Basis Swap (CCBS) (at t_0) in a Multi-Curve Setup**

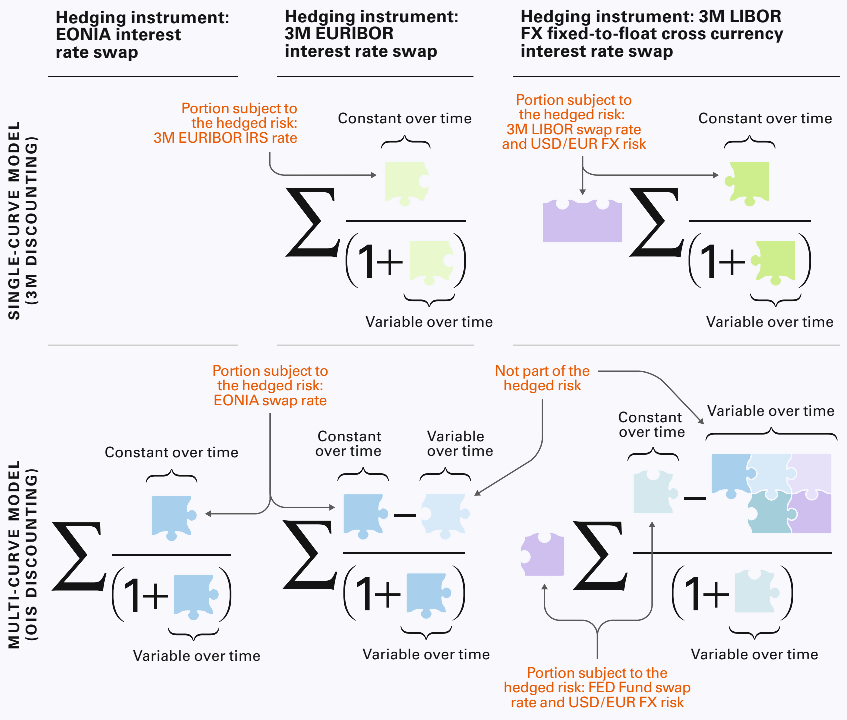


becomes apparent when considering the multi-curve model setup described in *Figure 8*. All parameters are tied to each other by the equilibrium conditions. A detailed discussion concerning the compliance of multi-curve models with IAS 39 hedge accounting requirements can be found in *Section 5*. But surprisingly this does not play any role in fair value hedge accounting, since:

- In a multi-curve model setup tenor basis risk cannot be designated as the "hedged risk" since, on one hand it is not an accepted benchmark on its own due to the float-to-float character of the generating instruments and, on the other hand the hedged risk is represented by the uniquely defined (per currency without consideration of cross currency products) discount curve (EONIA respectively FED Funds curve), and

- even if tenor basis risk were designated as the "hedged risk", the effectiveness test would potentially fail since the basis risk is represented by tenor basis swaps, which are basically float-to-float instruments, and changes in fair value result from changes in the nominator and the denominator (see *Figure 20*[21]).
- As illustrated in *Figure 20*[21], the risk factors "tenor risk" and "FX basis risk" are represented by float-to-float instruments: a 3-month EURIBOR/EONIA basis swap and a 3-month EURIBOR/3-month USD LIBOR cross currency basis swap.

FIGURE 21: Overview of the Construction of Risk-Equivalent Loans/Bonds in Single- and Multi-Curve Setups



Both derivatives change in value with respect to changes in the nominator and in the denominator. Therefore the construction of a "risk-equivalent bond/loan" representing the hedging costs of the hedged item economically includes "variable" cash flows in order to reflect the risk factors "tenor risk" and "FX basis risk" relative to the hedged risk over the lifetime. Thus the hedged item must in a way incorporate these "variable" cash flows in order to be in line with the markets' assessment of risk and valuation factors, since at any time only one risk factor is represented by the discount curve and recognized in the denominator. All remaining risk factors have their own dynamics which are measured relatively to the discount curve (hedged risk) and have to be recognized in the cash flow of the hedged item. This is admissible as a designation of a portion of cash flow according to IAS 39.81.

The "risk-equivalent loan/bond" determines the portion of cash flows of the hedged item subject to economic hedging derived from derivative prices, which are the only relevant prices in hedge accounting models. This concept is described and used throughout the following paper (refer to *Sections 4–6*). The determination of the "risk-equivalent loan/bond" requires mathematical modeling. In *Figure 21* the results are illustrated using the metaphorical language introduced above.

The major results shown in *Figure 21* are:

- In single-curve model setups the portion of the designated hedged risk (IAS 39.AG99F) and the portion of cash flows (IAS 39.81) coincide; in this case these portions are equal to the 3-month EURIBOR/3-month USD LIBOR interest rate swap rate.
- In multi-curve model setups the portion to the designated hedged risk (IAS 39.AG99F) and the portion of cash flows (IAS 39.81) do not coincide.
- Similar to single-curve model setups the portion subject to the hedged risk equals the EONIA resp. FED Funds interest rate swap rate.

- The portion of cash flows (IAS 39.81) is represented by a constant and a variable part, the constant part being the "designated hedged risk", while the variable part not being subject to the designated hedged risk but part of the designated portion of cash flow. In order to give a rough picture: the variable part is represented by the change in fair value of tenor basis spreads. This shows that the tenor risk is not subject to the designated hedged risk (EONIA or FED Funds benchmark interest rate risk).
- Both fair value hedge accounting models – whether single-curve or multi-curve – have in common that the designated hedged risk and the benchmark curve coincide!
- As can be seen in the left-hand elements in each row in *Figure 21,* if the discount and the forward curve coincide, fair value hedge accounting is performed by a static hedging strategy which is equal to the "traditional" fair value hedge accounting approach.
- The variable parts subject to the dynamic adjustment of the hedged item can be considered as a dynamic hedging strategy since in these cases the hedging instrument and the discount curve ("benchmark curve") do not coincide. An important feature of the dynamic hedging strategy is that the strategy is known at inception for the entire lifetime of the hedging relationship. The strategy is defined by changes in the fair value of tenor basis swaps – only the amount is unknown. For effectiveness testing this property can be used to simulate future fair value changes of tenor basis swaps in order to prove effectiveness.
- The alignment of the dynamic economic hedging model introduced by the multi-curve setup with the requirements of fair value hedge accounting according to IAS 39 requires the regular re-designation since the designated portion of cash flows changes in time. This introduces additional complexity with respect to effectiveness testing and the determination of booking entries, which requires additional modeling and assumptions (a description can be found in *Section 4*).

Given the derived rationale of the economic hedging and its corresponding concepts according to IAS 39 as described in the flow chart in *Figure 19,* the application of fair value hedge accounting to hedges involving fixed-to-float cross currency swaps is straightforward (Economic Hedging Relationship No. 2). *Figure 22* shows the impact of the re-assessment of market risk factors to the cash flows involved in the hedging relationship. The features of the approach, resolution of market segmentation etc. are identical to those in case of interest rate fair value hedge accounting (refer to *Figure 19* and the analysis of Economic Hedging Relationship No. 1). With respect to hedges of FX

FIGURE 22: Foreign Exchange (FX) and Interest Rate Fair Value Hedge Accounting according to IAS 39

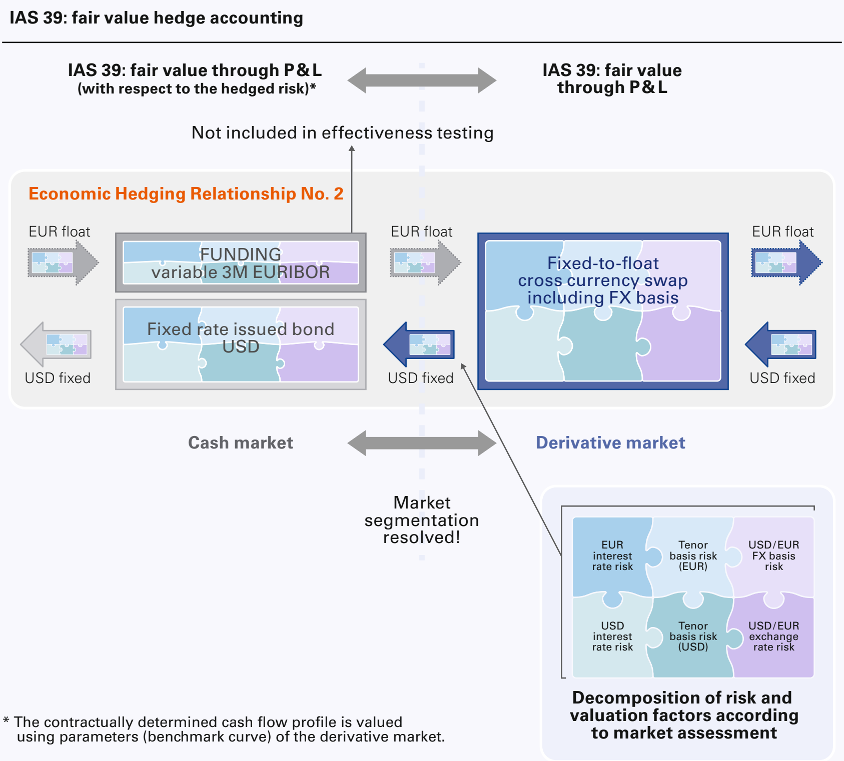
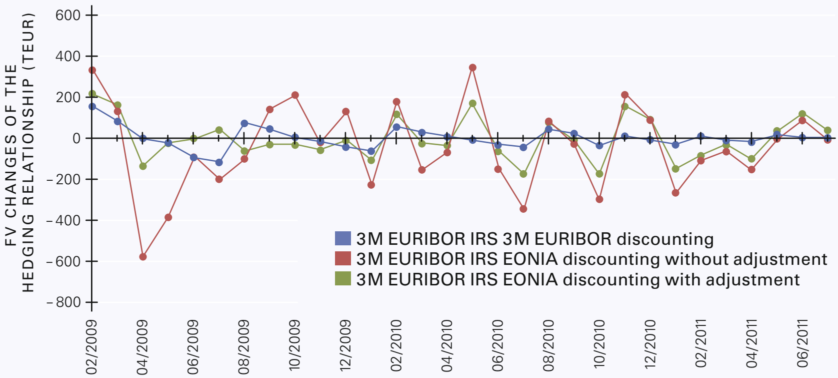


FIGURE 23: **Comparison of Different "Fair Value Hedge Accounting Strategies"**

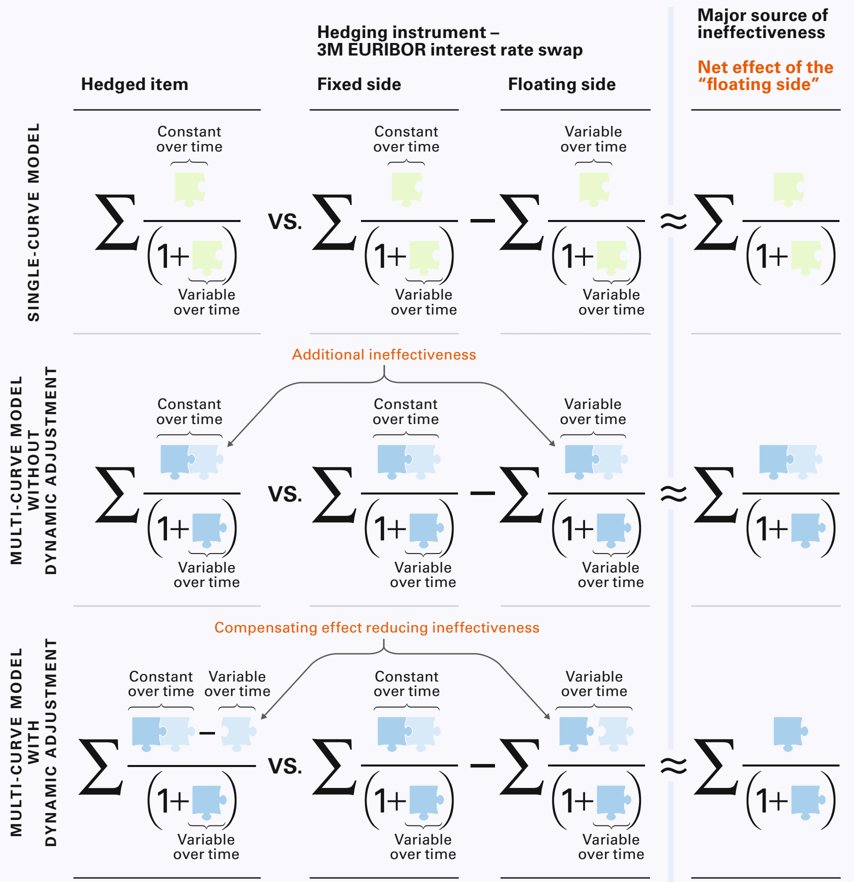


risk the interest rate parity plays a pivotal role in order to provide justifications for hedge accounting models according to IAS 39 (detailed description in *Section 5*). The interest rate parity – whether in single or multi-curve model setups – follows the absence of arbitrage principle and is, therefore, consistent with the economic rationale explained above.

In *Figure 23* three different "fair value hedge accounting strategies" determining the hedged item are compared. In all three approaches the hedging instrument is a 3-month EURIBOR interest rate swap. The blue line shows the results of the "traditional" fair value hedge accounting approach in a single-curve model, while the red line portrays the results if the benchmark curve (i.e. discount curve) is changed from the 3-month EURIBOR to the EONIA interest rate swap curve. The figure reveals the incremental ineffectiveness due to the two risk factors involved. The green line shows the results if the dynamic adjustment approach is applied; the ineffectiveness is lower than in the unadjusted case (red line).

A brief analysis of the major (structural[22]) sources of ineffectiveness shows the consistency with economic modeling. This is illustrated in *Figure 24*. In the rightmost column the “net effect of the floating side” of each fair value hedge accounting strategy is shown. Please note that

FIGURE 24: Comparison of Different “Fair Value Hedge Accounting Strategies” and the Major Source of Ineffectiveness



22 There will be also more technical sources of ineffectiveness due to e.g. different interest payment frequencies of the hedged item and the tenor basis swap.

when comparing the single-curve model with the multi-curve model including dynamic adjustment, the major (structural) source of ineffectiveness is similar. In the single-curve model the designated hedged risk ("3-month EURIBOR"), the designated portion of cash flow ("3-month EURIBOR interest rate swap rate") and the benchmark curve ("3-month EURIBOR interest rate swap curve") coincide. This results in the floating side of the 3-month EURIBOR as a major source of ineffectiveness. In the multi-curve model with dynamic adjustment the major source of ineffectiveness results from the floating side of an EONIA interest rate swap. This corresponds to the designated portion of risk "EONIA swap rate" and the benchmark curve ("EONIA interest rate swap curve").

Therefore on a net basis this fair value hedge accounting strategy corresponds to a fair value hedge accounting strategy using an EONIA interest rate swap as hedging instrument. Clearly the dynamic fair value hedge accounting strategy involves additional "ineffectiveness" due to amortizations of fair value adjustments recognized in interest result originating from regular re-designation, but this property is omitted for illustration purposes, as are more technical sources of ineffectiveness e.g. due to different interest payment frequencies of hedged item and inherent tenor basis swaps.

Table 2 summarizes the hedge accounting model according to IAS 39 in the multi-curve setup. Similar results can be derived for the EUR case (discounting with EONIA). The entire derivation of the results can be found in *Section 6.*

TABLE 2: Summary of Single- and Multi-Curve Models of Fair Value Hedge Accounting according to IAS 39

Hedged risk (bench-mark curve)	Discount curve	Portion of cash flow designated in the hedged item: Cash flow subject to the hedged risk	Portion of cash flow designated in the hedged item: Dynamic adjustment	Hedging instrument	Type of hedge	Sources of ineffectiveness
Model setup: single-curve						
3M LIBOR	3M LIBOR swap curve	3M LIBOR interest rate swap rate	None	3M LIBOR interest rate swap	Static	– Floating leg, – maturity mismatches, – incongruities in payment frequencies, – counterparty risk
3M LIBOR USD/EUR FX risk (spot rate)	3M LIBOR swap curve	3M LIBOR interest rate swap rate	None	USD/EUR fixed-to-float cross currency interest rate swap (no FX basis)	Static	Similar to the case above
Model setup: multi-curve						
FED Funds rate	FED Funds interest rate swap curve	FED Funds interest rate swap rate	None	FED Funds interest rate swap	Static	– Floating leg, – maturity mismatches, – incongruities in payment frequencies, – counterparty risk
FED Funds rate	FED Funds interest rate swap curve	FED Funds interest rate swap rate	Minus changes in 3M LIBOR/FED Funds basis swap	3M LIBOR interest rate swap	Dynamic	Additionally to above: amortizations of the recognized fair value adjustment in interest result due to regular designation/de-designation
FED Funds rate USD/EUR FX risk (spot rate)	FED Funds interest rate swap curve	FED Funds interest rate swap rate	Minus changes in 3M LIBOR/FED Funds basis swap and FX basis	USD/EUR fixed-to-float cross currency interest rate swap (with FX basis)	Dynamic	Similar to the case above

2.4 Implementation of Multi-Curve Hedge Accounting Models and Conclusions

The purpose of the paper is the analysis of the coherence of hedge accounting models under IAS 39 with multi-curve models being currently implemented by financial institutions to cope with changes in fair value valuation resulting from the developments in the course of the financial markets crises and institutional changes. With respect to the accounting model outlined above, there are two additional aspects of importance.

Portfolio Hedges:

- The derived results also hold for the two types of "portfolio hedges" permitted according to IAS 39. These two types of "portfolio hedges" are:
  - Hedges of a "group of items" (IAS 39.78 (b)): Assets, liabilities etc. can be summarized into one portfolio and designated as "hedged item". Forming such a portfolio presumes that the requirements of "similarity" under IAS 39.78 (b), IAS 39.83 and IAS 39.84 are met ("test of homogeneity"). With respect to this type of hedge the results derived above also hold requiring a regular re-designation. For the test of homogeneity the property of the known dynamic hedging at inception can be used (see above).
  - Hedges of "portfolio fair value hedges of interest rate risk" represent an approach applicable to IAS 39 only (IAS 39.78, IAS 39.81A, AG114(c), AG116 and AG118), which offers the possibility to form time buckets ("repricing dates") and allocate "cash flows" into these buckets. Since this hedge accounting approach requires regular re-designation, the "dynamic" hedge accounting model outlined above can be incorporated. Only the determination of the relevant tenor basis swap in order to derive the dynamic adjustment requires additional modeling.

IFRS 9 – Selected Topics:

- The following argumentation is based on ED 2010/13 and the discussions and tentative decisions of the International Accounting Standard Board (IASB) in the meantime. Therefore it is preliminary and subject to the upcoming review draft and the final standard.
- Since conceptually IAS 39 and IFRS 9 do not differ in respect of the definition of "separately identifiable" and "reliably measurable"[23], these analyses and results carry over to IFRS 9. With respect to effectiveness testing, changes under IFRS 9 are expected, in particular voluntary re-designation[24] is no longer permitted but a "re-balancing" approach[25] (varying hedge ratios during the lifetime of the hedge relationship without re-designation) is considered. Furthermore it seems that the application of the "hypothetical derivative" method might be allowed for fair value hedges[26], which, according to KPMG Insights 7.7.630.50, is not available under IAS 39, but this has to be confirmed by the forthcoming review draft and final standard respectively. As mentioned before, the final rules for hedge accounting under IFRS 9 have not yet been published and the effective date is scheduled for January 1, 2015.[27] The ongoing debate concerning the impairment rules under IFRS 9 may result in further delay. According to a similar framework of IAS 39 and IFRS 9 as well as potential relief in terms of effectiveness testing, no further analysis with respect to IFRS 9 is performed in this paper.

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23 ED 2010/13 §18.

24 ED 2010/13 B61 et seqq.; tentatively confirmed IASB Update May 31 – June 2, 2011.

25 ED 2010/13 B46 et seqq.; tentatively confirmed IASB Update May 31 – June 2, 2011.

26 ED 2010/13 B44 – 45.

27 IASB work plan projected targets on March 23, 2012.

Other Implementation Aspects:
The aim of the paper is not to provide an implementation guide, since the implementation of hedge accounting models represents a separate and complex undertaking. The complexity results from individual circumstances in financial institutions, which generally differ with respect to IT systems, corresponding processes etc. This document covers some basic examples of hedging relationships and for these, formulas for the adjusted internal coupon are presented. These cannot be carried over immediately to an arbitrary hedge accounting relationship, but a careful analysis has to be performed to adapt the general arguments presented to a specific case and certainly a number of further aspects and details have to be discussed. A generic plan to follow could involve the following steps:

1. Analysis of valuation models and parameters.
2. Measurement of the impact of pure change in valuation without further adjustments to the portfolio of hedging relationships.
3. Categorization of the hedge accounting portfolio in particular w.r.t. factors of the re-assessment of risk, e.g. collateralized/uncollateralized hedging instruments.
4. Identification of representatives for the different categories.
5. Analysis and generation of hedge accounting models for each representative taking into account the requirements of the standard, the specific features of the financial instruments involved as well as the individual environment of systems and processes and documenting assumptions/approximations and resulting limitations/restrictions/constraints.
6. Cost/benefit analysis of possible implementation variants including the assessment of non-recurring and recurring costs and effort.
7. Decision on an implementation variant supported/accepted by all parties involved.
8. Detailed concept for the chosen variant and detailed project plan for implementation.
9. Implementation with corresponding initial and regular tests and controls.

Practical Solutions:
In the course of the analysis of multi-curve models and their impact on hedge accounting practical solutions are considered in the financial industry. The issues of an approach used in practice for hedge accounting with a fixed-to-float cross currency swap as hedging instrument involving different kinds of hedging relationships to deal with the FX basis will be critically discussed in *Section 5.4*.

Another of these practical approaches applied to interest rate fair value hedge accounting is described briefly using the following example: The hedged item is discounted on 6-month EURIBOR interest rate swap curve, the hedging instrument – 6-month interest rate swaps – is discounted on EONIA ("mixed discount approach").[28]

This practical approach is conceptually questionable in several aspects:

- A "pure" 6-month EURIBOR interest rate swap curve in the interbank market in presence of collateralization does not exist. Liquidly traded 6-month interest rate swaps in the interbank market are mostly collateralized and subject to EONIA discounting and, therefore, interest rate swap rates are affected by EONIA. Consequently the designated hedged risk includes EONIA.
- It can also be shown that designating 6-month EURIBOR implicitly means designating EONIA and a 6-month EURIBOR/EONIA basis swap. This property is derived in connection with the change of discount curves in case of a 6-month EURIBOR to a 3-month EURIBOR discount rate (refer to *Section 4*).
- As will be shown, e.g. in *Section 6,* in a multi-curve model setup of collateralized derivatives a 3-month EURIBOR discount curve (and with the same arguments a 6-month EURIBOR discount curve) does not exist anymore. The inclusion of such a discount curve violates the absence of arbitrage principle and leads to an

28 PricewaterhouseCoopers (2011A) and PricewaterhouseCoopers (2011B).

inconsistent set of discount curves. A property which is harmful for performance measurement in financial institutions, since gains or losses can be generated artificially but might not exist in economic reality.

- This effect might even increase if optionalities such as prepayment options exist, since options embedded in the hedged item are discounted differently from options embedded in the derivative. Therefore this may result in ineffectiveness despite the fact that the hedge is economically sound in terms of matching cash flows.
- With respect to the definition of the hedged risk "6-month EURIBOR" the analysis in the paper has shown that in a multi-curve setup on a fair value basis only EONIA or FED Funds benchmark interest rate risk can be hedged. Therefore – although IAS 39 allows the designation of a 6-month EURIBOR as hedged risk – in a multi-curve setup with the EONIA curve as the only discount curve the 6-month EURIBOR interest rate risk does not represent the economically defined hedged risk.

Despite the criticism outlined the approach works quite well in terms of effectiveness testing in case of "plain vanilla" interest rate hedges. Consider the example shown in *Table 3*.

In the following single-curve (discounting the hedged item and the derivative by 6-month EURIBOR interest rate swap curve) and the "mixed discount" results are compared. Evaluating the effectiveness tests and applying periodic and cumulative dollar offset leads to the results shown in *Figures 25* and *26*.

The question is: why does this perform so well? The answer requires some mathematical calculus, which is described in *Section 4*. The differences in fair value using different discount curves are driven by the ratio of 6-month EURIBOR and EONIA annuities (sum of zero bonds). If this ratio becomes instable, as in the financial crisis, the ineffectiveness increases (refer to *Figure 25* and *Figure 26* above). As is known in

TABLE 3: Example of Different Discount Curves in the Hedged Item and Hedging Instrument – "Plain Vanilla" Interest Rate Hedges

	Bond (hedged item)	6M EURIBOR interest rate swap (collateralized) (hedging instrument)
Start date	07/13/2007	07/13/2007
Maturity	07/13/2011	07/13/2011
Cash flow (coupon)	Internal coupon 6M EURIBOR interest rate swap rate with 4 years maturity	Internal coupon 6M EURIBOR interest rate swap rate with 4 years maturity
Discount curve	6M EURIBOR interest rate swap curve	EONIA interest rate swap curve

FIGURE 25: Results of the Periodic Dollar Offset Method

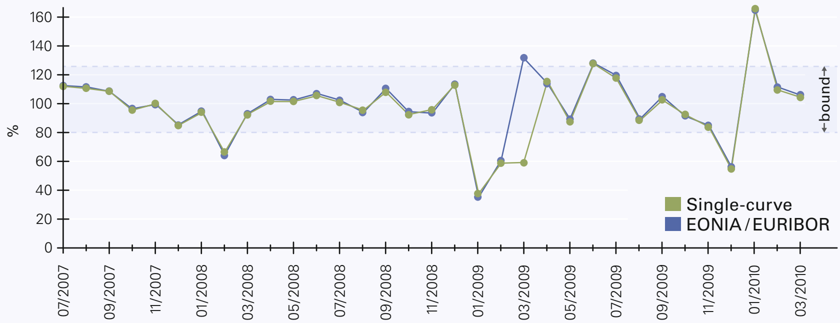
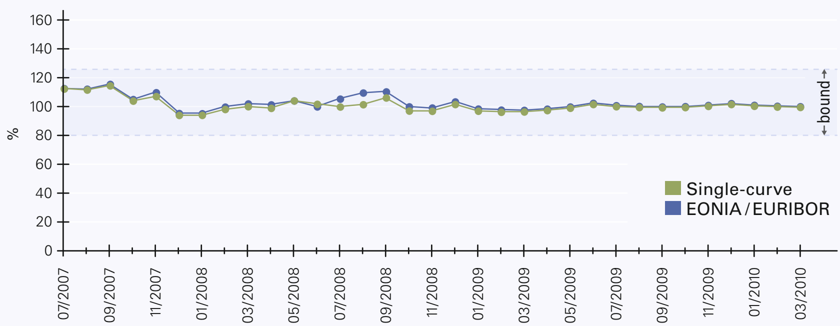


FIGURE 26: Results of the Cumulative Dollar Offset Method



practice the effectiveness results can be improved if the regression method is applied since in this case a smoothing effect occurs.

The situation is different if FX hedges with a fixed-to-float cross currency swaps are considered. The example is shown in *Table 4*.

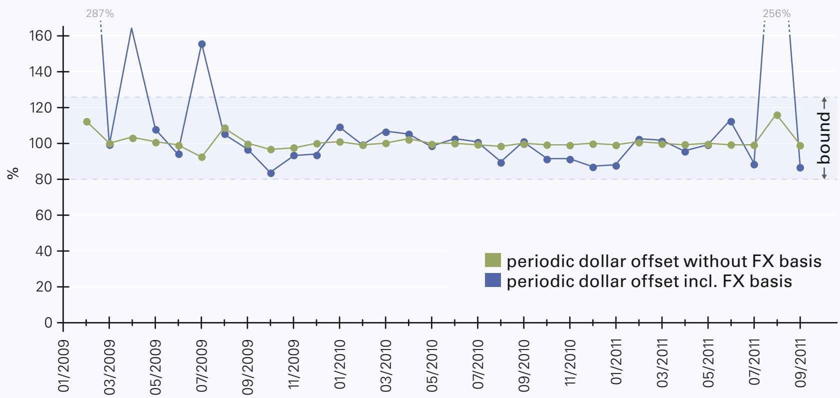
As before, according to the terms and conditions of the example, the effectiveness testing is performed. *Figure 27* shows the impact of the FX basis, the overall ineffectiveness increases and the entire hedge becomes ineffective if the volatility of the FX basis increases. The application of the regression method introduces some smoothing effects, so the ineffectiveness decreases. But as a result, if the FX basis in the hedged item is not taken into account the risk of becoming ineffective increases. But even if effectiveness is achieved the net P&L effect resulting from the application of fair value hedge accounting is exposed to volatility. Therefore unreasonable P&L volatility may occur despite a sound economic hedging relationship. *Figure 27* shows the hedge effectiveness valuing the hedged item on USD LIBOR and the derivative according to market conventions by OIS discounting and including the FX basis. Only for illustration purposes of the particular impact of the FX basis, the green line shows effectiveness measured on LIBOR/EURIBOR discounting and neglecting the FX basis. It is stressed at this point that there is no real cross currency swap without FX basis.

The brief description above shows the complexity of multi-curve models and their application in hedge accounting. The aim of the paper is the derivation of a consistent approach of multi-curve models and hedge accounting according to IAS 39 (IFRS 9). This consistency represents a contribution to future efforts in financial institutions to spread valuation methods applied in trading and treasury departments into the financial accounting in order to enable the alignment of reporting and economic hedging activities.

TABLE 4: **Example of Different Discount Curves in the Hedged Item and Hedging Instrument – FX Hedges with a Fixed-to-Float Cross Currency Swap**

	Bond (hedged item)	USD/EUR fixed-to-float cross currency swap (collateralized) (hedging instrument)
Start date	09/23/2009	09/23/2009
Maturity	09/23/2014	09/23/2014
Cash flow (coupon)	Internal coupon 3M USD LIBOR swap rate with 5 years maturity	Internal coupon 3M USD LIBOR swap rate with 5 years maturity
Discount curve	3M USD LIBOR interest rate swap curve	FED Fund interest rate swap curve/ EONIA interest rate swap curve/FX basis

FIGURE 27: **Results of the Periodic Dollar Offset Method – Fixed-to-Float USD/EUR Cross Currency Swap**



SECTION THREE

3

Cash, Money and Derivatives Markets – Market Conventions and Statistical Facts

In the following subsections some facts on different markets and markets segments are described. Before the crises and in the single-curve approach, generators of (discount) curves were taken from different markets or market segments (e.g. money and swap market) to construct the unique benchmark curve, neglecting the basis between markets and market segments because of its small size. Multi-curve setups aim to generate "homogenously generated" curves that are combined to price derivatives consistently.

3.1 Interest Rates (Money Market)

In *Table 5* the market conventions concerning the interest rates in the money market are listed, extending a similar presentation of Bianchetti (2011).

TABLE 5: Description of the Money Market for LIBOR, EURIBOR and EONIA

	Money market			
	LIBOR	**EURIBOR**	**EONIA**	**FED Funds (effective) Rate (US overnight rate)**
Definition	**L**ondon **I**nter **B**ank **O**ffered **R**ate	**Eur**o **I**nter **B**ank **O**ffered **R**ate	**E**uro **O**ver **N**ight **I**ndex **A**verage	Unsecured over-night lending rate **FED Funds Rate**
Market	London Interbank	Euro Interbank	Euro Interbank	USD Interbank
Side	Offer	Offer	Offer	Offer
Rate quotation specification	EUR LIBOR = EURIBOR, other currencies: minor differences (e.g. ACT/365, T+0, London calendar for GBP LIBOR).	TARGET calen-dar, settlement T+2, ACT/360, three decimal places, modified following, tenor variable.	TARGET calen-dar, settlement T+1, ACT/360, three decimal places, tenor 1 d.	ACT/360
Maturities	1 day (d) – 12 months (m)	1 week (w), 2 w, 3 w, 1 m, …, 12 m	1 day	1 day
Publication time	12.30 pm Central European Time (CET)	11:00 am Central European Time (CET)	6:45–7:00 pm Central European Time (CET)	Data is released by the Federal Reserve between 7:30 and 8:00 am for the prior business day
Panel banks[29]	8–16 banks (London based) per currency	43 banks from EU countries	same as EURIBOR	Banks which are members of the Fed-eral Reserve System
Calculation method	The top 4 and bottom 4 values are removed and an average taken of the middle eight.	The top and bottom 15% are eliminated and an average taken of the remaining quotes.	A weighted aver-age of all over-night unsecured lending transac-tions initiated by the panel banks.	FED Funds Rate is a daily overnight volume-weighted average that is calculated the day after closing for the previous day.
Calculation agent	Reuters	Reuters	European Central Bank	Bloomberg (FEDLO1), Reuters (FEDFUNS1)
Collateral	No (unsecured)	No (unsecured)	No (unsecured)	No (unsecured)
Credit risk / liquidity risk	Yes/yes	Yes/yes	Low/low	Low/low

29 The panel banks are the banks with highest volume of business in the USD/EUR money markets. The number of these banks or actually available offer rates may change from time to time.

Table 5 reveals differences in market participants (contributors) within the interbank market as well as different inherent risks associated with each rate. In particular for LIBOR and EURIBOR the market quotations represent "offer" rates, which have to be distinguished from "transaction" rates (lending rates) as in the case of EONIA. Another aspect in the determination process is that EONIA is the average of actually traded rates whereas for LIBOR and EURIBOR the rates are stated by the panel banks. Press reported on rumors of manipulations in the case of LIBOR.[30]

Due to the calculation method, the quoted rates also imply economic modeling. This stems from the fact that some quotes are eliminated and the average is formed from the remaining quotes. In *Figure 28* an example for the 3-month EURIBOR is provided. In order to calculate the 3-month EURIBOR the data from the Panel Bank Data (43 offer rates) is grouped, the highest and lowest 15% of the rates are eliminated and the average is formed. In the example the 3-month EURIBOR is 1.459%.

Since each rate represents the rate for unsecured lending in the interbank market, credit and liquidity risk is apparent. This is – to a certain extent – different from the derivative market as will be shown below. The quotations cover various maturities – up to one year. Consequently the quotations include not only credit risk but also some liquidity risk.[31] As EONIA has a maturity of one day, its credit risk and liquidity risk can be considered low in comparison to the different tenors of the EURIBOR. In *Figure 29* the differences between the EURIBOR rates with different tenors and the EONIA rates (for the corresponding time period) are shown. Before the financial crises (mid 2007) the

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30 See e.g. Bloomberg Market Magazine 11/22 or Wood, D. (2011), "Libor fix?", in: *Risk Magazine,* July 2011

31 As shown in Bianchetti, M. and Carlicchi, M. (2011) the credit and liquidity risk are the average default and liquidity risk of the interbank money market (of the LIBOR panel banks), not those associated to the specific counterparties involved in the financial contract.

differences in rates were low, but became significant afterwards. These differences can be regarded as a spread compensating for inherent credit and liquidity risk in short term lending in the interbank market. *Figure 29* also shows: the longer the maturity of the EURIBOR, the higher the difference to the EONIA rates. On the other hand, according to the market convention of these rates, credit risk has been always present in the quoted rates.

FIGURE 28: EURIBOR Panel Bank Data for November 14, 2011

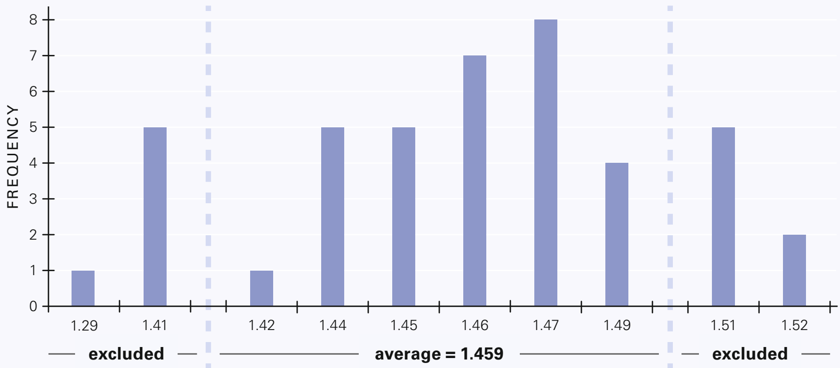
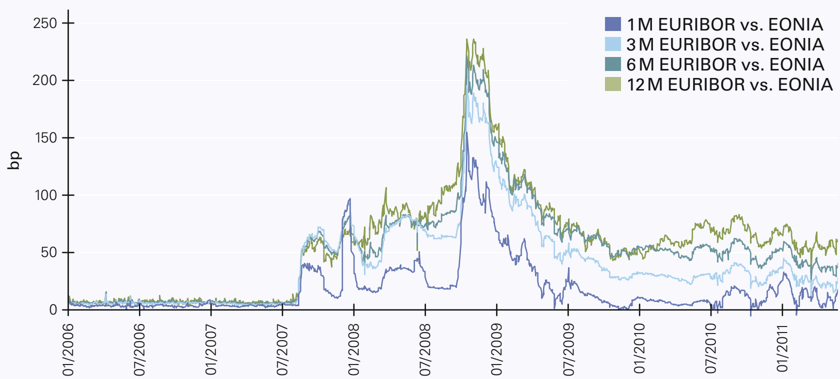


FIGURE 29: Differences of the EURIBOR with Different Tenors vs. EONIA (in bp)
(Source: Bloomberg)



A similar analysis can be performed for the USD: in *Figure 30* the USD LIBOR is compared with the Federal Funds Rate (FED Funds Rate), which represents the average rate of transactions during one day and is comparable to EONIA. FED Funds Rate is different from the Target FED Funds Rate, which is the rate decided on by members of the Federal Open Market Committee (FOMC).

With respect to the size of the money market, the European Central Bank ("Euro Money Market Survey", September 2011) provides a statistic for unsecured cash lending and borrowing. The statistic indicates a downward movement for unsecured cash lending and borrowing after the beginning of the financial crisis in 2008 (see *Table 6*).

3.2 Derivative Markets

3.2.1 Derivatives Based on OIS, EURIBOR and LIBOR including Tenor Basis Swaps

The market for interest rate derivatives plays a pivotal role for hedge accounting and is different from the money market described above. In contrast to the money market, the interest rate derivative market is based on individual legal contracts which are almost exclusively represented by the documentation issued by ISDA.[32]

The ISDA Master Agreement (2002), the ISDA Credit Derivatives Definitions (2003) and the ISDA Confirmation[33] listed in *Table 7* are standardized agreements with regard to derivatives transactions. The choice of law under the ISDA Master Agreement (2002) (i.e. English law or the laws of the State of New York, as applicable) are accepted in several jurisdictions. Legal opinions on the legal enforceability of claims on the basis of the ISDA Master Agreement (2002) exist for all major jurisdictions.[34]

FIGURE 30: **Differences of USD LIBOR with Different Tenors vs. FED Funds (in bp)** (Source: Bloomberg)

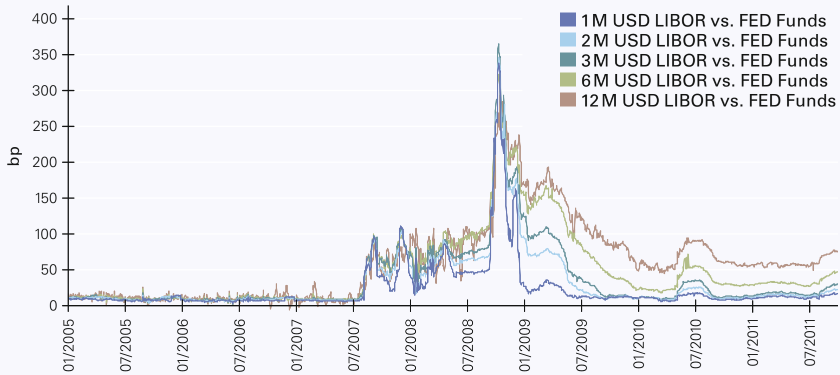


TABLE 6: **Average Daily Turnover Index for Unsecured Cash Lending and Borrowing** (Index: Cash Lending Volume in 2002 = 100)

Year	Lending	Borrowing	Year	Lending	Borrowing
2002	100.00	188.26	**2007**	139.07	296.95
2003	116.05	237.08	**2008**	145.27	247.57
2004	124.79	245.00	**2009**	107.45	172.20
2005	119.75	251.55	**2010**	110.47	135.48
2006	123.07	289.25	**2011**	104.48	170.83

32 Further legal documentations for derivatives are e.g. the German Master Agreement for financial transactions *(Rahmenvertrag für Finanztermingeschäfte)* with its annexes and supplements issued by the Association of German Banks *(Bundesverband Deutscher Banken)* and the European Master Agreement for financial transactions with its annexes and supplements issued by the Banking Federation of the European Union. The legal mechanism of these master agreements is similar to the ISDA Master Agreement (2002).

33 Documents and other confirming evidence exchanged between the parties or otherwise effective for the purpose of confirming or evidencing transactions entered into under the ISDA Master Agreement (2002).

34 Henderson, S.K. (2010) at p. 816, with further references.

Forming the legal basis of all derivative transactions, the ISDA Master Agreement (2002) includes for example the definition of "Events of Default" defining the default of the counterparty (counterparty risk). Credit events in connection with a credit default swap (CDS) are defined in the ISDA Credit Derivative Definitions. An ISDA Master Agreement (2002) is signed for various derivative transactions like IRS, FRA, tenor basis swaps, CDS and CCBS.

Table 7 shows the components of ISDA involved in derivative transactions.

The different legal setup of the money market in comparison to the interest rate derivative market also affects the credit risk inherent in these different markets. Since the derivative market is based on individual contracts between two counterparties, its definition of counterparty risk and the collateralization is defined by the documentation issued by ISDA (e.g. in the CSA). As shown in *Table 8,* for the majority of OTC derivative transactions, market participants enter into a CSA in order to mitigate the counterparty credit risk by posting collateral. Collateral is typically provided on a cash basis whereby the accrued interest is linked to the overnight rate. This resulted in recent changes for derivatives pricing and valuation, which are based on OIS for collateralized transactions. This will be analyzed below in connection with the multi-curve setup for derivative pricing and hedge accounting. But this does

TABLE 7: Legal Components of Derivative Contracts under ISDA

Contracts	Interest rate swap	Credit default swap	Components (examples)
ISDA Master Agreement (2002)	√	√	Events of default (see ISDA Master Agreement (2002) Chapter 5)
ISDA confirmation	√	√	
ISDA Credit Derivatives Definitions (2003)		√	Credit events (see Credit Derivatives Definitions (2003) Article IV)
Credit support annex	Optional	Optional	

TABLE 8: **Market Conventions for Interest Rate Derivatives**

	Interest rate derivatives market		
	OIS	**EURIBOR / LIBOR swap**	**Tenor basis swap**
Definition	Overnight index swap (EONIA swap, FED Funds swap): The floating rate of the swap = the geometric average of an overnight index over every day of the payment period.	Fixed-to-float IRS – examples: 6M EURIBOR floating vs. fix annual payments, 3M USD LIBOR floating vs. fix semi-annual payments.	Examples: 3M EURIBOR/ 6M EURIBOR as a combination of two fixed-to-float IRS, FED Funds/3M USD LIBOR as float-to-float instrument with constant spread on the shorter term leg.
Market	Worldwide	Worldwide	Worldwide
Side	Offer and bid	Offer and bid	Offer and bid
Rate quotation specification	Individual	Individual	Individual
Maturities	1 w–3-year (y) liquid quotes for maturities less than 3 years; longer maturities of EONIA swaps provided by contributors based on basis spreads.	1 y – 30 y	1 y – 30 y
Publication time	Intra-day	Intra-day	Intra-day
Panel banks	Different brokers (e.g. ICAP) and composites (London, Tokyo, New York)	same as OIS	same as OIS
Calculation method	Different composite rates, e.g. Composite Bloomberg	same as OIS	same as OIS
Calculation agent	Bloomberg, Reuters	Bloomberg, Reuters	Bloomberg, Reuters
Collateral	Secured and unsecured. Not all trades are collateralized (secured). 78% of all OTC derivatives transactions are covered by a CSA "Collateral Agreements". 80% of all collateral exchanges are in cash. Collateral interests depend on the agreement. (ISDA Margin Survey, 2010)		
Counterparty risk (credit risk) / liquidity risk	Yes/yes	Yes/yes	Yes/yes

not imply that there are different types of credit risk between the different types of swaps for the same counterparty since all are based on the identical legal framework. Because of the possible complexity of CSA by eligible collateral, eligible credit support, eligible currency, threshold and other terms that vary widely, ISDA is currently developing and discussing a "Standard Credit Support Annex (SCSA)"[35] aiming to facilitate booking and modeling of CSA terms as well as the novation of OTC derivatives to central counterparties (CCPs).

Based on a commitment of the G20 leaders to increase transparency and reduce risk in the OTC derivatives markets, the EU council and parliament agreed on the "European Market Infrastructure Regulation (EMIR)".[36] Two main aspects are that standardized OTC derivatives are required to be cleared through CCPs to reduce counterparty risk and – in order to give more transparency – that all derivative contracts are required to be reported to trade repositories (i.e. central data centers) surveyed by the European Securities and Markets Authority (ESMA).

Quotes of forward rate agreements (FRA) or futures are used as generators for the short end of rate curves of up to 2–3 years. In the multicurve setup also in this part of the curve homogeneity w.r.t. the tenor is intended as proposed by Ametrano, F. (2011). As shown in Bianchetti, M. and Carlicchi, M. (2011) there is a similar development between (quoted) FRA rates and (implied) forward rates showing a widening of spreads since the financial crisis. For the rate determination from future quotes due to the margining effect, convexity adjustments have to be taken into account.

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35 www2.isda.org/functional-areas/market-infrastructure/standard-csa.

36 ec.europa.eu/internal_market/financial-markets/derivatives/index_en.htm, eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=CELEX:52010PC0484:EN:NOT.

In *Table 8* the definitions and the market conventions of the various interest rate swap contracts are shown. It can be observed that the derivative market is segmented but each segment follows similar conventions. Furthermore tenor basis swaps represent individual and separate swap contracts and therefore constitute a separate (sub-)market. Additionally they play an important role in defining benchmark curves for discounting and are therefore analyzed in more detail at the end of this subsection. Like in case of the money market the "quotes" provided e.g. by market information and price providers (e.g. Bloomberg, Reuters) represent "composites", which are evaluated by forming the average of rates provided by the contributors to market information and price providers.

Example – Bloomberg Composite Rates for Derivatives (Composite London, Composite New-York):
The Bloomberg Composite Rate is a 'best market' calculation. At any given point in time, the composite bid rate is equal to the highest bid rate of all of the currently active, contributed bank indications. The composite ask rate is equal to the lowest ask rate offered by these same active, contributed bank indications. For rates to be accepted into the composite, they must come from data contributors who have been 'privileged'[37] to provide the data, and the pricing must be considered valid and current.

Some important features of the composite calculation:

- Mid rates: The system generates a trade value each time an ask rate (except for the first ask of the day) is received. This trade value is the mid value between the composite bid and the composite ask. Mid rates are not generated for individual contributors but only for composite bid and ask rates.

37 Each contributor is assessed for quality and consistency of the provided data, as well as for the consistency of the data with the market.

- ‘5 Minutes Rule’: If a best bid is accepted into the composite, a mid will not be generated until a best ask is then received by the composite. However, a mid will still not be generated if the best bid is more than 5 minutes old when a best ask is finally accepted into the composite.

When modeling benchmark curves, tenor basis swaps play an important role, since swap rates are not always directly available from quoted swap transactions in order to derive a discount curve (benchmark curve). With respect to multi-curve setups, tenor basis swaps have a significant impact, since the tenor basis represents an additional risk factor which needs to be considered in order to derive a consistent approach for discount and forward curves.

TABLE 9: Available Tenor Basis Swaps for the USD LIBOR and FED Funds Swap

	1M USD LIBOR	3M USD LIBOR	6M USD LIBOR	FED Funds swap
1M USD LIBOR	(1M USD LIBOR IRS) – no composite quotes for 1M USD LIBOR swap curve in Bloomberg/no broker composite quotes in Reuters. In Bloomberg the curve is constructed using the 3M USD LIBOR curve and 1M/3M tenor basis spread curve whereas in Reuters some broker quotes for the 1M USD LIBOR curve e.g. ICAP, Tullett Prebon exist.	There exists a tenor basis spread curve 1M/3M USD LIBOR. 1M/3M tenor basis swaps are directly quoted (e.g. Bloomberg, Reuters).	1M/6M USD LIBOR tenor basis swaps are directly quoted (e.g. Reuters).	Are not quoted (in Reuters on the page of LIBOR basis swaps: SWAP/12, there are no contributors who quote FED Funds/1M tenor basis swaps)
3M USD LIBOR		(3M USD LIBOR IRS) 3M USD LIBOR swap curve is directly quoted (e.g. Bloomberg, Reuters).	3M/6M USD LIBOR tenor basis swaps are directly quoted (e.g. Reuters).	FED Funds/3M USD LIBOR tenor basis swaps are quoted (e.g. Reuters).
6M USD LIBOR			(6M USD LIBOR IRS) 6M USD LIBOR swap curve is not directly quoted.	Are not quoted (no contributors who quote FED Funds/6M tenor basis swap).
FED Funds swap				FED Funds swap curve is directly quoted (e.g. Bloomberg, Reuters).

The analysis concerning tenor basis swaps needs to be performed for each rate and currency. Throughout the paper only USD and EUR are considered.

Table 9 shows the available tenor basis spreads for USD LIBOR rates. As shown, e.g. the 6-month USD LIBOR is not directly quoted by all market data and price providers[38], therefore the 6-month USD LIBOR is constructed by using the 3-month USD LIBOR and the 3-month/6-month USD LIBOR basis spread curve. Economically the construction of such a curve uses the absence of arbitrage principle and assumes an integrated market for the 3-month, 6-month USD LIBOR swaps as well as the 3-month/6-month USD LIBOR tenor basis swaps. The lack of quotes in some price information providers does not necessarily imply the illiquidity of the corresponding derivatives since they are traded OTC derivatives.

Remarks on Specifications:

- In the case of tenor basis swaps FED Funds vs. USD LIBOR it has to be noted that the average on the FED Funds leg is calculated as the arithmetic mean, not the geometric (compounded) one as in the OIS.
- For USD LIBOR swaps there are separate quotes referring to swaps with semi-annual payments on the fixed side.
- For swaps in the interbank market, US market makers use the quotation convention as spreads over the on-the-run treasury bonds; they even use treasury bonds to hedge swaps applying suitable hedge ratios.

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38 Reuters e.g. requires three contributors to form composite quotes.

To complete the presentation the available EUR tenor basis swaps are provided in *Table 10.*

As indicated in *Table 8* there are different quotations for tenor basis swaps e.g. for USD LIBOR and EUR LIBOR: in the former case a tenor basis swap is a float-to-float instrument with accrued payments to the longer tenor and a fixed (tenor basis) spread on the leg with shorter term, whereas in the latter case the quotation relates to the joint trade of two fixed-to-float IRS with different tenors, and the tenor basis spread is the difference of the rates on the fixed legs. As demonstrated in Filipović, D. and Trolle, A. (2011) the differences resulting from the different quotation conventions are small.

In the following sections multi-curve setups and their impact on hedge accounting (including effectiveness testing) are described. Since tenor basis spreads represent an additional risk factor, the questions arise how it will be characterized and what impact it will have on effectiveness testing. In the following an example for real quotes is given using

TABLE 10: Available Tenor Basis Swaps for the EURIBOR and EONIA Swap

	1M EURIBOR	3M EURIBOR	6M EURIBOR	EONIA swap
1M EURIBOR	1M EURIBOR interest rate swap curve is directly quoted (e.g. Reuters).	1M/3M EURIBOR tenor basis swaps are quoted (e.g. Reuters).	1M/6M EURIBOR tenor basis swaps are quoted (e.g. Reuters).	Not quoted.
3M EURIBOR		3M EURIBOR interest rate swap curve is directly quoted (e.g. Bloomberg, Reuters).	3M/6M EURIBOR tenor basis swaps are quoted (e.g. Reuters).	EONIA/3M EURIBOR tenor basis swaps are quoted (e.g. Reuters).
6M EURIBOR			6M EURIBOR interest rate swap curve is directly quoted (e.g. Bloomberg).	Not quoted.
EONIA swap				The EONIA swap curve is directly quoted (e.g. Reuters, Bloomberg).

those of Tullett Prebon quoted by Bloomberg: the difference of the individual rates for 2-year OIS (fixed vs. FED Funds) and 2-year swap with 3-month tenor (fixed vs. 3-month USD LIBOR) are compared to the quoted 2-year tenor basis swap spread FED Funds vs. 3-month USD LIBOR. Absolute differences are shown in *Figure 31* and the monthly changes of the differences in *Figure 32*. The regression reveals a close relationship between the differences of the monthly changes ($\text{correlation} = \sqrt{0.98} \approx 0.9899$).[39]

FIGURE 31: Regression Analysis 2-Year FED Funds / 3M USD LIBOR Tenor Basis Spread Quotes vs. Differences in the 2-Year FED Funds Swap and 3M USD LIBOR Swap Rates (in bp) (Source: Reuters)

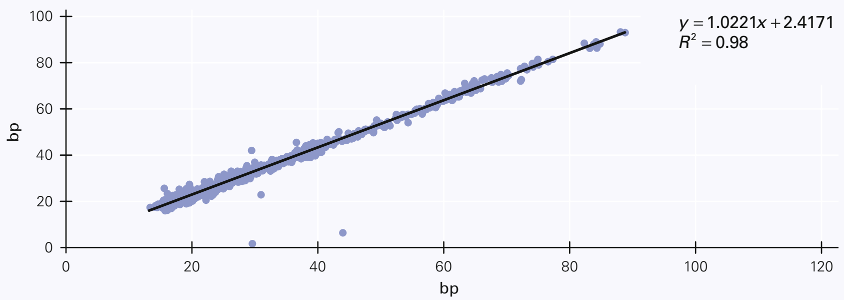
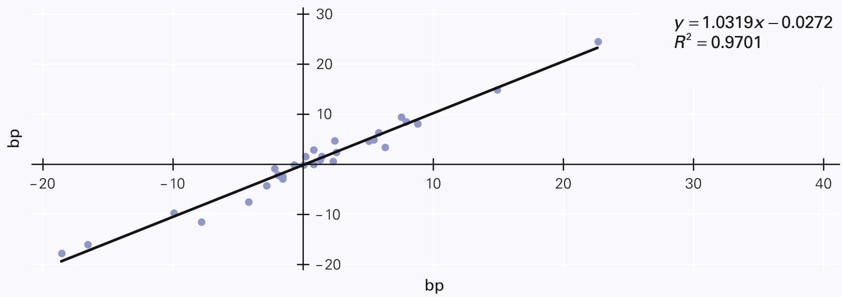


FIGURE 32: Regression Analysis of Monthly Changes of the Tenor Basis Spread Quotes and Differences in the Swap Rates (in bp) (Source: Reuters)



39 Depending on the convention the average calculation on the FED Funds leg of the tenor basis swap might differ from that of the floating leg in the FED Funds swap.

The corresponding development of the 2-year tenor basis spread FED Funds vs. 3-month USD LIBOR is given in *Figure 33*.

Figure 34 shows the 5-year 6-month EURIBOR vs. OIS basis spread, similarly to the other figures above the financial market crises caused an increase in basis spreads and the basis spreads did not return to an insignificant level in the post crises period. This corresponds to the picture drawn for the EURIBOR and EONIA rates themselves as depicted in *Figure 29*.

FIGURE 33: **Development of the 2-Year Tenor Basis Swap FED Funds vs. 3M USD LIBOR (in bp)** (Source: Reuters)

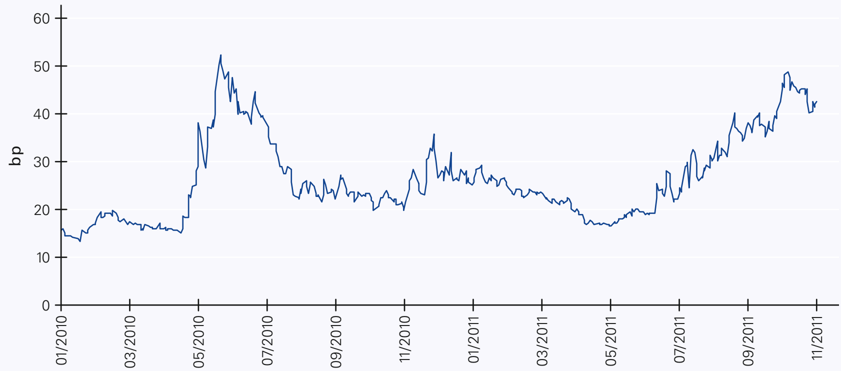


FIGURE 34: **Development of the 5-Year Tenor Basis Swap EONIA vs. 6M EURIBOR (in bp)** (Source: Bloomberg)

3.2.2 Cross Currency Basis Swaps

Hedging and hedge accounting of FX risk is a very important topic in the financial industry. Cross currency basis swaps represent one of the most important derivatives (hedging instruments) for this kind of risks. They exchange variable interest cash flows of two different currencies. The relevant quote is the constant running spread (cross currency basis spread or FX basis spread[40]) which has been proven to be significant and is added on the "less liquid" (mostly non USD) leg. As will be shown below, cross currency basis swaps in combination with interest rate swaps form the components of e.g. fixed-to-float cross currency swaps, which are used to hedge interest rate and FX risk simultaneously.

Cross currency basis swaps are traded separately in an individual derivative market and its conventions (refer to *Table 11*) as well as the legal framework follows the interest rate swaps listed above. To reduce the risk exposure, cross currency (basis) swaps are dealt with a resettable feature adjusting the notional of one leg to the current exchange rate on each reset date and generating a corresponding cash flow (mark-to-market-CCS)[41].

TABLE 11: Market Conventions for Cross Currency Basis Swaps

	FX derivative market
	Cross currency basis swap
Definition	Example: 3M EURIBOR/3M USD LIBOR with constant currency basis spread on the EUR leg
Market	Worldwide
Side	Offer and bid
Rate quotation specs	Individual
Maturities	1y–30y
Publication time	During the whole day
Panel banks	The same as OIS
Calculation method	The same as OIS
Calculation agent	Bloomberg, Reuters
Collateral	Secured and unsecured. Not all trades are collateralized (secured). 78% of all OTC derivatives transactions are covered by a CSA "Collateral Agreements". 80% of all collateral exchanges are in cash. Collateral interests depend on the agreement. (ISDA Margin Survey, 2010)
Counterparty risk/ liquidity risk	Yes/yes

40 The terms "cross currency basis spread" and "FX basis spread" will be used synonymously in this document.

41 Described in more detail e.g. in Fujii, M. et al. (2009).

Figure 35 shows the changes in the cross currency basis spread (FX basis spread) of a 3-month EURIBOR / 3-month USD LIBOR basis swap. As is apparent from the figure, the FX basis has become negative and significant after the financial crises. It even was significant in the early 90s. This statistical feature represents the starting point for multi-curve setups since the derivative market and its market participants demanded consistent pricings of interest rate swaps and cross currency basis swaps. Therefore *Section 5* begins with the pricing and hedge accounting of FX risk and the results are generalized with respect to OIS discounting in *Section 6*.

As stated in the Bank for International Settlement (BIS) Quarterly Review, March 2008[42] the cross currency basis swaps have greater liquidity than straight FX swaps throughout all maturities of one year or more. For that reason the cross currency market data was used for tests of long-term covered interest parity (CIP). The CIP is based on no-arbitrage arguments and states that the ratio of the FX forward and FX spot rate equals the ratio of the corresponding discount factors in the respective currencies. FX forward rates are also quoted for maturities of one year and more where the quotation is usually represented by swap or forward points in the unit of basis points that are added to the current spot rate similar to the quotation of forward rates for interest rates.

3.2.3 Statistical Facts of the Derivative Markets – Outstanding Notionals and Turnover

As OTC markets are not as transparent as exchange markets, it is more difficult to find statistical data. On the website of the BIS the amounts of outstanding OTC contracts are published to provide an overview over the number and notional amounts of interest rate contracts.[43] Because only some OTC parties and platforms provide their market data to aggregate to a financial market total, *Table 12* illustrates an indication of market activity in the derivative market.

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42 www.bis.org/publ/qtrpdf/r_qt0803h.pdf.

43 www.bis.org/publ/otc_hy1105.pdf, www.bis.org/publ/otc_hy1205.pdf.

FIGURE 35: 3M EURIBOR vs. 3M USD LIBOR Cross Currency Basis Spreads (5-Year Maturity, in bp) (Source: Bloomberg)

TABLE 12: Outstanding Notional Amount of Derivative Contracts (in Billions of USD per Half Year)

		H1 2009	H2 2009	H1 2010	H2 2010	H1 2011	H2 2011
	FX contracts	48,732	49,181	53,153	57,796	64,698	63,349
	Interest rate contracts	437,228	449,875	451,831	465,260	553,240	504,098
	Equity-linked contracts	6,584	5,937	6,260	5,635	6,841	5,982
	Commodity contracts	3,619	2,944	2,852	2,922	3,197	3,091
	Credit default swaps	36,098	32,693	30,261	29,898	32,409	28,633
FX contracts	Outright forwards and forex swaps	23,105	23,129	25,624	28,433	31,113	30,526
	Cross currency swaps	15,072	16,509	16,360	19,271	22,228	22,791
	Options	10,555	9,543	11,170	10,092	11,358	10,032
Interest rate contracts	FRAs	46,812	51,779	56,242	51,587	55,747	50,576
	Swaps	341,903	349,288	347,508	364,377	441,201	402,610
	Options	48,513	48,808	48,081	49,295	56,291	50,911

TABLE 13: Turnover Analysis – Notional Amounts of Derivative Contracts
(Index: the OTC derivatives volume in 2002 = 100)

Year	OIS	FX swaps	Other	CCY swaps	FRAs	Sum	Change relative to 2002
2002	35.98	41.91	10.47	0.66	10.98	100.00	
2003	80.26	65.03	17.25	1.37	17.13	181.05	81.05
2004	54.45	59.63	23.47	1.01	17.56	156.12	56.12
2005	58.78	57.48	24.70	0.71	13.08	154.75	54.75
2006	89.47	77.92	32.60	0.86	18.83	219.69	119.69
2007	73.56	80.23	30.20	0.94	17.44	202.37	102.37
2008	51.50	85.12	43.41	1.45	35.51	217.00	117.00
2009	43.37	86.14	35.50	2.40	44.62	212.01	112.01
2010	34.85	88.50	33.19	2.32	40.12	198.98	98.98
2011	49.57	89.31	45.54	3.90	47.00	235.33	135.33

TABLE 14: Average Daily Turnover in the Cross Currency Basis Swap Segment
(Index: the OTC derivatives volume in 2002 = 100)

Date	Up to 2 years	2 years to 5 years	5 years to 10 years	More than 10 years	Sum	Change relative to 2002
2002	44.46	34.58	16.03	4.93	100.00	
2003	108.90	23.88	48.04	26.39	207.21	107.21
2004	51.14	46.30	35.95	19.29	152.68	52.68
2005	25.25	42.98	29.43	10.52	108.18	8.18
2006	41.19	38.40	27.19	23.90	130.68	30.68
2007	26.90	51.12	43.06	21.84	142.92	42.92
2008	79.69	95.02	36.45	8.75	219.91	119.91
2009	82.20	153.41	91.43	35.59	362.63	262.63
2010	150.43	118.58	52.99	29.40	351.40	251.40
2011	351.86	134.13	72.96	31.56	590.51	490.51

On the website of the European Central Bank (ECB) the turnover analysis of the derivatives market is published, which serves as a measure of liquidity in the derivative market.[44] The panel comprised 85 credit institutions in 2000 and 2001 and 105 credit institutions thereafter. *Table 13* shows an increase in turnover of 135% in 2011 in comparison to 2002. In *Table 14* a detailed turnover analysis of the cross currency basis swap segments split into maturities is given.

The statistical evidence indicates the heterogeneity of derivative markets as well as its importance to the financial industry according to their traded volumes. With respect to valuation issues for financial accounting or economic P&L (e.g. trading P&L) purposes the market considers the derivative market as the most reliable source of prices.

The derivative markets have to be distinguished from the cash markets (e.g. bond markets) where market conventions and participants are notably different. In order to illustrate these differences between the derivative and bond market segment data and price providers are illustrated in the following subsection.

3.3 Cash Markets

Financial accounting and the evaluation of the economic P&L relies upon market data and price providers (e.g. Reuters, Bloomberg). Like in case of the money market rates, market data and prices for bonds are subject of economic modeling. In the following an example is provided in order to show that the prices of price providers do not only result from contributors. Additionally, prices or quotes are evaluated utilizing different models.

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44 www.ecb.int/pub/pdf/other/euromoneymarketsurvey201109en.pdf

Bond prices that Bloomberg receives from different price providers can be classified into

- **Indicative prices:** market makers have no obligation to execute trades at indicative prices, so it is not unusual to find indicative prices different from the actual market prices.
- **Executable prices:** available only for bonds traded on some electronic trading platforms.
- **Traded prices:** prices of actual trades.

This "input data" is utilized to construct the following two types of quotations for a bond:

- Bloomberg Generic (BGN) Price: The simple average of all kinds of prices, listed above, which are provided by the price contributors over a specified time window. The availability of BGN Prices for a bond is an indication of high liquidity for that bond.
- Bloomberg Fair Value (BFV): Model price of a bond. The BFV is calculated by utilizing bonds from Bloomberg Generic with similar characteristics (for example with comparable currency, market type, maturity, industry and credit rating). This serves as an indication where the price of a bond should trade.

Table 15 summarizes the market description and conventions of cash market instruments like bonds and FX spot rate.

Comparing derivative markets (e.g. outstanding notional *Table 12*) with outstanding debt for bond markets (refer to *Table 16*) shows the greater size of the derivative market measured by outstanding notional. According to publications of the Asset Allocation Advisor in December 2010[45] (November 2009[46], respectively) bond markets are even greater than stock markets but both much less in size (according to outstanding notional) than the swaps market (or derivative markets) as shown above.

TABLE 15: Conventions and Description of the Bond and FX Spot Rate Market

	Cash market		
	Bond market		FX market
	Government bond (treasury bond)	Corporate bond	FX spot rate
Description	Bond issued by a national government	Bond issued by a corporation	The rate of a FX contract for immediate delivery
Market	Worldwide	Worldwide	Worldwide
Side	Offer and bid	Offer and bid	Offer and bid
Maturities	3m–50y	Short and long term	...
Publication time	During the day	During the day	During the day
Panel banks	Different contributors	Different contributors	Different dealers
Calculation methods (bench-marks)	E.g. Bloomberg Generic Price, Barclays Capital Aggregate, Citigroup BIG, Merrill Lynch Domestic Master, Markit IBoxx Indices	E.g. BGN Price, Bloomberg Fair Value (BFV Barclays Corporate Bond Index, Dow Jones Corporate Bond Index)	E.g. effective exchange rate
Risks	Default risk, liquidity risk, currency risk, inflation risk	Default risk, liquidity risk, inflation risk, currency risk	...

TABLE 16: World Stock (Market Capitalization) and Bond Markets (Debt Outstanding) (in Billions of USD)

Market	Stock 2009	Bond 2009	Stock 2010	Bond 2010
USA	14.3	31.2	16.7	32.1
Euro area	6.5	23.6	6.0	24.9
Japan	3.5	10.7	3.5	12.9
China	4.9	2.4	6.7	3.0
Total	44.2	82.2	51.8	91.3

45 www.scribd.com/doc/63415142/world-stock-and-bond-markets-dec2010.

46 www.ohio.edu/people/prevost/fin%20443/chapter%201%20ancillary%20material/world_stock_and_bond_markets_nov2009.pdf.

3.4 First Results and Implications for Hedge Accounting under IAS 39

3.4.1 Market Segmentation

As discussed above market segmentation is characteristic for financial markets. In the following some examples concerning valuation and different representations of market segmentation are provided to demonstrate the difference of conventions and market pricings. In academic literature the predictive and explanatory power of interest rates with respect to bond prices is analyzed.[47] A review of term structure of interest rate models and its empirical analyses are beyond scope, but these studies have not revealed a direct connection between derivative and bond markets.

The following examples do not rely on sophisticated mathematical and statistical modeling, since this paper concentrates "only" on hedge accounting and derivative pricing. *Table 17* gives the terms and conditions of two floating rate notes denominated in USD and EUR.

TABLE 17: Example Terms and Conditions of EUR and USD Denominated Floating Rate Notes

Terms and conditions	EUR floating rate note	USD floating rate note
Country issuer	US	US
Currency	EUR	USD
Value date	04/05/2006	10/28/2005
Maturity date	04/05/2013	10/28/2015
1st coupon date	07/05/2006	01/30/2006
Rating	Moody's A3	Moody's A2
Coupon	3M EURIBOR	3M USD LIBOR
Issue price	99.809	100
Notional	1,000,000.00€	1,750,000.00$
Payment frequency	Quarterly	Quarterly
Day count convention	ACT/360	ACT/360

In the example the floating rate notes (defined in *Table 17*) and the floating part of interest rate derivatives are compared. By convention rating of the swap market is AA^{-}, the floating rates bear similar ratings as the swap market. The terms and conditions of the interest rate swaps are chosen similar to those of the floating rate notes.

47 Refer e.g. Fama, E. F., French, K. R. (1992), Fama, E. F., French, K. R. (1993) and Shiller, R. J. (1992).

FIGURE 36: **Regression Analysis of Monthly FFV Changes of the EURIBOR FRN and FV Changes of the Floating Side of an EONIA and a 3M EURIBOR Interest Rate Swap Respectively**

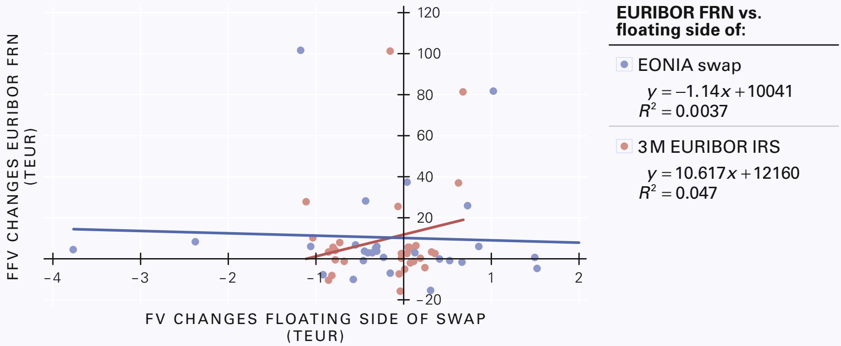

FIGURE 37: **Regression Analysis of Monthly FFV Changes of the USD LIBOR FRN and FV Changes of the Floating Side of a FED Funds and a 3M USD LIBOR Swap Respectively**

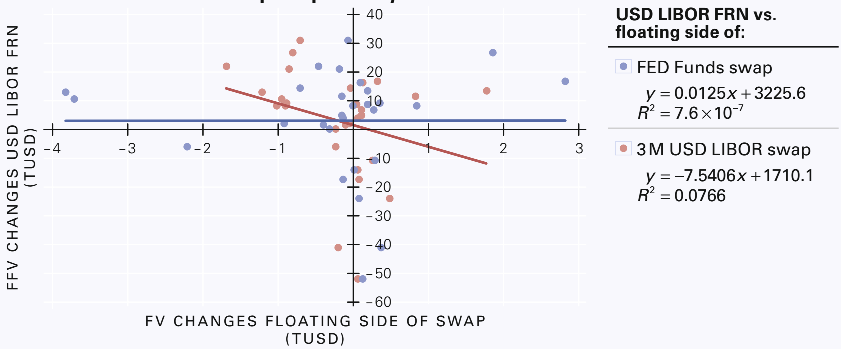

Figure 36 reveals that despite the fact that both – the 3-month EURIBOR floating side of the interest rate swap and the 3-month EURIBOR floating rate note – are tied to the identical 3-month EURIBOR rate, the pricings are different. The explanatory power of the monthly fair value changes of the 3-month floating side of interest rate swap with respect to the monthly full fair value (*FFV*) changes of the floating rate note is rather poor ($R^2 = 0.047$). This result does not change if an OIS is chosen, like *Figure 36* shows, and the slope is even negative. A similar analysis can be performed for the floating rate note denominated in USD – the results are shown in *Figure 37*.

The results are not surprising, since cash and derivative markets represent different markets with different market conventions, market participants, pricings etc. This difference is termed the "cash basis" for which the following aspects are of relevance:

- The poor explanatory power of interest rates derived from interest rate derivative for FFV changes will be of importance in connection with multi-curve approaches. As will be shown below, the absence of arbitrage principle works well in the derivative market, but statistically a direct connection between the cash market and the derivative market cannot be demonstrated (see *Figure 36* and *Figure 37*). So for example, if a cross currency basis swap can be replicated by interest rate swaps and the replication strategy can be statistically proven, so low explanatory power of the cross currency basis in the cash instruments is expected – apart from accidental statistical coincidence – since the cross currency basis "inherits" the low explanatory power of the interest rates derived from the derivative market. Similarly if the 3-month EURIBOR interest rate swap rate is of low explanatory power and the EURIBOR interest rate swap rate can be replicated by an OIS and a 3-month EURIBOR/OIS basis swap, both are – apart from accidental statistical coincidence – of low explanatory power to corresponding cash products.
- The "cash basis" is not only apparent in terms of interest rate hedging (hedge accounting according to IAS 39) using interest rate swaps but also for hedging models applying forwards, futures etc. Also in these cases the "payout" of the underlying coincides with the payout of the derivative but the prices and the pricings are different. This also affects other derivative contracts like cross currency basis swap that can be represented by means of FX forward rates as will be shown in *Section 5.*
- Please note that throughout the paper derivative markets and bond markets are empirically analyzed. Hedging and hedge accounting according to IAS 39/FAS 133 (US GAAP) is also performed for loans etc. for which hedge accounting is permitted if the

requirements are met, but since loans are not actively traded a "cash basis" cannot be analyzed.

3.4.2 Benchmark Curve Hedge Accounting Concept and Creation of Integrated Markets

For interest rate hedge accounting purposes according to IAS 39 or FAS 133 a "benchmark curve" needs to be defined. The "benchmark curve" defined from the derivative market is utilized to define the "hedged risk" (interest rate risk) and represents a discount curve which is used to evaluate the fair value of the hedged item (e.g. bond or loan) w.r.t. to the hedged risk and portion as well as for the determination of the corresponding booking entries. The benchmark curve[48] is also used as a discount curve evaluating the fair value of the hedging instrument (derivative) for the effectiveness testing as well as for the determination of the booking entries w.r.t hedge accounting.

In case of interest rate hedge accounting the "benchmark curve" is represented by the "swap curve" derived from interest rate derivatives. But the construction of the "swap curve" requires mathematical and economic modeling.[49] The "components" used in order to construct the "swap curve" are model-dependently chosen from money market rates, forward rate agreements, futures and interest rate derivatives like 3-month USD LIBOR swaps etc. As outlined above all these financial contracts are traded in separate market segments. The absence of arbitrage principle ("bootstrapping") is used to construct the entire "benchmark curve", and economically an integrated market amongst all the traded financial contracts used for the "benchmark curve" is created. It is important to note that the prices of the various financial contracts used for the "benchmark curve" are exogenously given and are not changed by the benchmark curve construction.

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48 Although IAS 39 does not explicitly prescribe to use the same benchmark curve for discounting the hedged item and the hedging instrument, the definition of the hedged risk and the application of a consistent multi-curve setup imply the usage of the identical discount curve for the hedged item and the hedging instrument.

49 Refer e.g. Schubert, D. (2011).

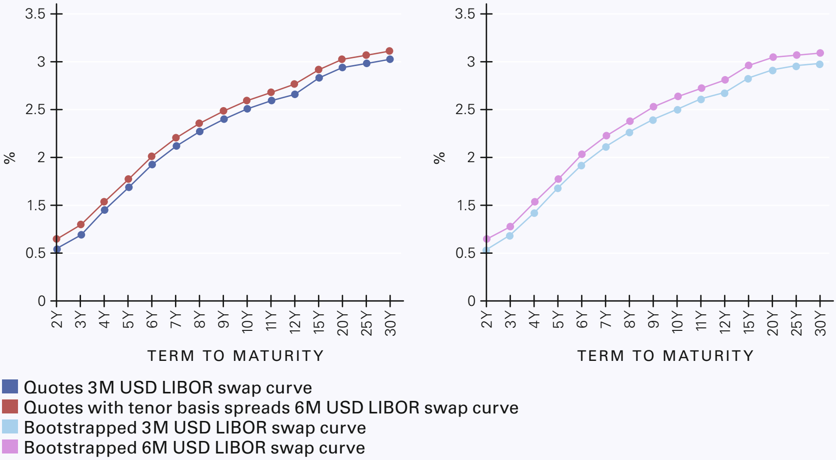
A simple example of the forming integrated markets: the 6-month USD LIBOR swap curve is not separately quoted in the market; it is constructed by using the 3-month USD LIBOR swap curve and the 3-month/6-month basis swap spreads. The 3-month/6-month basis spreads are added to the 3-month USD LIBOR curve and then the “bootstrapping” is performed to derive the 6-month USD LIBOR “benchmark curve”. Economically the 6-month USD LIBOR swap curve is constructed “synthetically” by forming an integrated market of two separately traded interest rate derivative contracts: 3-month USD LIBOR swaps and 3-month/6-month basis swaps. The absence of arbitrage principle is used twice: adding the tenor basis spreads on the 3-month USD LIBOR swap curve in order to arrive at the 6-month USD LIBOR swap rate[50] and the “bootstrapping” to model the entire 6-month USD LIBOR swap curve as discount curve (“benchmark curve”).

For illustration purposes only quotes and rates for maturities longer than 2 years are displayed in *Figure 38.* As mentioned above and also stressed in the presentation of Ametrano, F. (2011) for the generation of homogeneous curves, in particular for forward curves in a multi-curve setup, money market quotes and those of different tenor are avoided and in a more sophisticated approach homogeneous forward or future quotes are used to construct in particular the short end of homogeneous curves.

Creating an integrated market using economic modeling is not only a feature with respect to the construction of a “benchmark curve” but also a feature of hedge accounting models according to IAS 39/FAS 133.

Provided the requirements of IAS 39/FAS 133 are met, the “bench-mark curve” is utilized to determine the fair value of the bond or loan (hedged item) w.r.t. to the hedged risk and portion. Also the derivative (hedging instrument) is fair valued using the benchmark curve[51] as discount curve. Economically an integrated market for the hedging instrument and the hedged item is created.[52] Consequently the cash

FIGURE 38: Construction of the 6M USD LIBOR Swap Curve ("Benchmark Curve") Using 3M USD LIBOR Swap Curve and 3M/6M Tenor Basis Swaps

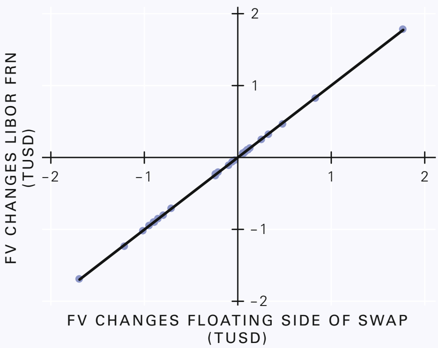


50 By definition an investor is indifferent between two portfolios: one consisting of 6-month USD LIBOR or a portfolio consisting of 3-month USD LIBOR and 3-month/6-month USD LIBOR basis swaps.

51 Although IAS 39 does not explicitly prescribe to use the same benchmark curve for discounting the hedged item and the hedging instrument, the definition of the hedged risk and the application of a consistent multi-curve setup imply the usage of the same discount curve for the hedged item and the hedging instrument.

52 For details of the derivation of this result and the underlying economic theory in connection with hedge accounting refer to Schubert, D. (2011).

FIGURE 39: Example Using One Benchmark Curve to Determine FV Changes of Derivative and Floating Rate Note (Cash Instrument)



basis is eliminated according to the hedge accounting model.[53] The absence of arbitrage principle represents the tool to define the portion of the hedged risk. Since the hedged item is "fair valued" according to the prices derived from the derivative market, the fair value adjustments equal the hedging costs associated with the hedged item.

In order to illustrate the impact of "fair valuing" using one discount curve ("benchmark curve") the example from above is continued (refer to *Figure 37*). As soon as the benchmark curve derived from the derivative market is used as a discount curve for the floating rate note, the cash basis is eliminated and, by construction, the fair value changes of the floating rate note and the floating side of the swap perfectly coincide (see *Figure 39*).[54]

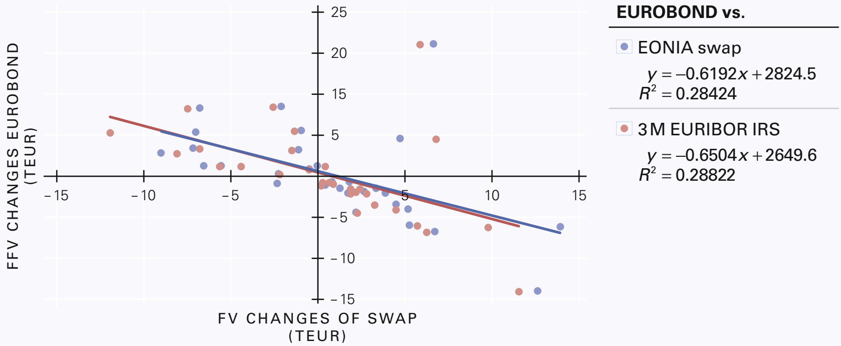
In the first step the fair value changes of the derivatives (3-month EURIBOR interest rate swap and OIS – hedging instruments) are compared with the FFV changes of the fixed rate bonds defined in *Table 18*.

Figure 40 reveals that the fair value changes of the interest rate derivatives (3-month EURIBOR and OIS) are of low explanatory power in comparison to the full fair value changes of the fixed rate bond (R^2 ranges between 0.2842 and 0.2882). The results are very similar to the regression analysis in connection the floating rate notes (refer e.g. to *Figure 36*). It reveals once more that there is no direct statistical connection between the derivative market and the cash market (bond market).

TABLE 18: Example Terms and Conditions of EUR and USD Denominated Fixed Rate Bonds

Terms and conditions	EUR fixed rate bond	USD fixed rate bond
Country issuer	DE	Netherlands
Currency	EUR	USD
Type	Fixed rate bond	Fixed rate bond
Value date	04/05/2005	10/28/2005
Maturity date	04/05/2013	10/28/2015
1st coupon date	04/05/2006	04/28/2006
Rating	Moody's A3	Moody's A3
Coupon	3.625%	5.3%
Issue price	99.45	99.785
Notional	1,000,000.00€	1,750,000.00$
Payment frequency	Annually	Semi-annually
Day count convention	ACT/ACT	ACT/ACT

FIGURE 40: Regression Analysis of the Fixed Rate EUROBOND vs. EONIA and 3M EURIBOR Interest Rate Swap



53 For brevity only fair value hedge accounting models are considered. Economic reasoning for cash flow hedge accounting models is similar. For details on the integrated market assumptions refer to Schubert, D. (2011) p. 15.

54 Clearly there is no fair value hedge accounting model for floating rate instruments defined according to IAS 39/FAS 133, but this illustrates the impact of "one" benchmark curve.

FIGURE 41: **Regression Analysis of the Fixed Rate Dollar Bond vs. FED Funds and 3M USD LIBOR Swap**

FIGURE 42: **Regression Analysis of the Fixed Rate EUROBOND FV Changes Due to the Hedged Risk (EONIA) vs. FV EONIA Swap and Due to the Hedged Risk (EURIBOR) vs. FV 3M EURIBOR Interest Rate Swap**

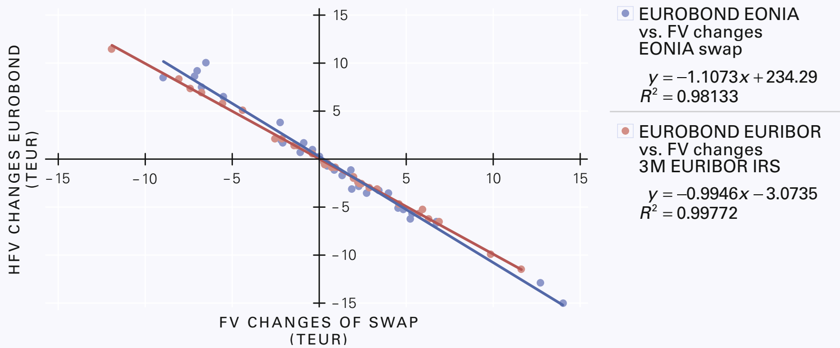
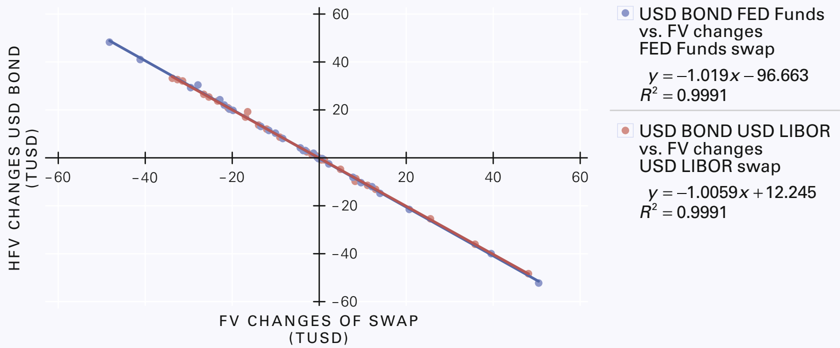


FIGURE 43: **Regression Analysis of the Fixed Rate USD BOND FV Changes Due to the Hedged Risk (FED Funds) vs. FV FED Funds Swap and Due to the Hedged Risk (3M USD LIBOR) vs. FV 3M USD LIBOR Swap**



As an addition a similar analysis is performed for the fixed rate USD bond which shows similar results (R^2 ranges between 0.4837 and 0.5038).

On the other hand, when the fixed or floating rate instruments are measured w.r.t. the hedged risk (this incorporates a valuation model!), i.e. using the swap curve in the discounted cash flow method, a high degree of effectiveness and explanatory power results. In the examples the slope ranges between −0.9946 and −1.1073 and R^2 between 0.9813 and 0.9977, as shown in *Figures 42* and *43*.

Observing the low explanatory power of the movements of the hedging swap with respect to the FFV of the hedged item as shown in *Figure 40* and *Figure 41,* the question poses itself whether the benchmark interest rate is actually contained in the contractual coupon. Although there might be arguments like the cash flows representation of asset and asset swaps as synthetic floater of a benchmark rate (e.g. 3-month EURIBOR) plus asset swap spread that vote for it. This is no strict proof since the focus is on interest payments, but the valuations of the asset and the "asset" leg of the interest rate swap represent the difference of the bond to the derivative market. The valuation of the floating leg of the interest swap including the asset swap spread, which is rather an element of the derivative market, will be done in the derivative market.

3.4.3 Summary of Implications to Hedge Accounting under IAS 39

The analysis above revealed that the utilization of "market prices" implies the use of economic models. The simplest example is the determination of money market quotes, which are subject to economic modeling.[55] As a consequence market prices from market data and price providers (like Reuters, Bloomberg) already represent "model

55 Additional and implicitly assumed economic models are used in financial accounting in connection with fair valuing: efficiency of market hypothesis and homogenous expectations of all market participants.

prices". These "prices" or "quotes" form the basis for constructing the benchmark curves for interest rate risk. Since interest rate risk is an unobservable risk, this risk has to be derived from "quoted" (liquid, traded) financial contracts.

There is a variety of generators derived from separate financial markets (financial contracts) necessary to construct the benchmark curve: money markets and future contracts, forward rate agreements, tenor basis swaps and interest rate swaps. These different financial contracts are tied to market conventions and a legal framework. For derivatives the ISDA Master Agreement (2002) forms the most common legal framework, which, amongst other important contracual features, defines e.g. counterparty credit risk. Since the construction of a benchmark curve requires separately traded and liquid financial contracts, credit risk and counterparty risk is an integral part of a benchmark curve, which cannot be separated. Consequently there is no "risk-free" rate available in the market. Even before the financial market crisis, in view of the definition of e.g. the EURIBOR rates interbank risk was present in the market quotes. According to the ISDA Master Agreement (2002) and the CSA collateral postings in the derivative market, especially in the interbank market, counterparty risk is effectively reduced. Consequently, provided two counterparties entered into a CSA, an EURIBOR interest rate swap or an OIS contract legally represent the same counterparty credit risk. This has to be distinguished from the question wether EURIBOR or EONIA rates represent a risk-free rate; according to the market conventions in the money market both are exposed to credit risk, since the money market represents a market for unsecured lending and borrowing.

As a consequence the derivative market represents the "best available" source for liquid traded and "risk-free" rates. The derivative market itself is subdivided into several "sub-" markets: OIS interest rate swaps, EURIBOR interest rate swaps, tenor basis swaps, cross currency basis swaps etc. As outlined above, the construction of a benchmark curve requires the construction of an integrated market model for derivatives utilizing the absence of arbitrage principle. If e.g. for the construction of short term forward rates and/or futures are chosen the benchmark curve represents a market model integrating the forward and/or future market.

The modeling idea of constructing integrated markets and a benchmark curve using the absence of arbitrage principle is a commonly applied conception for the valuation of derivatives. In applying hedge accounting of interest rate risk according to IAS 39/FAS 133 a benchmark curve is defined which is derived from the derivative market; in order to determine the hedged risk, the fair values of the hedging instrument as well as the hedged item and to perform the effectiveness testing, an integrated market model for derivatives and cash products (hedged items) is created. As shown above, the derivative market is different from the cash market, e.g. the bond market, and has a much higher trading volume. By assigning one benchmark curve to all financial contracts involved in the hedge accounting model according to IAS 39/FAS 133, the "Law of One Price" is defined, accordingly hedge accounting incorporates a relative valuation model: "Given" the benchmark curve, the hedge items and the hedging instruments (interest rate swaps, options, in-arrear features etc.) are fair valued relatively to the benchmark curve[56] derived from the derivative market. As a consequence e.g. the benchmark curve valuation conception does not cover the question of interest rate prediction or the fundamental analysis of interest rates. According to the reliance on the derivative market with respect to the benchmark curve conception, any speculation in the derivative market does not result in ineffectiveness in the hedge accounting model, since the cash basis (difference between derivative and cash market) has been eliminated and market segmentation has been resolved. Consequently market conventions in the derivative market are imputed into the hedged items (cash instruments). As outlined above, in statistical

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56 Although IAS 39 does not explicitly prescribe to use the same benchmark curve for discounting the hedged item and the hedging instrument, the definition of the hedged risk and the application of a consistent multi-curve setup imply the usage of the identical discount curve for the hedged item and the hedging instrument.

terms the explanatory power of the derivative market (interest rate swaps) in comparison to the cash market is very poor. So even if the payout of the cash product is tied to EURIBOR, the prices in the cash market do not reveal a direct connection to the derivative market. As a consequence it cannot be statistically proved that the benchmark risk is included in the hedged item (e.g. bond)[57]. This is an important statistical property of hedge accounting models, since the poor explanatory power in case of interest rate hedge accounting will also affect hedge accounting models involving FX risk as well as the multi-curve setup, which can be considered as a generalization of an integrated market model for all derivatives and cash products (hedged items).

In the following *Table 19* the major economic properties of interest rate hedge accounting are summarized, which hold for cash flow hedge accounting as well as fair value hedge accounting.[58]

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57 This is why the accounting standards have the concept of a benchmark risk, because "interest rate risk" cannot be separately identified by reference to the terms of the bond.

58 For more detailed description refer to Schubert, D. (2011).

TABLE 19: Summary of Major Economic Properties of Interest Rate Hedge Accounting and Financial Economics

Hedged risk	
Hedge accounting of interest rate risk	**Financial economics**
– Not a directly observable risk, since interest rate risk itself is not a traded instrument. – Not contractually specified (hedged item). – Defined by the benchmark curve derived from a liquid derivative market. Benchmark curves are homogeneous w.r.t. the tenor, i.e. also the hedged risk is tenor specific. – The reliance on the derivative market implies the reliance on the various definitions according to the ISDA Master Agreement (2002) (e.g. counterparty risk). – Does not cover the entire interest rate risk of the financial instrument.	The reliance to the derivative market implies: – Usage of economic models with respect to market quotations. – Introduction of the "Law of One Price" represented by the derivative market. – Assignment of market conventions from the derivative market into the cash markets (hedged items).

TABLE 19: **Summary of Major Economic Properties of Interest Rate Hedge Accounting and Financial Economics** *(continued)*

Hedged item (IAS 39.AG99F)

	Hedge accounting of interest rate risk	**Financial economics**
Portion (deterministic cash flow profiles)	E.g. LIBOR/EURIBOR/OIS (or a combination of these in a multi-curve setup) component only defined by the benchmark curve with its inherent rules of discounting and forwarding.	– Absence of arbitrage. – Completeness of markets. – Integrated market for hedged items and hedging instruments through the common benchmark curve (derived from liquid market of hedging instruments) leading to the elimination of basis risk between cash and derivative market. – Determination of a (cash flow) component attributable to the designated risk by the derivative market. – Statistics/econometrics: poor explanatory power of derivative induced prices in comparison to the cash market (market for the hedged items). – Statistically no direct link from the derivative market to the cash market can be shown (even in the classic single-curve approach!).
Separately identifiable		Identification of the portion of hedged risk by the derivatives defining the hedged risk that are used in the derivation of the "benchmark curve" – (derivative) zero EURIBOR/LIBOR/OIS rates utilized for discounting; coincides with the portion of cash flows in the single-curve approach.
Reliably measurable		Existence of a **liquid market for the** derivatives to derive the "benchmark curve", e.g. interest rate swaps (derivatives) based on EURIBOR/LIBOR/EONIA that covers all relevant market data to evaluate the portion of the hedged item attributable to the designated risk.

Construction of the benchmark curve

Hedge accounting of interest rate risk	**Financial economics**
Benchmark curve is derived from liquid market quotes: – Benchmark curve is derived from interest rate swaps and forward rate agreements/futures (for short term). – Using bootstrapping/interpolation to derive the "zero rates" using a valuation model (discounted cash flow model). – Calibration of the constructed benchmark to meet the quoted (input) market prices. – Benchmark curve bears credit risk (e.g. market standard for LIBOR/EURIBOR/OIS: AA^- rating). – Used for both: the valuation of the hedging instrument as well as the determination of the hedge fair value of the hedged item.	Creates an integrated market for the components of the benchmark curve chosen from money market products, futures, forward rate agreements, interest rate swaps and tenor basis swaps across different markets, market conventions (e.g. definitions of counterparty and credit risk). Only calibration techniques which recover initial data can be applied.

TABLE 19: Summary of Major Economic Properties of Interest Rate Hedge Accounting and Financial Economics *(continued)*

Valuation of the hedging instrument (deterministic cash flows)

Hedge accounting of interest rate risk	Financial economics
– Standard fair value measurement is applied using current market data. – For some tenors, market quotes are available for interest rate swaps. Market quotes for other tenors are determined by interpolation.	Fundamentals of derivative pricing: – Absence of arbitrage. – Completeness of markets. – Integrated (derivative) market for hedging instruments through the defined benchmark curve (derived from liquid market of hedging instruments).

Determination of the hedge fair value

Hedge accounting of interest rate risk	Financial economics
Determination of the changes in the fair value of the hedged item attributable to the hedged risk and relative to the state at inception: valuation carried out by using the current "zero" swap curve, which varies over time but leaving credit spread and margin constant as of inception date.	The cash basis is eliminated and the hedged items are fair valued relative to the benchmark curve defined by derivatives (see above). Therefore the hedged item is fair valued according to its hedging costs.

Effectiveness assessment / ineffectiveness measurement

Hedge accounting of interest rate risk	Financial economics
– The hedged item is priced according to its hedging costs ("fair value according to the hedged risk"): "cash flow profile of the hedged item times value derived from the benchmark curve". This is not a "short-cut" since the hedge fair value is calculated with respect to the terms and conditions of the hedged item! – Changes in the fair value of the hedged item attributable to the hedged risk (hedging costs) are compared to the fair value of the hedging instruments. – Ineffectiveness arises if terms or conditions of the hedged item do not correspond with those of the hedging instrument (or vice versa).	

SECTION FOUR

4

An Introduction: Relation of Multi-Curve Approaches and Hedge Accounting (IAS 39)

4.1 Introduction

In order to portray the basic mechanics of a multi-curve valuation approach using different curves for forwarding and discounting and its impact on hedge accounting, a simple example is considered: The example assumes a hedging relationship with deterministic cash flows and a change from the 6-month EURIBOR (assumed in the single-curve case) to the 3-month EURIBOR (multi-curve case) discount curve. As it will become apparent three "dimensions" of representations associated with the hedging relationship have to be distinguished:

1. Representation of contractual cash flow profiles,
2. Representation of risk and valuation factors,
3. Representation of the hedge accounting model.

All three types of representation need to be considered in order to assess the economic hedging and replication strategy associated with the hedge. Furthermore the three representations will serve to introduce the mathematical framework of multi-curve. Moreover it is assumed that the refinancing position is unaffected by the change in discount curve and remains on a 6-month EURIBOR basis.

4.2 Hedge Accounting in a Single-Curve Model

In order to assess the impact of the change of the discount curve, the decomposition of cash flows of a plain vanilla loan or bond is performed according to the 6-month EURIBOR and remaining components of the contractual coupon which is shown in *Figure 44*.[59]

In *Figure 44* a valuation model employing an equilibrium assumption is used to determine the risk-equivalent synthetic bond/loan bearing only the EURIBOR component subject to hedge accounting as fixed coupon. The cash flows of this synthetic bond/loan equal the coupon payments according to the fixed 6-month EURIBOR interest rate swap rate for the maturity of the bond/loan and the notional repayment of the original bond/loan.

4.2.1 Step 1: Representation of the Contractual Cash Flow Profiles Associated with the Hedging Relationship

Using the decomposition of the cash flow profile for the bond (hedged item) above and the decomposition of the 6-month EURIBOR interest rate swap into a fixed rate bond and a floating rate bond, the representation of the cash flow profile is associated with the economic hedging relationship including the floating funding on 6-month EURIBOR basis (see *Figure 45*).

In accordance with the interest rate risk considered the representation of interest rate sensitivity view is chosen: interest payments not yet fixed (floating funding and floating leg of the interest rate swap) are displayed by broken frames. According to the representation of cash flow profile apparently the sum of the overall cash flows sums up to zero.

FIGURE 44: Decomposition of Cash Flows and the Construction of a Risk-Equivalent Synthetic Bond / Loan

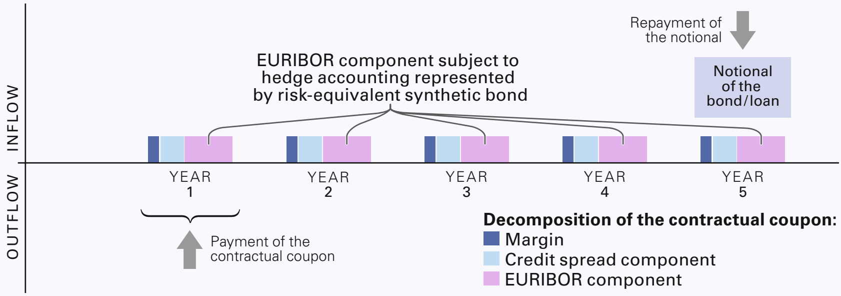
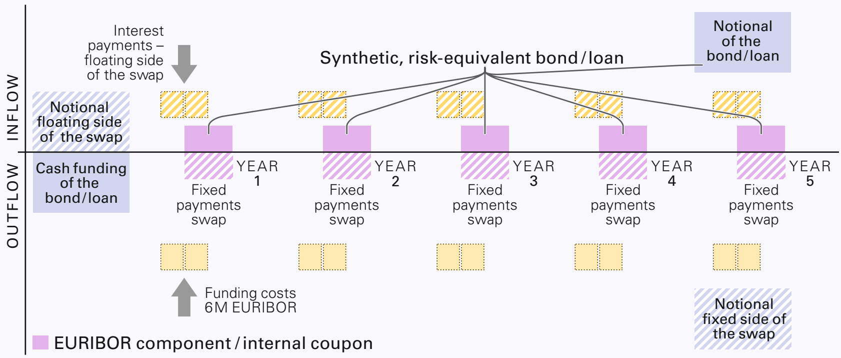


FIGURE 45: Representation of Cash Flows using the Construction of a Risk-Equivalent Synthetic Bond / Loan in a Single-Curve Model



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59 In other words, the cash flows exclude the credit spread and margin components of the coupon cash flows. For further details refer to Schubert, D. (2011).

4.2.2 Step 2: Representation of Risk and Valuation Factors

In order to portray the impact of valuation some notation is introduced:

In the example it is assumed that the discount curve is the 6-month EURIBOR interest rate curve, which is constructed using 6-month EURIBOR interest rate swaps for the maturities of more than one year, and the short end is constructed by money market instruments. Therefore forwarding and discounting is performed on the identical curve.[60]

Let

$$B^{6M}(t,T) := \frac{1}{\left(1+6\text{M EURIBOR zero swap}(t,T)\right)^{T-t}}$$

denote the zero swap rates ("zero bonds") in particular at times $t = T_0 = t_0, T_1 \ldots, T_N = T$ for annual payment or $t = t_0, t_1 \ldots, t_{2N} = T$ for semi annual payments, respectively, for the maturity in T. Accordingly the fixed 6-month EURIBOR interest rate swap rate at $t = t_0$ with maturity at T is defined by:

EQUATION 1: Definition of a 6M EURIBOR Interest Rate Swap Rate

$$c^{6M}(t_0,T) := \frac{\left(1 - B^{6M}(t_0,T)\right)}{\sum_{k=1}^{N} \Delta(T_{k-1},T_k) \cdot B^{6M}(t_0,T_k)} := \frac{\left(1 - B^{6M}(t_0,T)\right)}{A^{6M}(t_0,T)}$$

where $\Delta(T_{k-1},T_k)$ denotes the time fraction between the two dates indicated in the corresponding day count convention and

$$A^{6M}(t_0,T) := \sum_{k=1}^{N} \Delta(T_{k-1},T_k) \cdot B^{6M}(t_0,T_k)$$

is used as abbreviation for the (weighted) sum over the discount factors on the annual interest payment dates.

---

60 For simplification and because it has been the common approach so far, not a strictly "homogeneous" curve as stated in *Section 2,* but a curve made up of money market and swap quotes is used.

6-month EURIBOR forward rates $f^{6M}\left(t_0,t_{j-1},t_j\right), j \geq 2,3,4,\ldots,2N$, with $t_{2N}=T$ are defined for the 6-month tenor by:[61]

$$B^{6M}\left(t_0,t_j\right)=B^{6M}\left(t_0,t_{j-1}\right)\cdot\frac{1}{\left(1+\delta(t_{j-1},t_j)f^{6M}\left(t_0,t_{j-1},t_j\right)\right)},$$

where $\delta\left(t_{j-1},t_j\right)$ denotes the time fraction between the indicated reset dates according to the corresponding day count convention.

A floating rate note $FRN^{6M}\left(t_0,T\right)$ with respect to the 6-month tenor at $t=t_0$ with maturity at T can be valued by using the forward rates:

$$\begin{aligned}FRN^{6M}\left(t_0,T\right)=&\,\delta\left(t_0,t_1\right)\cdot\underbrace{r^{6M}\left(t_0\right)}_{\substack{\text{6M EURIBOR}\\ \text{spot rate}}}B^{6M}\left(t_0,t_1\right)\\&+\delta\left(t_1,t_2\right)f^{6M}\left(t_0,t_1,t_2\right)\cdot B^{6M}\left(t_0,t_2\right)+\ldots\\&+\delta\left(t_{j-1},t_j\right)f^{6M}\left(t_0,t_{j-1},t_j\right)\cdot B^{6M}\left(t_0,t_j\right)+\ldots\\&+\left(1+\delta\left(t_{2N-1},t_{2N}\right)f^{6M}\left(t_0,t_{2N-1},t_{2N}\right)\right)\cdot B^{6M}\left(t_0,t_{2N}\right)\\=&\left[\frac{1}{B^{6M}\left(t_0,t_1\right)}-1\right]\cdot B^{6M}\left(t_0,t_1\right)+B^{6M}\left(t_0,t_1\right)-B^{6M}\left(t_0,t_2\right)+\ldots\\&+B^{6M}\left(t_0,t_{j-1}\right)-B^{6M}\left(t_0,t_j\right)+\ldots+B^{6M}\left(t_0,t_{2N-1}\right)\\=&\,1\,.\end{aligned}$$

If the notional of the floating rate note is subtracted and only the interest payments are considered as in the floating leg $\Lambda^{6M}\left(t_0,T\right)$ of an interest rate swap at $t=t_0$ with maturity at T, then for:

$$\begin{aligned}\Lambda^{6M}\left(t_0,T\right):=&\,\delta\left(t_0,t_1\right)\cdot\overbrace{r^{6M}\left(t_0\right)}^{\substack{\text{6M EURIBOR}\\ \text{spot rate}}}B^{6M}\left(t_0,t_1\right)\\&+\sum_{j=2}^{2N}\delta\left(t_{j-1},t_j\right)\cdot f^{6M}\left(t_0,t_{j-1},t_j\right)\cdot B^{6M}\left(t_0,t_j\right)\end{aligned}$$

61 Since the tenor of 6-month represents the money market range, linear compounding is used.

$$
\begin{aligned}
&= \delta(t_0,t_1)\cdot \overbrace{r^{6M}(t_0)}^{\substack{\text{6M EURIBOR}\\ \text{spot rate}}} B^{6M}(t_0,t_1) \\
&\quad +\delta(t_1,t_2) f^{6M}(t_0,t_1,t_2)\cdot B^{6M}(t_0,t_2)+\ldots \\
&\quad +\delta(t_{j-1},t_j) f^{6M}(t_0,t_{j-1},t_j)\cdot B^{6M}(t_0,t_j)+\ldots \\
&\quad +\delta(t_{2N-1},t_{2N}) f^{6M}(t_0,t_{2N-1},t_{2N})\cdot B^{6M}(t_0,t_{2N}) \\
&= 1-B^{6M}(t_0,t_{2N}) \\
&= 1-B^{6M}(t_0,T).
\end{aligned}
$$

The evaluation above shows, that if forwarding and discounting is performed by identical curves, the floating rate part of the interest rate swap including the repayment is equal to par which is not necessarily the case in connection with multi-curve models.

Starting from the definition of the fair swap rate we have

$$
c^{6M}(t_0,T) := \frac{\left(1-B^{6M}(t_0,T)\right)}{A^{6M}(t_0,T)}
$$

$$
\Rightarrow\ B^{6M}(t_0,T) = 1-c^{6M}(t_0,T)\cdot \sum_{k=1}^{N} \Delta(T_k,T_{k+1})\cdot B^{6M}(t_0,T_k).
$$

Rearranging the terms yields the fair value of the 6-month EURIBOR interest rate swap at inception $t = t_0$:

EQUATION 2: Definition of Equilibrium Conditions for 6M EURIBOR Interest Rate Swap

$$
\overbrace{c^{6M}(t_0,T)\cdot \sum_{k=1}^{N} \Delta(T_{k-1},T_k)\cdot B^{6M}(t_0,T_k)}^{\substack{\text{PV of the fixed side of the}\\ \text{6M EURIBOR interest rate swap}}} \overset{!}{=} \underbrace{\left(1-B^{6M}(t_0,T)\right)}_{\substack{\text{PV of the floating side}\\ \text{of the 6M EURIBOR}\\ \text{interest rate swap}}}
$$

$$
\overbrace{c^{6M}(t_0,T)\cdot \sum_{k=1}^{N} \Delta(T_{k-1},T_k)\cdot B^{6M}(t_0,T_k) + B^{6M}(t_0,T)}^{\substack{\text{PV of the fixed synthetic bond}\\ \text{(hedged item) at inception}}} = \underbrace{1}_{\substack{\text{PV of the}\\ \text{floating rate}\\ \text{note at}\\ \text{inception}}}
$$

In *Equation 2* above an equilibrium condition is assumed, since the present value of the floating side is equal to the present value of the fixed side of the interest rate swap; the fair value of the 6-month EURIBOR interest rate swap is zero at inception. The present value of the floating side of the interest rate swap is derived by the following rationale: since it is assumed that forwarding and discounting is performed by identical curves, the floating rate note always equals par at each reset date (every 6-month), but since no notional is exchanged in a swap, the present value of receiving one EUR at time T has to be subtracted ($= -B^{6M}(t_0, T)$).

Particularly in view of fair value changes (and subsequent effectiveness measurement) it is important to note that a floating rate note is valued at par not only at inception $t = t_0$ but also on each reset date $t = t_j,\ j = 1, \dots, 2N$ in a single-curve setup.

The definition of $c^{6M}(t_0, T)$, which represents the "hedged portion" and the internal coupon, is made only at inception $t = t_0$, which implies that the present value of the fixed synthetic bond (hedged item) with rate $c^{6M}(t_0, T)$ is not equal to 1 on a valuation date after t_0 since market conditions and thus the discount factors have changed. The PV changes between the reset dates of a plain vanilla interest rate swap with inception date $t = t_0$ only results from the fixed side of the swap (including the repayment of the notional) in a single-curve setup. This will compensate for the corresponding PV changes of the fixed synthetic bond (hedged item) with identical terms and conditions.

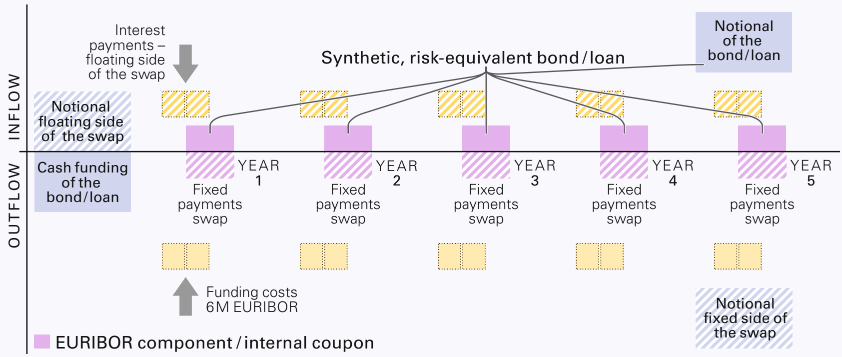
This important observation will be picked up in the multi-curve approach in *Section 4.3.2* when determining the dynamic adjustment.

It is important to note that forwarding and discounting with identical curves also relates to the cash flow profiles associated with the hedging relationship.

TABLE 20: Modeling of the Cash Flows of the Hedging Relationship

$t = t_0 = T_0, t_1, t_2 = T_1, \ldots, t_{2N} = T_n = T$	$t_0 = 0$	$t_1 = 6M$	$t_2 = 12M = T_1 = 1$ year	...	$t = T$
Hedged item (synthetic, risk-equivalent bond)					
	–1		$c^{6M}(t_0,T)$	...	$1+c^{6M}(t_0,T)$
Hedging instrument (6M EURIBOR interest rate swap)					
Fixed side	+1		$-c^{6M}(t_0,T)$	...	$-(1+c^{6M}(t_0,T))$
Floating side	–1	6M EURIBOR	6M EURIBOR	...	1+6M EURIBOR
Funding (6M EURIBOR)	+1	–(6M EURIBOR)	–(6M EURIBOR)	...	–(1+6M EURIBOR)
Sum	0	0	0	...	0

FIGURE 46: Representation of Risk and Valuation Factors of the Economic Hedging Model



Accordingly, using the model described above and the notation introduced, the cash flows can be represented as shown in *Table 20.*

According to the notation, the (dirty) fair value of the hedged item with respect to interest rate risk is evaluated as follows (discounted cash flows):

$$HFV_{t_0}^{6M}(t) := c^{6M}(t_0,T) \cdot \sum_{k=t+1}^{T} \Delta(T_{k-1},T_k) \cdot B^{6M}(t,T_k) + B^{6M}(t,T).$$

The fair value of the hedged item is the present value of the synthetic bond and coincides – by construction – with the present value of the fixed side of the corresponding 6-month interest rate swap.

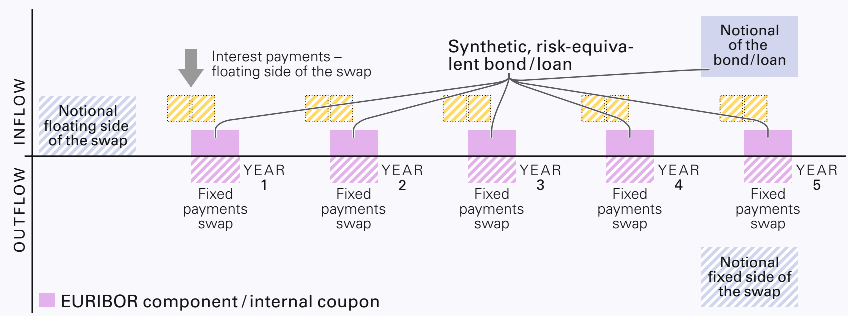
As a result, the economic hedging model comprises only one risk factor: 6-month EURIBOR interest rate. Accordingly the cash flow representation coincides with the representation of risk factors portrayed in *Figure 46*.

This corresponds to *Figure 45* since there is only one risk factor: 6-month EURIBOR.

4.2.3 Step 3: Representation of the Fair Value Hedge Accounting Model

According to IAS 39 the funding position is not included in the hedge accounting model, therefore for effectiveness testing purposes fair value changes of the floating side of the swap causes ineffectiveness.[62] The representation of the hedge accounting model is portrayed in *Figure 47*.

FIGURE 47: Representation of Hedge Accounting Model according to IAS 39

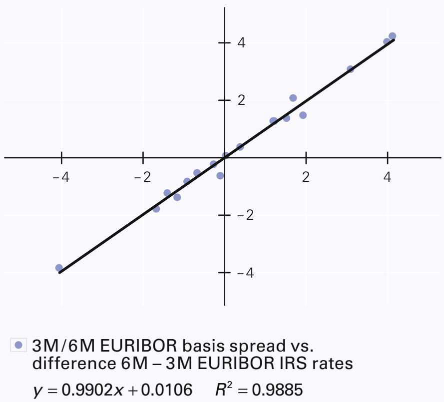


62 For further details refer to Schubert, D. (2011).

4.2.4 The Role of 3-Month/6-Month EURIBOR Basis Swaps

As shown in the previous *Section 3* tenor basis swaps have considerable influence in the construction of discount curves. In the example above the 6-month EURIBOR discount curve was constructed by "adding" the 3-month/6-month EURIBOR tenor basis swap spread to the 3-month EURIBOR zero swap rates following the "bootstrapping" procedure. Consequently the hedging relationship now contains two risk and valuation factors: 3-month EURIBOR zero swap rates and 3-month/6-month EURIBOR tenor basis swap spread. Since these effects are incorporated in one single-curve for discounting and forwarding, pricing and hedging work very easily.[63] Hence this type of hedging can be considered as implicit incorporation of risk and valuation factors. The multi-curve example below can therefore be considered as the "explicit" incorporation of risk and valuation factors and shows what happens in case the incorporation into one discount and forward curve is reversed. Multi-curve models create an integrated market for all derivatives using an equilibrium condition. Below it will be shown how the fair value of a 6-month EURIBOR interest rate swap is evaluated in case of using 3-month EURIBOR zero rates as discount curve. This immediately defines the derivatives in the "model" economy: 3-month EURIBOR interest rate swap, 6-month EURIBOR interest rate swap and the 3-month/6-month EURIBOR basis swap.

FIGURE 48: Regression Analysis of 3M/6M EURIBOR Basis Spread vs. Difference 6M – 3M EURIBOR Interest Rate Swap Rates



As the following analysis of quoted 3-month/6-month EURIBOR tenor basis spreads and the difference of quoted 6-month and 3-month EURIBOR interest rate swap quotes

63 For further details refer to Schubert, D. (2011).

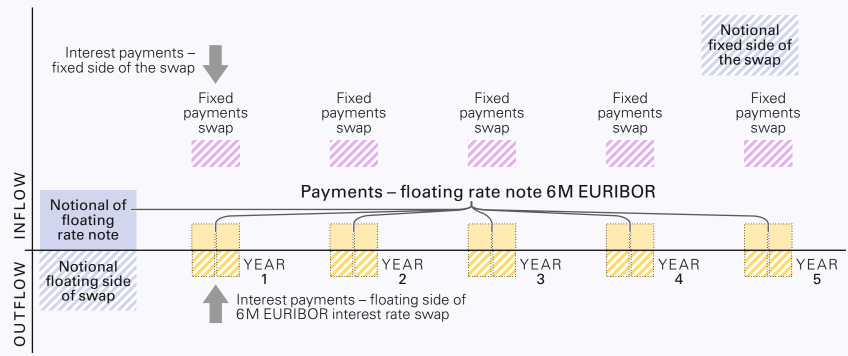
shows, the integrated market assumption of tenor and interest rate swaps is a good approximation of reality since the slope of regression is near 1 and the grade of determination close to 100% (see *Figure 48*).

Empirically it should be noted that neither the 6-month EURIBOR nor the 3-month EURIBOR zero swap rates have explanatory power with respect to the market quotes of bonds ("full fair value") and therefore this statistical property also holds – apart from accidental statistical coincidence – for the 3-month/6-month EURIBOR tenor basis swap spreads, since this is only the "difference" between these rates. Therefore it is a unpromising attempt to "prove" its statistical relevance, since the low explanatory power is inherited from the individual swap rates. This statistical fact is also present for cash flow hedge accounting as the following *Section 4.2.5* shows.

4.2.5 Cash Flow Hedge Accounting Using a 6-Month EURIBOR Interest Rate Swap

In *Figure 49* a typical example of a cash flow hedge accounting model is given. The example consists of a 6-month EURIBOR floating rate note and a 6-month EURIBOR interest rate swap both with identical

FIGURE 49: Representation of a Cash Flow Hedge Accounting Model according to IAS 39



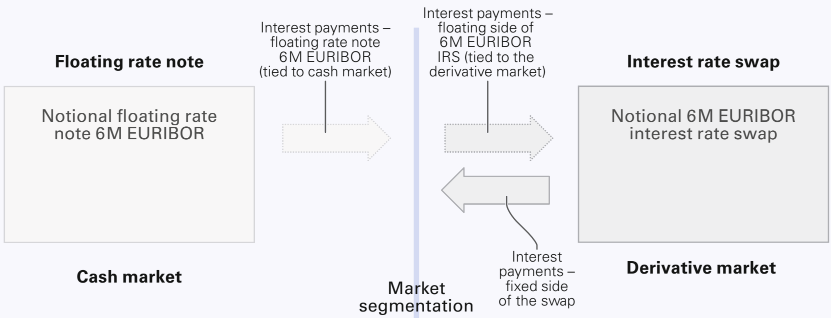
terms and conditions (equal maturities etc.) and similar credit risk (AA^{-} rating according to the market convention of the interest rate swaps).

Figure 49 also reveals that the cash flow hedge accounting model on its own, i.e. excluding the funding, is – in general – inconsistent with fair value based sensitivity interest rate risk management techniques of banks, since the fixed coupon of the interest rate swap is exposed to interest rate risk. Therefore the application of cash flow hedge accounting – considering only the hedging relationships without funding – increases economic sensitivity of interest rate risk, i.e. the risk in fair value change due to changes in market interest rates.

As described in the previous *Section 3* the cash market is different from the derivative market in products, pricings etc. This is shown in *Figure 50* and represented by the vertical blue line indicating market segmentation.

Referring to the analysis above, the fair value of the floating rate note (cash market) and the fair value of the floating side of the 6-month EURIBOR interest rate swap can be written in terms of forward rates.

FIGURE 50: Comparison of Cash Flows in a Cash Flow Hedge Accounting Model according to IAS 39



Floating leg of 6-month EURIBOR interest rate swap in $t = t_0$:

$$\begin{aligned}\Lambda^{6M}(t_0,T) = {} & \overbrace{\delta(t_0,t_1)\cdot r^{6M}(t_0)}^{\text{6M EURIBOR spot rate}}\cdot B^{6M}(t_0,t_1)\\ & +\delta(t_1,t_2)\cdot f^{6M}(t_0,t_1,t_2)\cdot B^{6M}(t_0,t_2)+\ldots\\ & +\delta(t_{j-1},t_j)\cdot f^{6M}(t_0,t_{j-1},t_j)\cdot B^{6M}(t_0,t_j)+\ldots\\ & +f^{6M}(t_0,t_{2N-1},t_{2N})\cdot B^{6M}(t_0,t_{2N})\\ = {} & 1-B^{6M}(t_0,t_{2N}).\end{aligned}$$

6-month EURIBOR floating rate note in $t = t_0$:

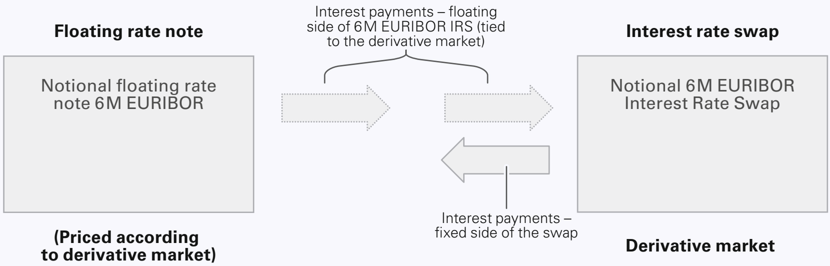
$$\begin{aligned}FRN^{cash}(t_0,T) = {} & \underbrace{\delta(t_0,t_1)\cdot r^{6M}(t_0)}_{\text{6M EURIBOR spot rate}}\cdot B^{6M/cash}(t_0,t_1)\\ & +\delta(t_1,t_2)\cdot f^{6M/cash}(t_0,t_1,t_2)\cdot B^{6M/cash}(t_0,t_2)+\ldots\\ & +\delta(t_{j-1},t_j)\cdot f^{6M/cash}(t_0,t_{j-1},t_j)\cdot B^{6M/cash}(t_0,t_j)+\ldots\\ & +\left(1+\delta(t_{2N-1},t_{2N})\cdot f^{6M/cash}(t_0,t_{2N-1},t_{2N})\right)\cdot B^{6M/cash}(t_0,t_{2N})\\ = {} & 1.\end{aligned}$$

However, keep in mind that in case of the floating rate note, discount and forward factors are derived from the cash market!

In a cash flow hedge accounting model, the requirements – besides others – are met, if the cash flows of the 6-month EURIBOR floating rate note and a 6-month EURIBOR interest rate swap match. For effectiveness testing purposes, the hypothetical derivative method is applied. In that case a "hypothetical" derivative reflecting the terms and conditions of the floating rate note is constructed and for effectiveness testing purposes its fair value changes are compared with the hedging and "real existing" 6-month EURIBOR interest rate swap. In this example no ineffectiveness is expected[64]. In case of the cash flow hedge accounting model it is assumed that the matching of cash flows in combination with the usage of the "hypothetical derivative method" for effectiveness testing implies that pricing for the floating rate note

64 For simplicity the impact of counterparty credit risk on the hedging instrument is neglected in this context.

FIGURE 51: Integrated Market Model Implied by the Cash Flow Hedge Accounting Model according to IAS 39



and the floating side of a swap are identical. Considering the pricing of these financial instruments in their respective markets (cash and derivative market) this is clearly not the case. It has been shown in the previous *Section 3* (*Figure 36* and *Figure 37*) that the explanatory power of the floating side of the interest rate swap in comparison to full fair value changes of a floating rate is rather poor. Consequently also the cash flow hedge accounting model incorporates a valuation model and relies on the derivative market as the relevant market with respect to pricing. The figures in the previous *Section 3* also reveal (*Figure 36* and *Figure 37*) that if cash prices (full fair value) of the floating rate note were compared with fair value changes of the floating side of the interest rate swap, the hedge would not meet the effectiveness criteria under IAS 39.

By application of this cash flow hedge accounting model an integrated market for the floating rate note and the 6-month EURIBOR interest rate swap is created and the issue of market segmentation has been resolved, which is similar to fair value hedge accounting models. Accordingly the cash forwards implied by the cash market (see above) are exchanged by the forwards derived from the set of 6-month EURIBOR interest rate swaps (derivative market) taking into account the 3-month/6-month EURIBOR tenor basis swaps. Furthermore it should be noted that equating cash flows in a cash flow hedge accounting model already implies economic modeling: no differences in pricing because the cash basis is eliminated and absence of arbitrage.

4.2.6 Dynamic Hedge Accounting in a Single-Curve Setup

In the following an example of a dynamic hedge accounting approach in a single-curve setup is described. This approach can be considered as a special case of hedge accounting a multi-curve setup and therefore serves as an illustration.

In view of the role of tenor basis swaps as described in *Section 4.2.4* and in the case of fair value hedging in the single-curve approach (taking the 3-month EURIBOR interest rate swap for this example) the two risk components – the 6-month EURIBOR interest rate swap and the 3-month/6-month EURIBOR tenor basis swap – are inherently given. By the application of two different strategies giving the same economical payoff it will be shown in the following that dynamic adjustments according to the two inherent risk factors can be considered as already inherent in the single-curve environment and thus giving a consistent approach.

Defining the abbreviations

$$A^{3M}(t_0,T) := \sum_{k=1}^{N} B^{3M}(t_0,T_k)\cdot\Delta(T_{k-1},T_k)$$

$$A^{6M}(t_0,T) := \sum_{k=1}^{N} B^{6M}(t_0,T_k)\cdot\Delta(T_{k-1},T_k)$$

the fair swap rates in t_0 are given as follows:

$$c^{3M}(t_0,T) := \frac{1-B^{3M}(t_0,T)}{A^{3M}(t_0,T)}$$

$$c^{6M}(t_0,T) := \frac{1-B^{6M}(t_0,T)}{A^{6M}(t_0,T)}.$$

Now the equilibrium conditions can be stated as follows:

6-month EURIBOR interest rate swap:

$$c^{6M}(t_0,T)\cdot A^{3M}(t_0,T)=\Lambda^{6M/3M}(t_0,T)$$

with:

$$\Lambda^{6M/3M}(t_0,T):=\delta(t_0,t_1)\cdot r^{6M}(t_0)\cdot B^{3M}(t_0,t_1)$$
$$+\sum_{j=2}^{2N}\delta(t_{j-1},t_j)\cdot f^{6M/3M}(t_0,t_{j-1},t_j)B^{3M}(t_0,t_j),$$

3-month EURIBOR interest rate swap:

$$c^{3M}(t_0,T)\cdot A_T^{3M}(t_0)=\Lambda^{3M}(t_0,T)=1-B^{3M}(t_0,T),$$

3-month / 6-month EURIBOR tenor basis swap (unsecured; for collateralized instruments and OIS discounting an analogous reasoning would be valid):

$$\left[c^{6M}(t_0,T)-c^{3M}(t_0,T)\right]\cdot A^{3M}(t_0,T)=\Lambda^{6M/3M}(t_0,T)-\Lambda^{3M}(t_0,T)$$
$$=\Lambda^{6M/3M}(t_0,T)-\left(1-B^{3M}(t_0,T)\right).$$

It should be noted that the tenor basis swap can be considered as a portfolio of the two plain vanilla swaps. This is consistent with market conventions in EUR, since the 3-month / 6-month EURIBOR tenor basis swap is defined by the difference of the 3-month EURIBOR and the 6-month EURIBOR interest rate swap as mentioned in *Section 3.* The economy defined above is also sufficient to describe the economy relevant for hedge accounting, since the funding position is not considered in the hedge accounting model but in reality.

According to IAS 39 the portion which is subject to hedge accounting is defined by means of the benchmark curve. In this case the benchmark curve is defined by the 3-month EURIBOR zero swap rates. As already shown in *Equation 2* in the case of the 6-month EURIBOR this "portion" is equal to the 3-month EURIBOR interest rate swap rate using the equilibrium condition:

$$c^{3M}(t_0,T)\cdot A^{3M}(t_0,T)+B^{3M}(t_0,T)\overset{!}{=}1.$$

The portion is defined by constructing a risk-equivalent synthetic bond with a notional of 1. In case of a multi-curve model this portion can now be stated in two ways, since two sets of interest rate swaps define the model economy. If the number of derivatives increases then the number of portion representations increases accordingly.

$$\begin{aligned}
&c^{3M}(t_0,T)\cdot A^{3M}(t_0,T)+B^{3M}(t_0,T)\\
=\;&\underbrace{c^{6M}(t_0,T)\cdot A^{3M}(t_0,T)-c^{6M}(t_0,T)\cdot A^{3M}(t_0,T)}_{=0}\\
&+c^{3M}(t_0,T)\cdot A^{3M}(t_0,T)+B^{3M}(t_0,T)\\
=\;&\left[c^{3M}(t_0,T)-c^{6M}(t_0,T)\right]\cdot A^{3M}(t_0,T)+c^{6M}(t_0,T)\cdot A^{3M}(t_0,T)\\
&+B^{3M}(t_0,T)\\
\underbrace{=}_{\text{Equilibrium condition}}\;&\left(1-B^{3M}(t_0,T)\right)-\Lambda^{6M/3M}(t_0,T)+\Lambda^{6M/3M}(t_0,T)+B^{3M}(t_0,T)\\
=\;&1.
\end{aligned}$$

Portion no. 1: $c^{3M}(t_0,T)$,
Portion no. 2: $c^{6M}(t_0,T)+\left[c^{3M}(t_0,T)-c^{6M}(t_0,T)\right]$.

Observe that portion no. 2 does not involve the designation of a tenor basis swap, but recognizes the tenor as an additional risk factor; both definitions of the portion are equal!

Consider two "strategies" to evaluate the hedge fair value:

Strategy no. 1:

$$HFV_{t_0}^{3M}(t_0) := c^{3M}(t_0,T)\,A^{3M}(t_0,T) + B^{3M}(t_0,T),$$

Strategy no. 2:

$$\begin{aligned} HFV_{t_0}^{6M\text{-}6M/3M}(t_0) := {} & c^{6M}(t_0,T)\cdot A^{3M}(t_0,T) \\ & -\left[c^{6M}(t_0,T) - c^{3M}(t_0,T)\right]\cdot A^{3M}(t_0,T) \\ & +B^{3M}(t_0,T). \end{aligned}$$

In the economy relevant for hedge accounting, both "strategies" should lead to the same fair value changes, otherwise this would create arbitrage possibilities and an inconsistent hedge accounting model.

Considering the hedged fair value changes from a reset date T_k to a reset date T_{k+1}[65], it can be shown that by inserting a fair current tenor basis swap for the remaining term (of value zero) at each reset date the presentation of the differences in each of the two risk factors (6-month EURIBOR interest rate swap and 3-month/6-month/tenor basis swap) lead to dynamic adjustments w.r.t. to the discount curve of 3-month EURIBOR that finally cancel out. The remaining terms just give the changes of the 3-month EURIBOR risk factor and the change in fair value of the repayment of the notional.

$$\begin{aligned} & HFV_{t_0}^{6M-6M/3M}(T_{k+1}) - HFV_{t_0}^{6M-6M/3M}(T_k) \\ = {} & c^{6M}(t_0,T)\cdot A^{3M}(T_{k+1},T) - \left[c^{6M}(t_0,T) - c^{3M}(t_0,T)\right]\cdot A^{3M}(T_{k+1},T) \\ & +B^{3M}(T_{k+1},T) - c^{6M}(t_0,T)\cdot A^{3M}(T_k,T) \\ & +\left[c^{6M}(t_0,T) - c^{3M}(t_0,T)\right]\cdot A^{3M}(T_k,T) + B^{3M}(T_k,T) \end{aligned}$$

65 For the sake of simplicity it is assumed that each interest payment date T_k of the fixed side is a reset date t_{2j} of the floating side.

$$
\begin{aligned}
&= c^{6M}(t_0,T)\cdot A^{3M}(T_{k+1},T)\underbrace{-\left[\Lambda^{6M/3M}(T_{k+1},T)-\Lambda^{3M}(T_{k+1},T)\right]}_{-\text{ Side of current tenor basis swap at } T_{k+1}} \\
&\quad -\left[c^{6M}(t_0,T)-c^{3M}(t_0,T)\right]\cdot A^{3M}(T_{k+1},T) \\
&\quad \underbrace{+\left[c^{6M}(T_{k+1},T)-c^{3M}(T_{k+1},T)\right]\cdot A^{3M}(T_{k+1},T)}_{+\text{ Side of current tenor basis swap at } T_{k+1}}+B^{3M}(T_{k+1},T) \\
&\quad -c^{6M}(t_0,T)\cdot A^{3M}(T_k,T)\underbrace{+\left[c^{6M}(T_k,T)-c^{3M}(T_k,T)\right]\cdot A^{3M}(T_k,T)}_{+\text{ Side of current tenor basis swap at } T_k} \\
&\quad +\left[c^{6M}(t_0,T)-c^{3M}(t_0,T)\right]\cdot A^{3M}(T_k,T) \\
&\quad \underbrace{-\left[\Lambda^{6M/3M}(T_k,T)-\Lambda^{3M}(T_k,T)\right]}_{-\text{ Side of current tenor basis swap at } T_k}-B^{3M}(T_k,T)
\end{aligned}
$$

$$
\begin{aligned}
&= c^{3M}(t_0,T)\cdot A^{3M}(T_{k+1},T) \\
&\quad \left.\begin{aligned}&+\left[c^{6M}(t_0,T)-c^{3M}(t_0,T)\right]\cdot A^{3M}(T_{k+1},T)\\ &-\left[c^{6M}(T_{k+1},T)-c^{3M}(T_{k+1},T)\right]\cdot A^{3M}(T_{k+1},T)\end{aligned}\right\}\ +\text{ Dynamic adjustment } k+1 \\
&\quad \left.\begin{aligned}&-\left[c^{6M}(t_0,T)-c^{3M}(t_0,T)\right]\cdot A^{3M}(T_{k+1},T)\\ &+\left[c^{6M}(T_{k+1},T)-c^{3M}(T_{k+1},T)\right]\cdot A^{3M}(T_{k+1},T)\end{aligned}\right\}\ -\text{ Dynamic adjustment } k+1 \\
&\quad +B^{3M}(T_{k+1},T)-c^{3M}(t_0,T)\cdot A^{3M}(T_k,T) \\
&\quad \left.\begin{aligned}&-\left[c^{6M}(t_0,T)-c^{3M}(t_0,T)\right]\cdot A^{3M}(T_k,T)\\ &+\left[c^{6M}(T_k,T)-c^{3M}(T_k,T)\right]\cdot A^{3M}(T_k,T)\end{aligned}\right\}\ -\text{ Dynamic adjustment } k \\
&\quad \left.\begin{aligned}&+\left[c^{6M}(t_0,T)-c^{3M}(t_0,T)\right]\cdot A^{3M}(T_k,T)\\ &-\left[c^{6M}(T_k,T)-c^{3M}(T_k,T)\right]\cdot A^{3M}(T_k,T)\end{aligned}\right\}\ +\text{ Dynamic adjustment } k \\
&\quad -B^{3M}(T_k,T)
\end{aligned}
$$

$$
\begin{aligned}
&= c^{3M}(t_0,T)A^{3M}(T_{k+1},T)+\underbrace{B^{3M}(T_{k+1},T)}_{\substack{\text{Repayment of}\\ \text{the notional}}} \\
&\quad -c^{3M}(t_0,T)\cdot A^{3M}(T_k,T)-\underbrace{B^{3M}(T_k,T)}_{\substack{\text{Repayment of}\\ \text{the notional}}} \\
&= HFV_{t_0}^{3M}(T_{k+1})-HFV_{t_0}^{3M}(T_k).
\end{aligned}
$$

The analysis above reveals that both strategies result in the same fair value changes. Although the theoretical underpinning of the dynamic adjustment will be portrayed below, the dynamic adjustments relate the 6-month EURIBOR risk factor to the 3-month EURIBOR discount curve and are represented by the 3-month / 6-month tenor basis swap with opposite sign. The adjustment will be dynamic since the differences in evolution of risk factors 6-month EURIBOR and 3-month EURIBOR have become more significant are not negligible any more.

Strategy no. 1 entails the definition of a static portion, while the application of strategy no. 2 involves dynamic adjustments in order to compensate the 3-month / 6-month tenor basis spread that is not the hedged risk with respect to the portion. In the case of a multi-curve setup with different forwarding and discounting it will be shown in more detail that the dynamic adjustments require the designation of different portions of cash flows over time of the hedged item and re-designation of the hedging relationship.

4.3 Hedge Accounting in a (Simple) Multi-Curve Model

In the following the "mechanics" of a simple multi-curve model is introduced. Let's continue the example above and assume for illustration purposes that the 3-month EURIBOR zero swap curve is chosen as discount curve (instead of the 6-month EURIBOR). For the example it is presumed that the 3-month EURIBOR interest rate swaps are liquid and the choice of discount curve is market practice, but economic hedging is still performed by the 6-month EURIBOR interest rate swap.[66] It should be pointed out that the valuation of hedging instruments and hedged items is always performed relatively to the discount curve.

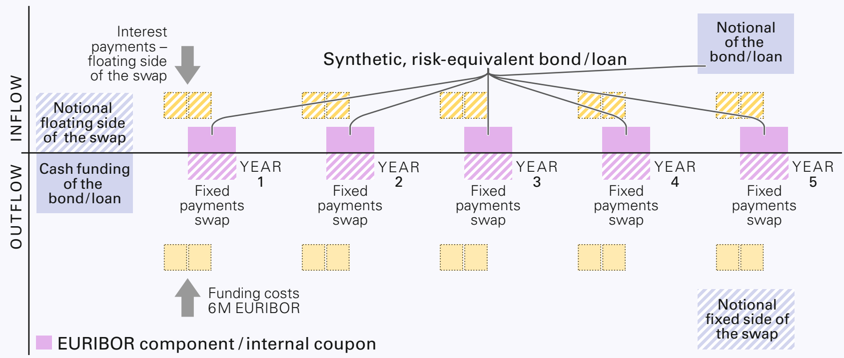
66 For collateralized derivatives OIS discounting is about to become the commonly applied discount rate. For sake of simplicity this fact is ignored in this example, so a 3-month EURIBOR zero swap curve is used for discounting.

Thus, changing the discount curve changes the benchmark i.e. the reference. In the given example the interest rate risk is still economically hedged with respect to the 6-month EURIBOR tenor on a cash flow basis but not on a fair value basis anymore. But due to the change of the discount curve, the 6-month EURIBOR interest rate swap is measured relatively to the 3-month EURIBOR interest rate swap curve, i.e. to the 3-month EURIBOR risk factor. In general the difference between these curves – i.e. the tenor basis spread curve – will not remain constant but evolve over time. It will be shown how a dynamic adjustment of the designated portion with respect to this dynamic of the 3-month EURIBOR interest rate swap curve can be determined. Technically the dynamics results from forwarding and discounting with different curves.

4.3.1 Step 1: Representation of the Cash Flow Profile Associated with the Hedging Relationship

In a multi-curve model the cash flow profiles of the hedging relationship do not change, since all (except the synthetic, risk-equivalent bond/loan) are contractually specified. Please note that the funding position of the economic hedge does not change, either. Accordingly the cash flow table shown in *Figure 52* is identical to the one in *Figure 45*.

FIGURE 52: Representation of Cash Flows Using the Construction of a Risk-Equivalent Synthetic Bond / Loan in a Multi-Curve Model



4.3.2 Step 2: Representation of Risk and Valuation Factors

The analysis due to changes in the discount curve commences with the evaluation of the 6-month EURIBOR interest rate swap discounted with 3-month EURIBOR zero swap rates.

The notation with respect to the 3-month EURIBOR interest rate swap is used similar to the notation above:

EQUATION 3: Definition of an Equilibrium Condition for a 6M EURIBOR Interest Rate Swap Rate Discounted with 3M EURIBOR Zero Swap Rates

$$\underbrace{c^{6M}(t_0,T)\cdot\sum_{k=1}^{T}\Delta(T_{k-1},T_k)\cdot B^{3M}(t_0,T_k)}_{\text{PV of the fixed side of the 6M EURIBOR interest rate swap discounted with 3M EURIBOR}} \overset{!}{=} \underbrace{\left(1-B^{3M}(t_0,T)\right)+\hat{TS}_T}_{\text{PV of the floating side of the 6M EURIBOR interest rate swap discounted with 3M EURIBOR}}$$

In order to evaluate the fair value of an 6-month EURIBOR interest rate swap discounted with 3-month EURIBOR zero swap rates, the fixed side of the 6-month EURIBOR interest rate swap (6-month swap rate) is discounted with the 3-month EURIBOR zero swap rates. By definition (market convention) the initial fair value (present value) of the 6-month EURIBOR interest rate swap equals zero, so therefore the fair value of the fixed side of the interest rate swap is equal to the present value of the floating side of the interest rate swap. But now, according to the application of the 3-month EURIBOR zero swap rates, the floating rate side of this interest rate swap is no longer equal to par minus the discounted repaid notional like in the case above *(Equation 2)*. Therefore $\hat{TS}_T$ is different from zero.

Rearranging the terms and using *Equation 1* yields:

$$\hat{TS}_T := \left(c^{6M}(t_0,T)-c^{3M}(t_0,T)\right)\cdot\sum_{k=1}^{T}\Delta(T_{k-1},T_k)\cdot B^{3M}(t_0,T_k)$$
$$=: TS_T\cdot\sum_{k=1}^{T}\Delta(T_{k-1},T_k)\cdot B^{3M}(t_0,T_k).$$

It should be noted that the presentation above using $\hat{TS}_T$ is for illustration purposes only; we will shortly derive the forward rate representation of the floating rate side. But there are several important results:

- The swap rates $c^{6M}(t_0,T)$ and $c^{3M}(t_0,T)$ are taken from market quotes. Consequently the multi-curve approaches use the prices of interest rate swaps (derivatives) with different tenors as input parameters. The market quotes of derivatives do not change because of the application of multi-curve models!
- "Differences" resulting from discounting the fixed side of the interest rate swap are "compensated" by the floating rate side following market conventions. Accordingly at inception $t = t_0$ the usage of a different discount curve yields a simultaneous change in the fixed and the floating side of a fair interest rate swap by the same amount.
- The term $TS_T = \left(c^{6M}(t_0,T) - c^{3M}(t_0,T)\right)$ is termed the tenor basis spread between the 6-month and the 3-month EURIBOR for maturity T; it represents the rate of the "fixed side"[67] of a 3-month/6-month EURIBOR tenor basis swap[68].
- Essentially the change of discount curves only means that the fair value (present value) of the 6-month EURIBOR interest rate swap is expressed in terms of a 3-month EURIBOR interest rate swap. The set of 3-month EURIBOR interest rate swaps serves as currency in the sense of a relative measure as mentioned above, and the price of every other derivative, like the 6-month EURIBOR interest rate swap, is expressed in 3-month EURIBOR terms.

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67 As "fixed side" of a tenor in this paper the difference of the fixed payments of the two interest rate swap constituting the tenor basis swap denoted; correspondingly the difference of the two floating sides is named "floating side" of the tenor basis swap.

68 Usually OIS discounting would be expected for collateralised tenor basis swaps but to keep things simple in this introductory example it will be discounted on the 3-month EURIBOR zero swap curve.

- In this example the 3-month EURIBOR zero swap rates serve as a "benchmark curve", but this can also be the other way around: a 3-month EURIBOR interest rate swap can be discounted with 6-month EURIBOR zero swap rates without affecting the market price (quote) of a 3-month EURIBOR interest rate swap. This shows – similar to the single-curve case due to the choice of financial instruments from which the discount curve is derived and the interpolation techniques used – that deriving the discount curve in multi-curve setups is individual to each balance sheet preparer, but taking corresponding market practice and conventions into account. So despite similar mathematical and economic reasoning there are differences across market participants.
- Considering the 3-month EURIBOR as "the" discount curve is in financial economics termed "choosing a numeraire" as the benchmark curve, all other financial contracts in the model economy are measured relatively to this benchmark curve.

So far, we have not finalized the analysis of the "new" floating rate part and the impact after changing the discount curve. Currently we are only aware of the fact that the floating rate side of the interest rate swap discounted by the 3-month EURIBOR zero swap rates differs from par minus the discounted repayment.

In order to determine the new representation of the floating rate part of the interest rate swap, the following iterative procedure "bootstrapping" is performed.

For $t = T_1 = t_2$:

$$\overbrace{c^{6M}(t_0,T_1)\cdot\Delta(t_0,T_1)\cdot B^{3M}(t_0,T_1)}^{\substack{\text{PV of the fixed side of the 6M EURIBOR}\\ \text{interest rate swap discounted with}\\ \text{3M EURIBOR with 1 year maturity}}}$$

$$\overset{!}{=}\delta(t_0,t_1)\cdot\overbrace{f^{6M/3M}(t_0,t_0,t_1)}^{\substack{\text{6M EURIBOR spot rate}\\ \text{at } t=t_0}}\cdot B^{3M}(t_0,t_1)+\underbrace{\delta(t_1,t_2)\cdot\overbrace{f^{6M/3M}(t_0,t_1,t_2)}^{\text{Unknown}}\cdot B^{3M}(t_0,t_2)}_{\substack{\text{PV of the floating side of the 6M EURIBOR}\\ \text{interest rate swap discounted with 3M EURIBOR}}}.$$

- The first iterative step is performed at $t = T_1 = 1$ year.
- $c^{6M}(t_0, T_1)$ is the 6-month EURIBOR interest rate swap rate with one year maturity. It is taken from market quotes! This shows that the initial market price of the 6-month EURIBOR interest rate swap does not change.
- Since the iterative procedure requires a swap rate for each tenor (every 6 months), modeling like e.g. spline interpolation is required.
- The floating side of the 6-month EURIBOR interest rate swap resets every 6 months. We already know that the floating rate note does not reset to 1. In order to "adjust" the floating rate side to cover the change of discount curve of the fixed side, the forward rates are "calibrated" since these represent the missing variables.
- For the example denote $T = T_1$, $t_0 = 0$, $t_1 = 6\text{M}$, $t_2 = 12\text{M} = T_1$ and $(t_1 - t_0) = (t_2 - t_1) = 6\text{M}$.
- In the equation above the forward

  $$f^{6\text{M}/3\text{M}}(0,0,6\text{M}) = f^{6\text{M}/3\text{M}}(t_0, t_0, t_1)$$

  represents the spot interest for 6-month (known at $t = t_0$), while

  $$f^{6\text{M}/3\text{M}}(0,6\text{M},12\text{M}) = f^{6\text{M}/3\text{M}}(t_0, t_1, t_2)$$

  represents the forward rate between the 6-months and 12-months at time $t = t_0$, both discounted with the 3-month EURIBOR zero swap rates. The later rate is unknown and is derived from the equation above.

Using *Equation 1* we can rewrite:

$$c^{6\text{M}}(t_0,T_1)\cdot\Delta(t_0,T_1)\cdot B^{3\text{M}}(t_0,T_1)$$

$$\stackrel{!}{=}\delta(t_0,t_1)\cdot f^{6\text{M}/3\text{M}}(t_0,t_0,t_1)\cdot B^{3\text{M}}(t_0,t_1) + \delta(t_1,t_2)\cdot f^{6\text{M}/3\text{M}}(t_0,t_1,t_2)\cdot B^{3\text{M}}(t_0,t_2)$$

$$c^{6\text{M}}(t_0,T_1)\cdot\Delta(t_0,T_1)\cdot B^{3\text{M}}(t_0,T_1)$$

$$\stackrel{!}{=}\delta(t_0,t_1)\cdot r^{6\text{M}}(t_0)\cdot B^{3\text{M}}(t_0,t_1) + \delta(t_1,t_2)\cdot f^{6\text{M}/3\text{M}}(t_0,t_1,t_2)\cdot B^{3\text{M}}(t_0,t_2)$$

$$c^{6M}(t_0,T_1)\cdot\Delta(t_0,T_1)\cdot B^{3M}(t_0,T_1)$$

$$\overset{!}{=}\left[\frac{1}{B^{6M}(t_0,t_1)}-1\right]B^{3M}(t_0,t_1)+\delta(t_1,t_2)\cdot f^{6M/3M}(t_0,t_1,t_2)\cdot B^{3M}(t_0,t_2)$$

$$c^{6M}(t_0,T_1)\cdot\Delta(t_0,T_1)$$

$$\overset{!}{=}\left[\frac{1}{B^{6M}(t_0,t_1)}-1\right]\frac{B^{3M}(t_0,t_1)}{B^{3M}(t_0,t_2)}+\delta(t_1,t_2)\cdot f^{6M/3M}(t_0,t_1,t_2)$$

$$c^{6M}(t_0,T_1)\cdot\Delta(t_0,T_1)-\underbrace{\left[\frac{1}{B^{6M}(t_0,t_1)}-1\right]}_{\text{6M EURIBOR spot rate}}\cdot\left[1+\delta(t_1,t_2)\cdot\overbrace{f^{3M}(t_0,t_1,t_2)}^{\text{3M EURIBOR forward rate}}\right]$$

$$\overset{!}{=}\delta(t_1,t_2)\cdot\underbrace{f^{6M/3M}(t_0,t_1,t_2)}_{\text{6M forward rate discounted with 3M EURIBOR}}.$$

The equation above reveals that in equilibrium the tenor basis spread with a maturity of one year can be expressed as a function of forward rates. It is important to distinguish between $f^{6M/3M}(t_0,t_1,t_2)$ and $f^{3M}(t_0,t_1,t_2)$ forward rates. The former is a result of the "bootstrapping" algorithm described above, "compensates" the 6-month EURIBOR interest rate swap rates quotes discounted on the 3-month EURIBOR zero swaps rates and denotes the "mixed" 3-month/ 6-month EURIBOR forward rate, whereas the latter is the forward rate only based on the 3-month EURIBOR.

This set of $f^{6M/3M}(t_0,t_1,t_2)$ and $f^{3M}(t_0,t_1,t_2)$ forward rates for all $t_j, j=1,\ldots,2N$ tenors generates the forward surface using suitable interpolation techniques, so in comparison to the single-curve model, the forward curve is not a single-curve anymore. Accordingly all 3-month and 6-month EURIBOR interest rate swaps depend on each other due to the equilibrium condition (the present value of the fixed side of the interest rate swap equals the present value of the floating side of the interest rate swap!).

$$c^{6M}(t_0,T_1)\cdot\Delta(t_0,T_1)-\underbrace{\left[\frac{1}{B^{6M}(t_0,t_1)}-1\right]}_{\text{6M EURIBOR spot rate}}\left[1+\delta(t_1,t_2)\cdot\overbrace{f^{3M}(t_0,t_1,t_2)}^{\text{3M EURIBOR forward rate}}\right]$$

$$\overset{!}{=}\delta(t_1,t_2)\cdot\underbrace{f^{6M/3M}(t_0,t_1,t_2)}_{\text{6M forward rate discounted with 3M EURIBOR}}.$$

$f^{6M/3M}(t_0,t_1,t_2)$ adjusts so that the equilibrium condition is met.

The next step is performed at $t=T_1+6M$ (equal to the subsequent tenor of the 6-month EURIBOR interest rate swap). For this step it is important to note that it requires modeling of the fixed rate coupon with maturity $t=T_1+6M=t_3$. Apart from an interpolation assumption on the quoted fixed rates, a short first period is assumed to derive the next forward rate. The calculations are carried out in the appendix in *Section 7* leading to the following general formulas:

It follows for $j\geq 2,\dots,2N$:

$$\delta(t_{j-1},t_j)\cdot f^{6M/3M}(t_0,t_{j-1},t_j)$$

$$\overset{!}{=}c^{6M}(t_0,t_j)\cdot\frac{A^{3M}(t_0,t_j)}{B^{3M}(t_0,t_j)}-c^{6M}(t_0,t_{j-1})\cdot\frac{A^{3M}(t_0,t_{j-1})}{B^{3M}(t_0,t_j)}$$

$$\overset{!}{=}c^{6M}(t_0,t_j)\cdot\sum_{\substack{i\geq 1\\ i,j\text{ either pair}\\ \text{or unpair}}}^{j}\Delta(t_{i-i_0},t_{i-i_0+2})\frac{B^{3M}(t_0,t_{i-i_0+2})}{B^{3M}(t_0,t_j)}$$

$$-c^{6M}(t_0,t_{j-1})\cdot\sum_{\substack{i\geq 1\\ i,j-1\text{ either pair}\\ \text{or unpair}}}^{j-1}\Delta(t_{i-i_0},t_{i-i_0+2})\frac{B^{3M}(t_0,t_{i-i_0+2})}{B^{3M}(t_0,t_j)}.$$

In the last formula the 6-month EURIBOR interest rate swap rates (quoted from the market) are multiplied by the quotient of the corresponding "annuity" $A^{3M}(t_0,t_j)$ and the discount factor of the maturity. These quotients will result in (weighted) sums of forward rates which are derived from the 3-month discount curve. In the case of pair j, swaps have no irregular periods and the swap rates are given by market quotes only; for impair j interpolation is needed.

Using equilibrium conditions and notations as introduced in *Section 4.2.6* the corresponding representation can be derived as follows:

EQUATION 4: 6M EURIBOR Interest Rate Swap Discounted with 3M EURIBOR

$$\overbrace{c^{6M}(t_0,T)\cdot\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B^{3M}(t_0,T_k)}^{\text{PV of the fixed side of the 6M EURIBOR interest rate swap discounted with 3M EURIBOR}}$$

$$\overset{!}{=}\underbrace{\delta(t_0,t_1)\cdot r^{6M}(t_0)\cdot B^{3M}(t_0,t_1)+\sum_{j=2}^{2N}\delta(t_{j-1},t_j)\cdot f^{6M/3M}(t_0,t_{j-1},t_j)\cdot B^{3M}(t_0,t_j)}_{\text{PV of the floating side of the 6M EURIBOR interest rate swap discounted with 3M EURIBOR}}$$

In correspondence with the notation in the previous *Sections 4.2.2* or *4.2.6* the following abbreviations are used:

The "annuity" of the fixed side of the 6-month EURIBOR interest rate swap discounted on the 3-month EURIBOR zero swap curve which only depends on the interest payment dates and the discount factors:

$$A^{3M}(t_0,T):=\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B^{3M}(t_0,T_k).$$

The floating side of the 6-month EURIBOR interest rate swap discounted on the 3-month EURIBOR zero swap curve:

$$\Lambda^{6M/3M}(t_0,T):=\delta(t_0,t_1)\cdot r^{6M}(t_0)\cdot B^{3M}(t_0,t_1)$$
$$+\sum_{j=2}^{2N}\delta(t_{j-1},t_j)\cdot f^{6M/3M}(t_0,t_{j-1},t_j)B^{3M}(t_0,t_j).$$

Thus the equilibrium condition of *Equation 4* can be written as:

$$c^{6M}(t_0,T)\cdot A^{3M}(t_0,T)=\Lambda^{6M/3M}(t_0,T).$$

Rearranging the terms yields the following important result:

$$\overbrace{c^{6M}(t_0,T)\cdot A^{3M}(t_0,T)}^{\substack{\text{PV of the fixed side of the}\\ \text{6M EURIBOR interest rate swap}\\ \text{discounted with 3M EURIBOR}}}\underbrace{-c^{3M}(t_0,T)\cdot A^{3M}(t_0,T)+c^{3M}(t_0,T)\cdot A^{3M}(t_0,T)}_{=0}$$

$$\overset{!}{=}\Lambda^{6M/3M}(t_0,T)\underbrace{-\Lambda^{3M}(t_0,T)+\Lambda^{3M}(t_0,T)}_{=0}$$

$$\overbrace{\left(c^{6M}(t_0,T)-c^{3M}(t_0,T)\right)\cdot A^{3M}(t_0,T)}^{\text{Fixed side of the 6M/3M EURIBOR basis swap}}+\underbrace{c^{3M}(t_0,T)A^{3M}(t_0,T)}_{\substack{\text{Fixed side of the 3M EURIBOR}\\ \text{interest rate swap}}}$$

$$\overset{!}{=}\underbrace{\Lambda^{6M/3M}(t_0,T)-\Lambda^{3M}(t_0,T)}_{\substack{\text{=Floating side of a}\\ \text{6M/3M EURIBOR basis swap}}}+\underbrace{\Lambda^{3M}(t_0,T)}_{\substack{\text{Floating side of}\\ \text{the 3M EURIBOR}\\ \text{interest rate swap}}}.$$

The calculation shows that a 6-month EURIBOR interest rate swap discounted on 3-month EURIBOR zero swap rates can be represented (decomposed) as a 3-month/6-month EURIBOR basis swap and a 3-month EURIBOR interest rate swap.

In the multi-curve environment it is no longer true that a floater values at par on reset dates: if forwarding is performed w.r.t. the 6-month EURIBOR zero curve and discounting w.r.t. the 3-month EURIBOR curve, we get:

$$\begin{aligned}FRN^{6M/3M}(t_0,T)&=\Lambda^{6M/3M}(t_0,T)+B^{3M}(t_0,T)\\&=\Lambda^{6M/3M}(t_0,T)-\Lambda^{3M}(t_0,T)+\Lambda^{3M}(t_0,T)+B^{3M}(t_0,T)\\&=\left(\underbrace{c^{6M}(t_0,T)-c^{3M}(t_0,T)}_{\text{Tenor basis spread}}\right)\cdot A_T^{3M}(t_0)+1.\end{aligned}$$

Thus also the floating leg of a swap is no longer valued at par minus the discounted repaid notional. The floating leg of a 6-month EURIBOR interest rate swap discounted on the 3-month EURIBOR zero swap curve can be written as:

$$\Lambda^{6M/3M}(t_0,T)=\left(\underbrace{c^{6M}(t_0,T)-c^{3M}(t_0,T)}_{\text{Tenor basis spread}}\right)\cdot A^{3M}(t_0,T)+1-B^{3M}(t_0,T).$$

Another useful representation of the floating side of a 6-month EURIBOR interest rate swap discounted at 3-month EURIBOR is the following:

EQUATION 5: Representation of the Floating Side of a 6M EURIBOR Interest Rate Swap Discounted on a 3M EURIBOR Curve

$$\Lambda^{6M/3M}(t_0,T) = c^{6M}(t_0,T)\cdot A^{3M}(t_0,T)$$
$$= c^{6M}(t_0,T)\cdot \frac{A^{6M}(t_0,T)}{A^{6M}(t_0,T)}\cdot A^{3M}(t_0,T)$$
$$= \Lambda^{6M}(t_0,T)\cdot \frac{A^{3M}(t_0,T)}{A^{6M}(t_0,T)}$$

Summary of the features from the analysis:

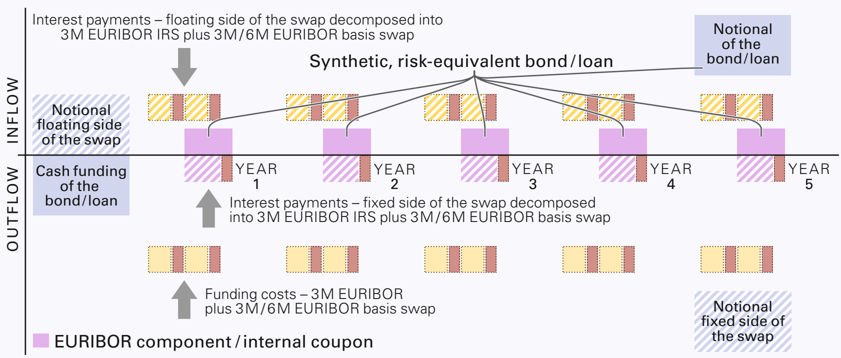
- According to *Equation 4* the floating side of the swap is represented by 3-month/6-month forward rates; furthermore its fair value is different from par.
- Changing the discount curve implies decoupling forwarding (rolling out cash flows) from discounting cash flows.
- Market participants are forming cash flow expectations differently from discounting them.
- Changing the discount curve from 6-month EURIBOR is identical to entering into a 3-month EURIBOR interest rate swap and a 3-month/6-month EURIBOR basis swap.
- At $t = t_0$ the economic hedging by replication using derivatives is still valid, but according to the decomposition into a 3-month EURIBOR interest rate swap and a 3-month/6-month EURIBOR tenor basis swap the hedging strategy is no longer static but dynamic! This results from the fact that the 3-month/6-month EURIBOR tenor basis swap basis is not perfectly correlated with the 3-month EURIBOR interest rate swap throughout time.

- Note that only the representation of risk and valuation factors has changed, the contractual cash flows, terms and conditions etc. remain unchanged.
- The prices for interest rate swaps for multi-curve models are taken from market quotes, so the prices do not change, but the dynamic of the prices changes over time.
- According to the representation of risk and valuation factors, also the representation of the 6-month EURIBOR funding changes. The funding is now represented by 3-month EURIBOR plus 3-month / 6-month EURIBOR tenor basis swap.

Figure 53 summarizes the economic hedging strategy using 6-month EURIBOR interest rate swap and discounting with 3-month EURIBOR zero swap rates.

In *Figure 53* the EURIBOR component is not yet specified. Since the forwarding of cash flows is different from discounting cash flows, the cash flows (EURIBOR component / internal coupon) of a synthetic, risk-equivalent bond are no longer constant over time.

FIGURE 53: Representation of Valuation and Risk Factors in a Multi-Curve Model ($t = t_0$)



One approach to quantify the dynamic adjustment that compensates for the different evolution of the forwarding based on the 6-month EURIBOR zero swap curve and the discounting on the 3-month EURIBOR zero swap curve as explained at the beginning of this section is to proceed similarly to the single-curve case described in *Section 4.2.2:*

EQUATION 6: Definition of Equilibrium Conditions for 6M EURIBOR Interest Rate Swap

$$\overbrace{c^{6M}(t_0,T)\cdot\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B^{6M}(t_0,T_k)}^{\text{PV of the fixed side of the 6M EURIBOR interest rate swap}} \overset{!}{=} \underbrace{\left(1-B^{6M}(t_0,T)\right)}_{\text{PV of the floating side the 6M EURIBOR interest rate swap}}$$

$$\overbrace{c^{6M}(t_0,T)\cdot\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B^{6M}(t_0,T_k)+B^{6M}(t_0,T)}^{\text{PV of the fixed sythetic bond (hedged item) at inception}} = \underbrace{1}_{\text{Notional of the hedged bond at inception =PV of the floater}}$$

The second relation of *Equation 6* defines the risk-equivalent synthetic bond, it is important to note that the risk-equivalent bond is derived from the fair value determination of the interest rate swap, but its economic interpretation is different. The equation determines the hedging cost of a "plain vanilla" bond with notional 1. Only in this case this coincides with PV of the floating leg of the interest rate swap. Then the cost of hedging equals the swap rate $c^{6M}(t_0,T)$. Conversely, if the notional differs from 1 (e.g. premium/discount) then the cost of hedging changes accordingly. It is important to distinguish between these economic perceptions.

In the multi-curve setup the determination of the "risk-equivalent, synthetic bond" follows the same economic rationale, but it is more complicated, since the equilibrium condition is defined according to the entire set of derivatives: 6-month EURIBOR interest rate swap and 3-month EURIBOR interest rate swap and the 3-month/6-month EURIBOR basis swap is derived from both. The following two equilibrium conditions for 3-month EURIBOR and 6-month EURIBOR interest rate swaps at inception (cf. *Equation 3* or *Equation 4*) will be used. With the abbreviations defined above these can be stated as follows:

EQUATION 7: Equilibrium Conditions in a Multi-Curve Setup

3-month EURIBOR interest rate swap:

$$c^{3M}(t_0,T)\cdot A^{3M}(t_0,T)=\Lambda^{3M}(t_0,T)=1-B^{3M}(t_0,T)$$

6-month EURIBOR interest rate swap:

$$c^{6M}(t_0,T)\cdot A^{3M}(t_0,T)=\Lambda^{6M/3M}(t_0,T)$$

3-month/6-month EURIBOR tenor basis swap:

$$\begin{aligned}\left[c^{6M}(t_0,T)-c^{3M}(t_0,T)\right]\cdot A^{3M}(t_0,T)&=\Lambda^{6M/3M}(t_0,T)-\Lambda^{3M}(t_0,T)\\&=\Lambda^{6M/3M}(t_0,T)-\left(1-B^{3M}(t_0,T)\right)\end{aligned}$$

Using the equilibrium conditions from *Equation 4* also for $t=T_k$, the value of a 6-month EURIBOR interest rate swap with the fair swap rate from $t=t_0$, maturity T and discounted on the 3-month EURIBOR zero swap curve is considered on a reset date $T_k=t_{2j}$[69]:

$$\begin{aligned}&c^{6M}(t_0,T)\cdot A^{3M}(T_k,T)-\Lambda^{6M/3M}(T_k,T)\\&=c^{6M}(t_0,T)\cdot A^{3M}(T_k,T)-c^{6M}(T_k,T)\cdot A^{3M}(T_k,T)\\&=\left[c^{6M}(t_0,T)-c^{6M}(T_k,T)\right]\cdot A^{3M}(T_k,T)\\&=\left[c^{6M}(t_0,T)-c^{6M}(T_k,T)\right]\cdot A^{3M}(T_k,T)\\&\quad+c^{3M}(T_k,T)\cdot A^{3M}(T_k,T)-\Lambda^{3M}(T_k,T)\\&=\left[c^{6M}(t_0,T)\underbrace{-c^{6M}(T_k,T)+c^{3M}(T_k,T)}_{\substack{\text{Tenor basis spread}\\ \text{at } T_k \text{ for maturity } T}}\right]\cdot A^{3M}(T_k,T)-1+B^{3M}(T_k,T).\end{aligned}$$

69 It is assumed for the sake of simplicity that the repricing dates of the fixed side coincide with the repricing dates of the floating side.

This reveals that the fair value of the current tenor basis spread for the remaining time to maturity represents the difference to the valuation with respect to the 3-month EURIBOR discount curve. Rearranging and using the definition of the equilibrium condition of the 3-month/ 6-month basis swap yields the following representation:

$$
\begin{aligned}
& c^{6M}(t_0,T)\cdot A^{3M}(T_k,T)-\Lambda^{6M/3M}(T_k,T) \\
&= c^{3M}(t_0,T)\cdot A^{3M}(T_k,T)-\underbrace{\left(\Lambda^{6M/3M}(T_k,T)-\Lambda^{3M}(T_k,T)\right)}_{\substack{\text{Floating side of the}\\ \text{6M/3M EURIBOR basis swap}}} \\
&\quad +\underbrace{\left(c^{6M}(t_0,T)-c^{3M}(t_0,T)\right)\cdot A^{3M}(T_k,T)}_{\text{Fixed side of the 6M/3M EURIBOR basis swap}}+B^{3M}(T_k,T)-1 \\
&= \underbrace{\left(c^{6M}(t_0,T)-c^{3M}(t_0,T)\right)\cdot A^{3M}(T_k,T)}_{\text{Fixed side of the 6M/3M EURIBOR basis swap}}+c^{3M}(t_0,T)\cdot A^{3M}(T_k,T) \\
&\quad -\underbrace{\left(\Lambda^{6M/3M}(T_k,T)-\Lambda^{3M}(T_k,T)\right)}_{\substack{\text{Floating side of the}\\ \text{6M/3M EURIBOR basis swap}}}+B^{3M}(T_k,T)-1.
\end{aligned}
$$

The derivation uses the equilibrium condition for the 6-month EURIBOR interest rate swap and the 3-month EURIBOR interest rate swap. This representation reveals that the 6-month EURIBOR interest rate swap is decomposed into two risk factors: 3-month EURIBOR rate and tenor basis spread between 6-month and 3-month EURIBOR.

Inserting $t = t_0$ (where the swap value is zero at inception) results in the representation of the "synthetic and risk-equivalent bond":

EQUATION 8: Determination of the Risk-Equivalent Bond in the Multi-Curve Case

$$
\left.
\begin{aligned}
& c^{3M}(t_0,T)\cdot A^{3M}(T_k,T)-\underbrace{\Lambda^{6M/3M}(T_k,T)-\Lambda^{3M}(T_k,T)}_{\substack{\text{Dynamic adjustment:}\\ \text{floating side of the tenor basis swap}}} \\
& +\underbrace{\left(c^{6M}(t_0,T)-c^{3M}(t_0,T)\right)\cdot A^{3M}(T_k,T)}_{\text{Fair value of the initial tenor basis spread}}
\end{aligned}
\right\} \text{Internal coupon } *A^{3M}(T_k,T)
$$

$$
+B^{3M}(T_k,T) \quad \} \text{Discounted repayment}
$$

EQUATION 8: Determination of the Risk-Equivalent Bond in the Multi-Curve Case *(continued)*

$$
\begin{aligned}
&= c^{3M}(t_0,T)\cdot A^{3M}(T_k,T) \\
&\quad - \Big[\underbrace{\big(c^{6M}(T_k,T)-c^{3M}(T_k,T)\big)-\big(c^{6M}(t_0,T)-c^{3M}(t_0,T)\big)}_{\text{Adjustment to the current 6M/3M EURIBOR basis spread}}\Big] \\
&\quad \cdot A^{3M}(T_k,T) \qquad \Big\} \text{ Internal coupon } * A^{3M}(T_k,T) \\
&\quad + B^{3M}(T_k,T) \qquad \Big\} \text{ Discounted repayment} \\
&= 1 \quad \text{only if} \quad T_k = t_0
\end{aligned}
$$

According to *Equation 8* the following results can be derived:

- In a multi-curve setup the hedged risk and the portion of cash flows subject to hedge accounting according to IAS 39 cannot be defined independently from the entire set of derivatives, which constitutes the model-economy.
- In the example different forwarding and discounting result into a two risk factor representation of the "internal coupon", which corresponds to the portion of cash flows which is subject to hedge accounting:
  As indicated in *Equation 8* the EURIBOR component represented by the internal coupon $c_{\text{int},t_0}^{6M,3M}(T_k,T)$ is defined by

EQUATION 9: Determination of the EURIBOR Component in the Multi-Curve Setup

$$
c_{\text{int},t_0}^{6M,3M}(T_k,T) = c^{3M}(t_0,T) - \Big[\big(c^{6M}(T_k,T)-c^{3M}(T_k,T)\big)-\big(c^{6M}(t_0,T)-c^{3M}(t_0,T)\big)\Big]
$$

For $t = t_0$ the internal coupon $c_{\text{int},t_0}^{6M,3M}(t_0,T)$ coincides with the $c^{3M}(t_0,T)$ swap rate which represents the hedged risk. Additionally this definition does not introduce a "sub-LIBOR" issue.

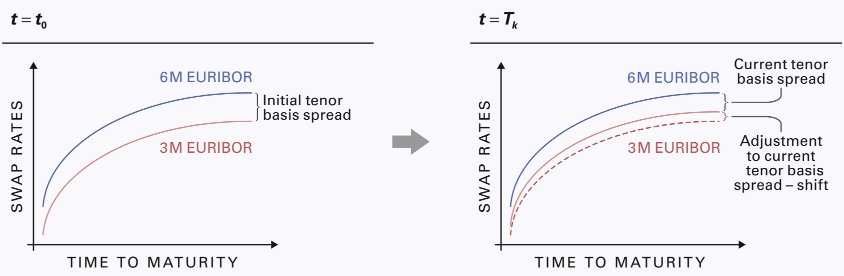
- The definition of the internal coupon is time-dependent; the internal coupon changes its value according to the tenor basis swap. It represents the time dependent adjustment of the 3-month EURIBOR interest rate swap rate to the 6-month EURIBOR interest rate swap rate at inception of the hedge.

  Please note that the definition of the internal coupon is a risk-equivalent and arbitrage-free representation of the swap rate $c^{6M}(t_0,T)$ discounted with the 3-month EURIBOR curve. The 6-month EURIBOR interest rate swap rate is now decomposed into a 3-month EURIBOR interest rate swap rate plus and the difference between the current and the initial tenor basis spread. This can be regarded as the periodic adjustment due to differences in forwarding the cash flows on a different basis than discounting them. Thus it can be termed (concept formation) the "3-month EURIBOR component hedged by and 6-month EURIBOR interest rate swap". This also reveals the impact of "reversing" the procedures of summarizing the 3-month/6-month EURIBOR basis spread and the 3-month EURIBOR into the 6-month EURIBOR interest rate swap curve.
- The time-dependence of the internal coupon results in a dynamic hedge accounting model under IAS 39.

  According to the time-dependent definition of the EURIBOR component, the change of the hedged risk 3-month EURIBOR component relative to the 6-month EURIBOR interest rate swap rate is adjusted. The adjustment refers to the initial tenor basis spread $\left(c^{6M}(t_0,T)-c^{3M}(t_0,T)\right)$ relative to the current tenor basis spread $\left(c^{6M}(T_k,T)-c^{3M}(T_k,T)\right)$. Approximately this can be regarded as a shift of the 3-month EURIBOR, which preserves the distance between the current 3-month EURIBOR curve and the 6-month EURIBOR curve at time T_k (see *Figure 54*).

This follows the same rationale as the calculation of dynamic adjustment in *Section 4.2.6.* The definition of the hedge fair value will be provided in the next *Section 4.3.3.*

FIGURE 54: Illustrative Example of the Dynamic Adjustment of the Internal Coupon

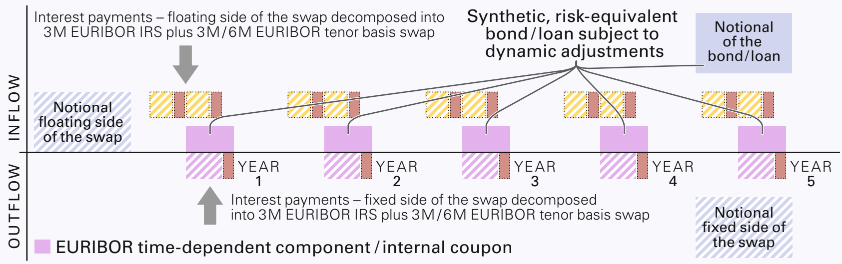


- Special cases of *Equation 8* are those where the term of the hedging instrument and that of the discount curve are identical (3-month or 6-month EURIBOR). In these cases the EURIBOR component equals the 3-month or the 6-month EURIBOR interest rate swap rate.
- If the difference between the tenor basis spread (curve) between the 6-month EURIBOR and the 3-month EURIBOR interest rate swap curve is small, then the EURIBOR component is close to the 3-month EURIBOR interest rate swap rate.
- If the differences in tenor basis spreads widen relative to the initial tenor basis spread, the EURIBOR component decreases and vice-versa.
- If the tenor basis spread (curve) is constant over time, then the dynamic hedging strategy becomes a static correction for the EURIBOR component.

4.3.3 Step 3: Representation of the Fair Value Hedge Accounting Model

According to the analysis of the economic hedging relationship in the multi-curve setup, the explicit decomposition into 3-month EURIBOR interest rate swap and 3-month/6-month EURIBOR tenor basis swap results in a dynamic hedging strategy. The dynamic hedging strategy is driven by the dynamics resulting from the differences in forwarding

FIGURE 55: **Representation of Hedge Accounting Model according to IAS 39 in the Multi-Curve Case**



and discounting. This dynamic can be reflected by the hedge accounting model, but it requires the determination of the "portion" of cash flows at each designation (usually at the beginning of each month) and the re-designation at each subsequent month as is already common practice in the application of macro hedge accounting models.

According to the definition of the EURIBOR component above, the fair value according to the hedged risk – "hedge fair value" (*HFV*) – is represented by:

$$HFV_{t_0}^{3M}(t,T) := c_{\text{int},t_0}^{6M,3M}(t,T) \cdot A^{3M}(t,T) + B^{3M}(t,T),$$

using the definition of the internal coupon

$$c_{\text{int},t_0}^{6M,3M}(t,T)$$
$$= c^{3M}(t_0,T) - \left[\left(c^{6M}(t,T) - c^{3M}(t,T)\right) - \left(c^{6M}(t_0,T) - c^{3M}(t_0,T)\right)\right].$$

Like with hedge accounting in the single-curve setup, ineffectiveness results from the floating side of the 3-month EURIBOR interest rate swap between reset dates. The effect of the floating side of the tenor basis swap (changes of the present value of the basis) according to the changed representation of risk and valuation factors is covered by the dynamically adjusted cash flows of the hedged item.

In the following the dynamic adjustment of the portion is demonstrated for a couple of periods (that are assumed to be months) in formulas forming the base for the calculation of an explicit example.

1st Period $\left(t_0, t_1^m\right)$**:** Let $t = t_0$ be the inception date, then the hedge fair value is defined as (refer to the analysis above)

$$HFV_{t_0}^{3M}\left(t_0, T\right) := c^{3M}\left(t_0, T\right) \cdot A^{3M}\left(t_0, T\right) + B^{3M}\left(t_0, T\right) = 1.$$

In order to distinguish the hedge fair value at the same point in time before de-designation and after re-designation, an additional notation is introduced: superscript D and R respectively. Thus for the hedge fair value at the first measurement date $t = t_1^m$ the dirty hedge fair value (indicated by the bar above) is calculated to be

$$\overline{HFV}_{t_0}^{3M,D}\left(t_1^m, T\right) = c^{3M}\left(t_0, T\right) \cdot A^{3M}\left(t_1^m, T\right) + B^{3M}\left(t_1^m, T\right).$$

As usual the clean hedge fair value is obtained by subtracting the accruals (acc):

$$HFV_{t_0}^{3M,D}\left(t_1^m, T\right) = \overline{HFV}_{t_0}^{3M,D}\left(t_1^m, T\right) - acc_{\overline{HFV}_{t_0}^{3M,D}\left(t_1^m, T\right)}\left(t_0, t_1^m\right).$$

Thus the first fair value change for the time period $\left(t_0, t_1^m\right)$ is

$$\begin{aligned}
&HFV_{t_0}^{3M,D}\left(t_1^m, T\right) - HFV_{t_0}^{3M}\left(t_0, T\right) \\
&= c^{3M}\left(t_0, T\right) \cdot A^{3M}\left(t_1^m, T\right) + B^{3M}\left(t_1^m, T\right) - acc_{HFV_{t_0}^{3M,D}\left(t_1^m, T\right)}\left(t_0, t_1^m\right) - 1 \\
&= \left[c^{3M}\left(t_0, T\right) - c^{3M}\left(t_1^m, T\right)\right] \cdot A^{3M}\left(t_1^m, T\right) - acc_{HFV_{t_0}^{3M,D}\left(t_1^m, T\right)}\left(t_0, t_1^m\right).
\end{aligned}$$

Then the hedge is de-designated.

2nd Period $\left(t_1^m, t_2^m\right)$**:** The hedge is re-designated with the dynamically adjusted internal coupon

$$\begin{aligned}
&c_{\text{int},t_0}^{6M,3M}\left(t_1^m, T\right) \\
&= c^{3M}\left(t_0, T\right) - \left[\left(c^{6M}\left(t_1^m, T\right) - c^{3M}\left(t_1^m, T\right)\right) - \left(c^{6M}\left(t_0, T\right) - c^{3M}\left(t_0, T\right)\right)\right].
\end{aligned}$$

Resulting in the following hedge fair value for period $\left(t_1^m, t_2^m\right)$:

$$\overline{HFV}_{t_0}^{3M,R}\left(t_1^m,T\right)=c_{\text{int},t_0}^{6M,3M}\left(t_1^m,T\right)\cdot A^{3M}\left(t_1^m,T\right)+B^{3M}\left(t_1^m,T\right)$$

$$HFV_{t_0}^{3M,R}\left(t_1^m,T\right)=\overline{HFV}_{t_0}^{3M,R}\left(t_1^m,T\right)-acc_{\overline{HFV}_{t_0}^{3M,R}\left(t_1^m,T\right)}\left(t_0,t_1^m\right).$$

At the end of this period we have at t_2^m:

$$\overline{HFV}_{t_0}^{3M,D}\left(t_2^m,T\right)=c_{\text{int},t_0}^{6M,3M}\left(t_1^m,T\right)\cdot A^{3M}\left(t_2^m,T\right)+B^{3M}\left(t_2^m,T\right)\cdot A^{3M}\left(t_2^m,T\right)$$

$$HFV_{t_0}^{3M,D}\left(t_2^m,T\right)=\overline{HFV}_{t_0}^{3M,D}\left(t_2^m,T\right)-acc_{\overline{HFV}_{t_0}^{3M,D}\left(t_2^m,T\right)}\left(t_0,t_2^m\right).$$

Thus the second fair value change for the time period $\left(t_1^m, t_2^m\right)$ is

$$HFV_{t_0}^{3M,D}\left(t_2^m,T\right)-HFV_{t_0}^{3M,R}\left(t_1^m,T\right)$$

$$=\overline{HFV}_{t_0}^{3M,D}\left(t_2^m,T\right)-acc_{\overline{HFV}_{t_0}^{3M,D}\left(t_2^m,T\right)}\left(t_0,t_2^m\right)-\overline{HFV}_{t_0}^{3M,R}\left(t_1^m,T\right)$$
$$+acc_{\overline{HFV}_{t_0}^{3M,R}\left(t_1^m,T\right)}\left(t_0,t_1^m\right)$$

$$=acc_{\overline{HFV}_{t_0}^{3M,R}\left(t_1^m,T\right)}\left(t_0,t_1^m\right)-acc_{\overline{HFV}_{t_0}^{3M,D}\left(t_2^m,T\right)}\left(t_0,t_2^m\right)$$
$$+\left[c^{3M}\left(t_0,T\right)-\left[\begin{matrix}\left(c^{6M}\left(t_1^m,T\right)-c^{3M}\left(t_1^m,T\right)\right)\\-\left(c^{6M}\left(t_0,T\right)-c^{3M}\left(t_0,T\right)\right)\end{matrix}\right]\right]\cdot A^{3M}\left(t_2^m,T\right)+B^{3M}\left(t_2^m,T\right)$$
$$-\left[c^{3M}\left(t_0,T\right)-\left[\begin{matrix}\left(c^{6M}\left(t_1^m,T\right)-c^{3M}\left(t_1^m,T\right)\right)\\-\left(c^{6M}\left(t_0,T\right)-c^{3M}\left(t_0,T\right)\right)\end{matrix}\right]\right]\cdot A^{3M}\left(t_1^m,T\right)-B^{3M}\left(t_1^m,T\right)$$

$$=acc_{\overline{HFV}_{t_0}^{3M,R}\left(t_1^m,T\right)}\left(t_0,t_1^m\right)-acc_{\overline{HFV}_{t_0}^{3M,D}\left(t_2^m,T\right)}\left(t_0,t_2^m\right)+B^{3M}\left(t_2^m,T\right)-B^{3M}\left(t_1^m,T\right)$$
$$+\left[c^{3M}\left(t_0,T\right)-\left[\begin{matrix}\left(c^{6M}\left(t_1^m,T\right)-c^{3M}\left(t_1^m,T\right)\right)\\-\left(c^{6M}\left(t_0,T\right)-c^{3M}\left(t_0,T\right)\right)\end{matrix}\right]\right]\cdot\left[A^{3M}\left(t_2^m,T\right)-A^{3M}\left(t_1^m,T\right)\right].$$

The hedge is then de-designated in order re-designate it with the new portion.

j^{th} Period $\left(t_{j-1}^m, t_j^m\right)$: The hedge is re-designated with the dynamically adjusted internal coupon

$$c_{\text{int},t_0}^{6M,3M}\left(t_{j-1}^m, T\right)$$
$$= c^{3M}\left(t_0, T\right) - \left[\left(c^{6M}\left(t_{j-1}^m, T\right) - c^{3M}\left(t_{j-1}^m, T\right)\right) - \left(c^{6M}\left(t_0, T\right) - c^{3M}\left(t_0, T\right)\right)\right]$$

giving the following hedge fair value for period $\left(t_{j-1}^m, t_j^m\right)$:

$$\overline{HFV}_{t_0}^{3M,R}\left(t_{j-1}^m, T\right)$$
$$= \left[c^{3M}\left(t_0, T\right) - \left[\begin{array}{l}\left(c^{6M}\left(t_{j-1}^m, T\right) - c^{3M}\left(t_{j-1}^m, T\right)\right) \\ -\left(c^{6M}\left(t_0, T\right) - c^{3M}\left(t_0, T\right)\right)\end{array}\right]\right] \cdot A^{3M}\left(t_{j-1}^m, T\right) + B^{3M}\left(t_{j-1}^m, T\right).$$

Assuming that t_{j*} was the last reset date:

$$HFV_{t_0}^{3M,R}\left(t_{j-1}^m, T\right) = \overline{HFV}_{t_0}^{3M,R}\left(t_{j-1}^m, T\right) - acc_{\overline{HFV}_{t_0}^{3M,R}\left(t_{j-1}^m, T\right)}\left(t_{j*}, t_{j-1}^m\right).$$

At the end of this period we have at t_j^m:

$$\overline{HFV}_{t_0}^{3M,D}\left(t_j^m, T\right)$$
$$= \left[c^{3M}\left(t_0, T\right) - \left[\begin{array}{l}\left(c^{6M}\left(t_{j-1}^m, T\right) - c^{3M}\left(t_{j-1}^m, T\right)\right) \\ -\left(c^{6M}\left(t_0, T\right) - c^{3M}\left(t_0, T\right)\right)\end{array}\right]\right] \cdot A^{3M}\left(t_j^m, T\right) + B^{3M}\left(t_j^m, T\right)$$

$$HFV_{t_0}^{3M,D}\left(t_j^m, T\right) = \overline{HFV}_{t_0}^{3M,D}\left(t_j^m, T\right) - acc_{\overline{HFV}_{t_0}^{3M,D}\left(t_j^m, T\right)}\left(t_{j*}, t_j^m\right).$$

Thus the j^{th} fair value change for the time period $\left(t_{j-1}^m, t_j^m\right)$ is

$$HFV_{t_0}^{3M,D}\left(t_j^m, T\right) - HFV_{t_0}^{3M,R}\left(t_{j-1}^m, T\right)$$
$$= acc_{\overline{HFV}_{t_0}^{3M,R}\left(t_{j-1}^m, T\right)}\left(t_{j*}, t_{j-1}^m\right) - acc_{\overline{HFV}_{t_0}^{3M,D}\left(t_j^m, T\right)}\left(t_{j*}, t_j^m\right) + B^{3M}\left(t_j^m, T\right) - B^{3M}\left(t_{j-1}^m, T\right)$$
$$+ \left[c^{3M}\left(t_0, T\right) - \left[\begin{array}{l}\left(c^{6M}\left(t_{j-1}^m, T\right) - c^{3M}\left(t_{j-1}^m, T\right)\right) \\ -\left(c^{6M}\left(t_0, T\right) - c^{3M}\left(t_0, T\right)\right)\end{array}\right]\right] \cdot \left[A^{3M}\left(t_j^m, T\right) - A^{3M}\left(t_{j-1}^m, T\right)\right].$$

The hedge is then de-designated in order re-designate it with the new portion.

TABLE 21: Comparison of Hedge Effectiveness in Different Hedge Accounting Setups

	Discount curve	Swap	Adjustment	Regression		Cumulative dollar offset		
				slope	R^2	Min.	Max.	Mean
Single-curve hedge accounting	6M	6M	N.a.	–1.0373	0.9925	81.09%	101.74%	96.04%
	3M	3M	N.a.	–1.0222	0.9974	92.14%	100.00%	98.70%
	OIS	OIS	N.a.	–1.0878	0.9895	–33.31%	128.01%	90.12%
Multi-curve hedge accounting	3M	6M	No	–1.0876	0.9766	78.27%	100.64%	92.32%
	OIS	6M	No	–1.2040	0.9190	–1222.61%	5484.40%	306.97%
	3M	6M	Yes	–1.0428	0.9944	91.79%	100.00%	97.29%
	OIS	6M	Yes	–1.1568	0.9726	73.77%	129.91%	97.39%

In *Table 21* the results of different model setups and effectiveness measurement methods are portrayed. The results include an example using OIS discounting, which are derived similarly to the analysis above: the 3-month EURIBOR discount curve is exchanged for the OIS discount curve.

- The results of the single-curve setups are shown for comparison purposes, the outliers in case of OIS discounting result from the well-known problem of small numbers.
- If the multi-curve setup is applied, the table indicates that without any adjustments the hedge effectiveness decreases. The dynamic adjustments increase the results for hedge effectiveness, which are close to – not identical with – the results in the single-curve setup.
- The multi-curve setup involving regular re-designation is not without cost, since the recognized fair value adjustments are amortized into interest. Therefore overall the multi-curve approach induces "ineffectiveness" shown in the interest P & L.
- Furthermore the techniques for effectiveness testing do not necessarily use the same values as those determined for the booking entries. This results from the dynamic adjustment feature, since like in single-curve hedge accounting[70], fair values evaluated according

to the hedged risk in general do not coincide with the book values of the recognized assets / liabilities. These differences are taken into account for effectiveness test purposes, but do not enter into the booking entries in order to avoid double recognition.

Continuing the example above, an extract of the relevant booking entries is provided in the following. The economic hedging is performed more frequently than the hedge accounting process which is typically accomplished on a monthly basis and is thus discrete in time. Therefore the economic hedging model (dynamic adjustment) in continuous time reflects the economic result of the hedging relationship. The formulas derived for the hedged item above carry over to continuous time, representing the correct economic result. Accordingly the booking entries have to take into account a discretization effect in order to reflect the correct economic result. Since the focus is on the method of dynamic adjustment of the portion with de- and re-designation for each period, the following simplifications and assumptions for the representation are made:

- Fair Values are clean fair values.
- Fair rates for swaps where the term to maturity is not an integer factor of the repricing cycle are calculated with a short first period, but on the tenor rate.
- No calendars, business day conventions or fixing days are considered.
- Curves setup of MM and swap rates are used with linear interpolation.
- No interest payments or accruals are booked.
- Separate line item (SLI) is used for hedged adjustment.
- For each amortization a single sub-account is created.
- Monthly reporting.

70 E.g. due to amortizations, installment, impairment (not part of the hedged risk) or the consideration of clean and dirty fair values.

Contracts with the data shown in *Table 22* are considered.

Table 23 shows clean fair values that are calculated for the first periods (effectiveness testing results are shown in *Table 21*).

Following the outline above, this will result to the booking entries shown in *Table 24*.

Here the sub-accounts of the loan account are denoted as follows:

- Loan SLI: sub-account for the separate line item containing the hedge adjustment of the current period.
- Loan t_x_A_yz: sub-account for amortization of the hedge adjustment of the period ending with de-designation at date t_x with yz month(s) remaining to maturity.

TABLE 22: Dynamic Adjustment Example – Contracts

Terms and conditions	Hedged item	6M EURIBOR IRS fixed side	6M EURIBOR IRS floating side
Value date / inception date	01/23/2009	01/23/2009	01/23/2009
Maturity	01/23/2014	01/23/2014	01/23/2014
Interest payment frequency	Annually	Annually	Semi-annually
Notional	100,000,000.00€	–100,000,000.00€	100,000,000.00€
Day count convention	30/360	30/360	ACT/360

TABLE 23: Dynamic Adjustment Example – Clean Fair Values

Date	Internal coupon	$HFV^{3M,R}$	$HFV^{3M,D}$	Fair value swap
t_0	2.9620%	100,000,000.00€		0.00€
t_1	2.9436%	100,990,043.45€	101,072,774.98€	–908,810.38€
t_2	2.9693%	101,183,582.84€	101,071,726.67€	–1,095,888.32€
t_3	2.8976%	100,993,644.95€	101,294,359.24€	–979,524.17€

TABLE 24: Dynamic Adjustment Example – Booking Entries

Date/period	Reason	Debit	Credit	Amount
t_0 (designation)	Grant of the loan	Loan	Cash	100,000,000.00€
	Purchase of the swap	Cash	Trading derivative	0.00€
	Reclass of the swap	Trading derivative	Hedging derivative	0.00€
$t_0 - t_1$	Hedge fair value adjustment $HFV_{t_0}^{3M,D}(t_1,T) - HFV_{t_0}^{3M,R}(t_0,T)$	Loan SLI	P&L– hedge result	1,072,774.98€
	Hedge fair value adjustment $HFV_{t_0}^{3M,R}(t_1,T) - HFV_{t_0}^{3M,D}(t_1,T)$ due to discretization	P&L– hedge result	Loan SLI	82,731.53€
	Fair value change of the swap	P&L– trading result	Hedging derivative	908,810.38€
	Reclass of fair value change of the swap	P&L– hedge result	P&L– trading result	908,810.38€
t_1 (de-/re-designation)	Reclass for amortization of recognized fair value change of the hedged item	Loan t_1_A_59	Loan SLI	1,072,774.98€
	Reclass for amortization of recognized fair value change of the hedged item	Loan SLI	Loan t_1_A_59	82,731.53€
$t_1 - t_2$	Hedge fair value adjustment $HFV_{t_0}^{3M,D}(t_2,T) - HFV_{t_0}^{3M,R}(t_1,T)$	Loan SLI	P&L– hedge result	81,683.22€
	Hedge fair value difference $HFV_{t_0}^{3M,R}(t_2,T) - HFV_{t_0}^{3M,D}(t_2,T)$ due to discretization	Loan SLI	P&L– hedge result	111,856.17€
	Fair value change of the swap	P&L– trading result	Hedging derivative	187,077.94€
	Reclass of fair value change of the swap	P&L– hedge result	P&L– trading result	187,077.94€
	Amortization of hedge result of period $t_0 - t_1$ over remaining term to maturity 59M	P&L– interest result	Loan t_1_A_59	16,780.40€
t_2 (de-/re-designation)	Reclass for amortization of recognized fair value change of the hedged item	Loan t_2_A_58	Loan SLI	81,683.22€
	Reclass for amortization of recognized fair value change of the hedged item	Loan t_2_A_58	Loan SLI	111,856.17€

TABLE 24: Dynamic Adjustment Example – Booking Entries *(continued)*

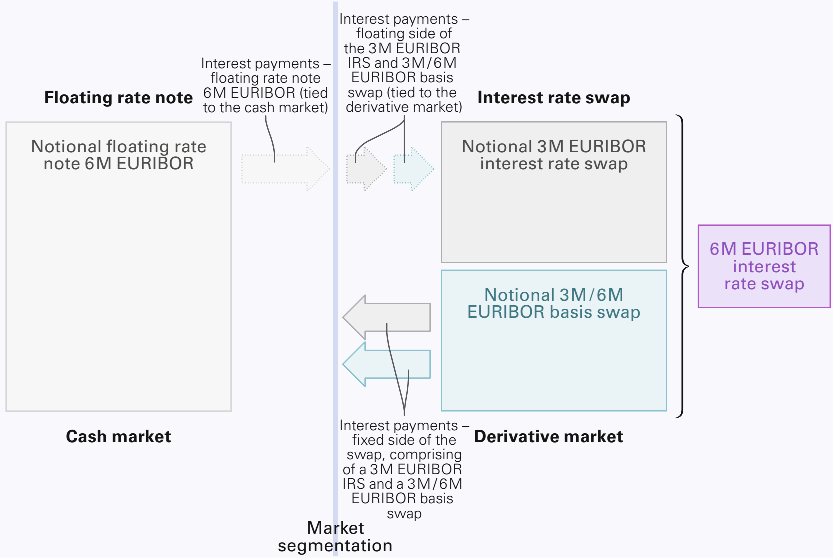
Date/period	Reason	Debit	Credit	Amount
$t_2 - t_3$	Hedge fair value adjustment $HFV_{t_0}^{3M,D}(t_3,T) - HFV_{t_0}^{3M,R}(t_2,T)$	Loan SLI	P&L– hedge result	110,776.40€
	Hedge fair value difference $HFV_{t_0}^{3M,R}(t_3,T) - HFV_{t_0}^{3M,D}(t_3,T)$ due to discretization	P&L– hedge result	Loan SLI	300,714.29€
	Fair value change of the swap	Hedging derivative	P&L– trading result	116,364.15€
	Reclass of fair value change of the swap	P&L– trading result	P&L– hedge result	116,364.15€
	Amortization of hedge result of period $t_0 - t_1$ over remaining term to maturity 59M	P&L– interest result	Loan t_1_A_59	16,780.40€
	Amortization of hedge result of period $t_1 - t_2$ over remaining term to maturity 58M	P&L– interest result	Loan t_2_A_58	3,336.89€
t_3 (de-/re-designation)	Reclass for amortization of recognized fair value change of the hedged item	Loan t_3_A_57	Loan SLI	110,776.40€
	Reclass for amortization of recognized fair value change of the hedged item	Loan SLI	Loan t_3_A_57	300,714.29€

4.3.4 Cash Flow Hedge Accounting

As shown above, the economic rationale of multi-curve models applies similarily to the cash flow hedge accounting model. *Figure 56* portrays the decomposition of the 6-month EURIBOR interest rate swap into the 3-month EURIBOR interest rate swap and the 3-month/6-month EURIBOR tenor basis swap.

Like in the example in *Section 4.2.5* the cash flows of the floating rate note and the floating side of the 6-month EURIBOR interest rate swap match, but now, according to the changes in discount curves, the fair value of the floating rate side of the swap is different. As shown above:

FIGURE 56: Representation of Risk and Valuation Factors in the Cash Flow Hedge Accounting Model according to IAS 39



- The fair value of the floating rate side of the swap is not par at inception, but the entire swap is!
- The fair value changes of the floating rate side of the interest rate swap are tied to two risk and valuation factors: the 3-month forward and the 3-month/6-month forward rate.
- According to the economic rationale implied by the "hypothetical derivative" method, the forward rates of the floating rate note are exchanged by the 3-month forward and the 3-month/6-month forward rates.
- "Economically" the 6-month EURIBOR money market cash flow of the floating rate note is decomposed into a 3-month EURIBOR cash flow and a 3-month/6-month EURIBOR cash flow money market cash flow. As can be seen from the previous *Section 2,* such a cash flow decomposition does not exist in money markets, this is entirely synthetic.

The application of the "hypothetical derivative" method implies that there is no ineffectiveness according to the change in discount curves, since the changes in fair value of the hypothetical derivative exactly mirror the changes in fair value of the 6-month EURIBOR interest rate swap discounted with the 3-month EURIBOR interest rate swap rates. As shown in the previous *Section 2,* the pricing of the derivative market is applied to the cash market with respect to the effectiveness testing in the case of the cash flow hedge accounting model. *Figure 57* summarizes the economic rationale of the cash flow hedge accounting model in the multi-curve setup.

As mentioned before, the tenor basis swap in this context can be considered as a separate risk factor but not necessarily as a separate hedging instrument. If otherwise the inherent tenor basis swap were not considered as part of the hedging instrument, the cash flow hedge accounting would have to be performed with a 3-month EURIBOR interest rate swap. The requirements of cash flow hedge accounting are considered met if the variability in cash flows shows a close correlation between the 3-month and 6-month EURIBOR. *Figure 58* shows the regression of monthly changes of the 3-month EURIBOR versus the 6-month EURIBOR over the last two years with a correlation of 95 %.

But considering the case of OIS discounting which is the standard for collateralized trades, this argument would fail in the sense that correlation even versus the 3-month compounded EONIA is only about 75 % (see *Figure 59*).

FIGURE 57: **Market Integration Model Implied by the Cash Flow Hedge Accounting Model according to IAS 39 in the Multi-Curve Setup**

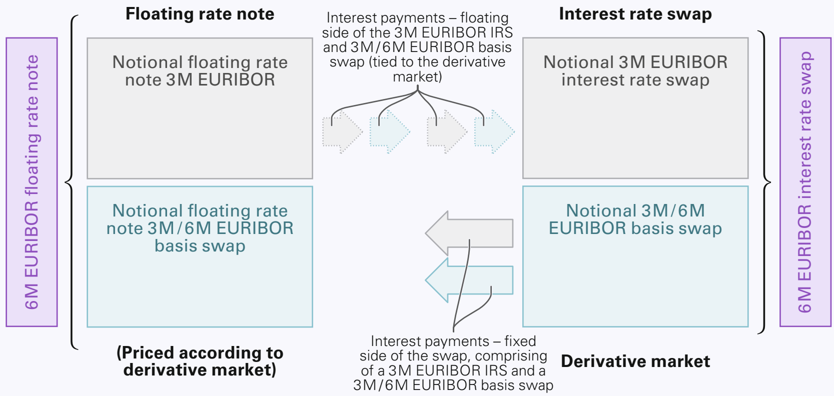


FIGURE 58: **Regression of Monthly Changes 3M vs. 6M EURIBOR Money Market Rates, 2010 – 2011** (Source: Reuters)

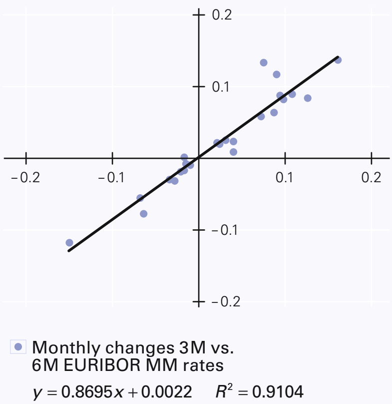
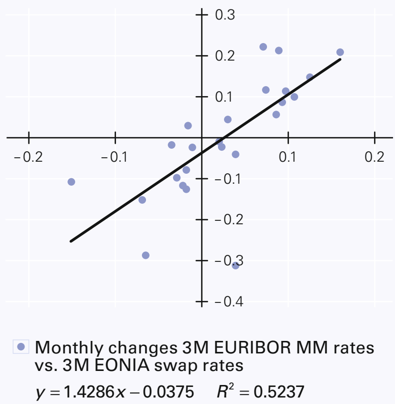


FIGURE 59: **Regression Analysis of 3M EURIBOR Money Market Rates vs. 3M EONIA Swap Rates, 2010 – 2011** (Source: Reuters)



4.4 Comparison of Fair Value Hedge Accounting according to IAS 39 in a Single- and Multi-Curve Model

In *Table 25* the hedge accounting models in a single- and multi-curve setup are summarized. *Table 19* in the previous *Section 3* showing the coherence of the financial economics with hedge accounting requirements according to IAS 39 is still valid in both model setups. For the sake of brevity the conceptions like absence of arbitrage, completeness, elimination of basis risk etc. are omitted. Cash flow hedge accounting model follows a similar economic rationale. It should be noted that none of the "risk factors" considered have proven to be of statistical explanatory power – apart from accidental statistical coincidence. This fact is irrespective of the applied hedge accounting model.

The major difference between the single- and the multi-curve model setups is the handling with respect to tenor basis swaps. In a single-curve model the tenor basis swaps are incorporated into one benchmark curve, whereas in a multi-curve setup the tenor basis swaps are modeled explicitly. This results in a dynamic hedge accounting model, since the 3-month EURIBOR benchmark curve is not fully correlated with 3-month/6-month basis swaps. In the case of fair value hedge accounting this dynamic has to be covered by regular adjustments in the cash flows of the hedged item, so that regular designation and re-designation is necessary.

TABLE 25: Summary Hedge Accounting according to IAS 39 in a Single- and Multi-Curve Model

Requirement IAS 39.AG99F	Financial economics (single-curve model)	Financial economics (multi-curve model)
(Hedged item)	Hedging instrument: 6M EURIBOR interest rate swap, funding: 6M EURIBOR	Hedging instrument: 6M EURIBOR interest rate swap, funding: 6M EURIBOR
Portion	– 6M EURIBOR interest rate swap rate. – "Implicit representation of risk factors": incorporation of the 3M EURIBOR and 3M/6M EURIBOR tenor basis swap into one benchmark curve (swap rate). – "Implied" integrated market for hedging instruments 3M EURIBOR and 3M/6M EURIBOR tenor basis swap. – Non-dynamic due to: forwarding = discounting. – Determination of a (cash flow) component attributable to the designated risk by the hedging instrument: 6M EURIBOR interest rate swap rate.	– 3M swap rate and dynamic adjustment to the current tenor. – "Explicit" integrated market for derivatives: 3M EURIBOR and 3M/6M EURIBOR tenor basis swap. – "Explicit" presentation of two risk factors: 3M/6M EURIBOR tenor basis swaps and 3M EURIBOR interest rate swaps. – Dynamic due to: forwarding ≠ discounting. – Determination of a (cash flow) component attributable to the designated risk by the hedging instrument: 6M EURIBOR interest rate swap.
Separately identifiable	Identification by the hedging instruments and derivation of the "benchmark curve" – (derivative) zero 3M EURIBOR/3M/6M EURIBOR tenor basis swap resulting in a "single" 6M EURIBOR zero swap rate curve. 6M EURIBOR zero swap rates utilized for discounting.	Identification by the hedging instruments: zero 3M EURIBOR/3M/6M EURIBOR tenor basis swap and derivation of the "benchmark curve" – (derivative) 3M EURIBOR zero swap rate curve. 3M EURIBOR zero swap rates utilized for discounting.
Reliably measureable	Existence of a liquid market for the hedging instrument to derive the "benchmark curve", market for hedging instruments 3M EURIBOR and 3M/6M EURIBOR tenor basis swap.	Existence of a liquid market for the hedging instrument to derive the "benchmark curve", market for hedging instruments 3M EURIBOR and 3M/6M EURIBOR tenor basis swap.

SECTION FIVE

5

Hedge Accounting of FX Risk and the FX Basis Risk

5.1 Introduction – FX Risk is an Unobservable Overlay Risk

FX risk represents a prominent example for the application of multi-curve model setups. This stems from the fact that foreign and domestic currency discount curves cannot be modeled independently from each other. Furthermore economic modeling of FX risk involves a model for the exchange of cash payments. Consequently the modeling of FX risk consists of a model concerning interest rate, liquidity (in terms of the exchange of cash) and FX risk. Therefore FX risk is a typical example of an overlay risk, since it requires a model with several different, but interdependent risk factors. Similar to the modeling of interest rate risk, FX risk is an unobservable risk, since some risk factors need to be derived from traded market instruments. FX contracts represent common OTC derivative contracts to hedge against FX risk, which require a simultaneous modeling of different risk factors and are used to determine the foreign currency forward rates.

5.2 Hedge Accounting of FX Risk without FX Basis Risk

5.2.1 Cash and Carry Arbitrage Relationship – Interest Rate Parity

If a domestic based company buys or sells goods in a foreign currency and wants to hedge against movements of the FX rate, it enters into an FX forward contract. An FX forward contract is a derivative contract which gives the holder the right to buy or sell a specified foreign currency amount at a pre-specified forward-price on a future date T.

An EUR based company wants to buy goods from a US based company and has to pay USD (\$) at some future time T. The EUR based company is exposed to currency risk since, if the USD appreciates against the EUR, the EUR based company has to pay more for the goods and vice versa.

A possible way to eliminate the currency risk for the EUR based company is to enter into an FX contract to buy USD (long position).

Notation:

- $S_{€}^{\$}(t)$ defines the spot exchange rate of EUR vs. USD at time t.
- $B_{€}(t_0,t)$, $B_{\$}(t_0,t)$: price of EUR resp. USD denominated zero coupon bond;
- $f_{€}^{\$}(t_0,t)$: forward-price of EUR vs. USD at time t_0 for delivery at time t;
- $F_{€}^{\$}\left[f_{€}^{\$}(t_0,T),t\right]$: (fair) present value of a foreign exchange (FX) forward contract at time t with forward price $f_{€}^{\$}(t_0,t)$.

In order to derive the price or present value of a $F_{€}^{\$}\left[f_{€}^{\$}(t_0,T),t\right]$ FX forward contract the following two strategies are compared:

At time $t=t_0$:

Strategy no. 1:
Enter into a long position of an FX forward contract $F_{€}^{\$}\left[f_{€}^{\$}(t_0,T),t_0\right]$ at time $t=t_0$ to buy USD at time $t=T$. By market convention the price of $F_{€}^{\$}\left[f_{€}^{\$}(t_0,T),t_0\right]$ is zero.

Strategy no. 2:
Buy USD zero coupon bond at a price $B_{\$}(t_0,T)$ or $S_{€}^{\$}(t_0)\cdot B_{\$}(t_0,T)$ EUR. Borrow for the period $[t_0,T]$ the present value of the forward-price $B_{€}(t_0,T)\cdot f_{€}^{\$}(t_0,T)$.

Total cost of strategy no. 2: $S_{€}^{\$}(t_0)\cdot B_{\$}(t_0,T)-B_{€}(t_0,T)\cdot f_{€}^{\$}(t_0,T)$.

At time $t=T$ the payoffs of both strategies are compared:

Strategy no. 1:

$$F_{€}^{\$}\left[f_{€}^{\$}(t_0,T),T\right]=S_{€}^{\$}(T)-f_{€}^{\$}(t_0,T),$$

Strategy no. 2:

$$\begin{cases} \quad S_{€}^{\$}(T)\cdot B_{\$}(T,T)=S_{€}^{\$}(T)\cdot 1 \\ -f_{€}^{\$}(t_0,T)\cdot B_{€}(T,T)=f_{€}^{\$}(t_0,T)\cdot 1 \\ \hline \qquad\qquad =S_{€}^{\$}(T)-f_{€}^{\$}(t_0,T). \end{cases}$$

Since both strategies result in the identical payoffs at $t=T$, both strategies have the same present value at time $t=t_0$ given the absence of arbitrage. This implies:

EQUATION 10: Foreign Currency Interest Rate Parity

$$0 \overset{!}{=} S_{€}^{\$}(t_0) \cdot B_{\$}(t_0,T) - B_{€}(t_0,T) \cdot f_{€}^{\$}(t_0,T)$$

$$S_{€}^{\$}(t_0) \cdot B_{\$}(t_0,T) = B_{€}(t_0,T) \cdot f_{€}^{\$}(t_0,T)$$

$$\text{or} \quad S_{€}^{\$}(t_0) \cdot \frac{B_{\$}(t_0,T)}{B_{€}(t_0,T)} = f_{€}^{\$}(t_0,T)$$

Equation 10 is termed "foreign currency interest rate parity", which links the domestic term-structure curve to the foreign term structure curve. If the domestic interest rates are higher than the foreign interest rates then the FX forward rates tend to be higher than the spot exchange rate and vice versa.

Observe that the *Equation 10* also holds for any $t < T$,

$$S_{€}^{\$}(t) \cdot B_{\$}(t,T) = B_{€}(t,T) \cdot f_{€}^{\$}(t,T),$$

$$\begin{aligned} 0 &= F_{€}^{\$}\left[f_{€}^{\$}(t,T),t\right] \\ &= PV_t\left[S_{€}^{\$}(T) - f_{€}^{\$}(t,T)\right] \\ &= PV_t\left[S_{€}^{\$}(T)\right] - f_{€}^{\$}(t,T) \cdot B_{€}(t,T) \end{aligned}$$

$$\Rightarrow \quad PV_t\left[S_{€}^{\$}(T)\right] = f_{€}^{\$}(t,T) \cdot B_{€}(t,T)$$

so the present value (fair value) of the foreign exchange forward is given by:

EQUATION 11: Fair Value of Foreign Exchange Forward

$$\begin{aligned} F_{€}^{\$}\left[f_{€}^{\$}(t_0,T),t\right] &= PV_t\left[S_{€}^{\$}(T) - f_{€}^{\$}(t_0,T)\right] \\ &= PV_t\left[S_{€}^{\$}(T)\right] - f_{€}^{\$}(t_0,T) \cdot B_{€}(t,T) \\ &= f_{€}^{\$}(t,T) \cdot B_{€}(t,T) - f_{€}^{\$}(t_0,T) \cdot B_{€}(t,T) \\ &= \left[f_{€}^{\$}(t,T) - f_{€}^{\$}(t_0,T)\right] \cdot B_{€}(t,T) \end{aligned}$$

Please note that so far the "zero coupon" bonds have not been specified. Since FX forward contracts represent OTC derivatives, the "zero coupon" bonds are derived from interest rate swaps. Therefore the interest rate swaps denominated in USD and EUR as well as the spot exchange rate describes the entire model economy. In the following we assume that the 3-month EURIBOR and the 3-month USD LIBOR form the model economy (see *Equation 12*):

EQUATION 12: Definition of Equilibrium Conditions for Interest Rate Swaps in an Exchange Rate Economy

EUR:

$$\overbrace{c_{€}^{3M}(t_0,T)\cdot\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B_{€}^{3M}(t_0,T_k)}^{\text{PV of the fixed side of the 3M EURIBOR interest rate swap}} \overset{!}{=} \underbrace{\left(1-B_{€}^{3M}(t_0,T)\right)}_{\text{PV of the floating side of the 3M EURIBOR interest rate swap}}$$

USD:

$$\overbrace{c_{\$}^{3M}(t_0,T)\cdot\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B_{\$}^{3M}(t_0,T_k)}^{\text{PV of the fixed side of the 3M LIBOR interest rate swap}} \overset{!}{=} \underbrace{\left(1-B_{\$}^{3M}(t_0,T)\right)}_{\text{PV of the floating side of the 3M LIBOR interest rate swap}}$$

EUR /USD exchange rate: $S_{€}^{\$}(t)$

Note that according to *Equation 12* the domestic and the foreign interest rate curve cannot be modeled independently from each other.

In the following an example of a cash flow hedge for an FX hedge of a highly probable forecast transaction by an FX forward contract will be provided in order to demonstrate the interaction of FX and interest rate risk. This interaction is already considered in IAS 39.74(b) (analog to example IAS 39 IGF.5.6) by allowing the separation of the interest element and the spot price of an FX forward contract. There are two possibilities to perform hedge accounting:

1. Designating the forward FX risk, i.e. taking the whole fair value changes of the FX forward contract as hedging instrument into account.

TABLE 26: FX Hedge Example – (Fair) FX Forward Contract

Terms and conditions	Value date	Maturity T	Inverse FX Forward rate	Amount purchased	Amount sold	Day count convention
FX Forward contract	06/08/20X0	06/08/20X1	1.193346	100,000.00$	83,797.99€	ACT/360

TABLE 27: FX Hedge Example – Market Data

Date	Days to maturity	Inverse exchange rate $\left(S_{€}^{\$}\right)^{-1}$ USD for 1 EUR	EUR zero rate date – maturity	USD zero rate date – maturity	Inverse FX forward rate $\left(f_{€}^{\$}\right)^{-1}$ date – maturity
t_0 **= 06/08/20X0**	365	1.1942	1.2680%	1.1966%	1.193346
t_1 **= 12/08/20X0**	182	1.3200	1.2590%	0.4584%	1.314691
T **= 06/08/20X1**	0	1.4608	1.0650%	0.1265%	1.460800

2. Designating only the spot component, i.e. taking only the fair value changes of the FX forward contract attributable to the changes of the spot rate into account.

The example assumes an EUR based company which intends to purchase goods in an amount of USD 100,000.00 on a future date 06/08/20X1. The company demonstrates that this future purchase meets the requirements of a highly probable forecast transaction (cf. IAS 39.88(c), IAS 39 IGF.3.7, KPMG Insights 7.7.230, 240). Economically the company hedges this future purchase by entering into a long position of the (fair) FX forward contract shown in *Table 26.* The corresponding market data are shown in *Table 27.*

Here the FX forward rates are calculated from the given zero and exchange rates using the foreign currency interest rate parity of *Equation 10* with linear discounting, since the term to maturity is equal or less than one year:

$$\left(f_{€}^{\$}(t_0,T)\right)^{-1} = \left(S_{€}^{\$}(t_0)\right)^{-1} \cdot \frac{B_{€}(t_0,T)}{B_{\$}(t_0,T)}$$

$$= 1.1942 \frac{\$}{€} \cdot \frac{(1 + 1.2680\% \cdot 365 / 360)^{-1}}{(1 + 1.1966\% \cdot 365 / 360)^{-1}}$$

$$= 1.193346 \frac{\$}{€},$$

$$\left(f_€^\$(t_1, T)\right)^{-1} = \left(S_€^\$(t_1)\right)^{-1} \cdot \frac{B_€(t_1, T)}{B_\$(t_1, T)}$$

$$= 1.32 \frac{\$}{€} \cdot \frac{(1 + 1.2590\% \cdot 182 / 360)^{-1}}{(1 + 0.4584\% \cdot 182 / 360)^{-1}}$$

$$= 1.314691 \frac{\$}{€}.$$

At maturity T the FX forward rate coincides with the spot exchange rate of 1.4608.

Using the formula derived in *Equation 11,* for the FX forward contract this yields the fair values and changes shown in *Table 28:*

$$F_€^\$\left[f_€^\$(t_0, T), t_1\right] = N_\$ \cdot \left[f_€^\$(t_1, T) - f_€^\$(t_0, T)\right] \cdot B_€(t_1, T)$$

$$= 100{,}000.00\,\$ \cdot \left[\frac{1}{1.314691} - \frac{1}{1.193346}\right] \frac{€}{\$}$$

$$\cdot \frac{1}{(1 + 1.2590\% \cdot 182 / 360)}$$

$$= -7{,}685.60\,€,$$

$$F_€^\$\left[f_€^\$(t_0, T), T\right] = N_\$ \cdot \left[f_€^\$(T, T) - f_€^\$(t_0, T)\right] \cdot B_€(T, T)$$

$$= 100{,}000.00\,\$ \cdot \left[\frac{1}{1.4608} - \frac{1}{1.193346}\right] \frac{€}{\$}$$

$$= -15{,}342.35\,€.$$

TABLE 28: FX Hedge Example – FV and FV Changes of the FX Forward Contract

Date	FX Forward contract – FV	FX Forward contract – FV changes
t_0 = 06/08/20X0	0.00€	0.00€
t_1 = 12/08/20X0	–7,685.60€	–7,685.60€
T = 06/08/20X1	–15,342.35€	–7,656.76€

TABLE 29: FX Hedge Example – Forward Liability / OCI

Date	Forward liability	OCI
t_0 = 06/08/20X0	0.00€	0.00€
t_1 = 12/08/20X0	–7,685.60€	7,685.60€
T = 06/08/20X1	–7,656.76€	7,656.76€

Since a cash flow hedge is applied, the effectiveness will be measured using the hypothetical derivative method. Because the hedging instrument exactly matches the terms and conditions of the hedged item (critical term match), it also corresponds exactly to the hypothetical derivative according to the assumptions of this method. The hypothetical derivative therefore equals the hedging FX forward contract with opposite sign and initial value of zero. Prospective effectiveness is performed by critical terms match, and retrospective effectiveness measurement will show that fair value changes of the hypothetical derivative and the hedging instrument in this ideal case will perfectly offset each other[71]. Since the same calculations are performed for the effectiveness testing, hedge accounting will be described for both methods of hedge accounting mentioned above.

Using the first possibility of hedge accounting, designating the forward FX risk would result in the booking entries for the accounts "forward liability", other comprehensive income (OCI) and P&L shown in *Table 29.*

71 For reasons of simplicity the impact of counterparty credit risk on the hedging instrument is neglected in this context.

Applying the second method would only designate the spot component (measured relatively to the initial spot exchange rate) as hedged risk (see *Table 30*), which is calculated by the formula

$$Spot\_component_{€}^{\$}(t,T) = N_{\$} \cdot \left[S_{€}^{\$}(t,T) - S_{€}^{\$}(t_0,T) \right] \cdot B_{€}(t,T):$$

$$Spot\_component_{€}^{\$}(t_1,T)$$
$$= 100{,}000.00\,\$ \cdot \left[\frac{1}{1.32} - \frac{1}{1.1942} \right] \cdot \frac{1}{(1+1.2590\,\% \cdot 182\,/\,360)}$$
$$= -7{,}930.02\,€,$$

$$Spot\_component_{€}^{\$}(T,T)$$
$$= 100{,}000.00\,\$ \cdot \left[\frac{1}{1.4608} - \frac{1}{1.1942} \right]$$
$$= -15{,}282.43\,€.$$

Designating only the spot component of the FX forward contract results in the recognition of the interest component in P&L. The interest component is just calculated as the remainder of the FV changes minus those of the spot component, which gives the booking entries shown in *Table 31*.

TABLE 30: FX Hedge Example – Spot Component

Date	Spot component	Changes spot component
t_0 = 06/08/20X0	0.00€	0.00€
t_1 = 12/08/20X0	–7,930.02€	–7,930.02€
T = 06/08/20X1	–15,282.43€	–7,352.41€

TABLE 31: FX Hedge Example – Forward OCI (Spot) / P&L (Interest)

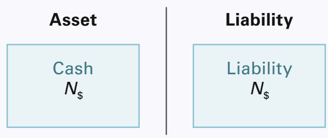
Date	Forward liability	OCI (spot)	P&L (interest)
t_0 = 06/08/20X0	0.00€	0.00€	0.00€
t_1 = 12/08/20X0	–7,685.60€	7,930.02€	–244.42€
T = 06/08/20X1	–7,656.76€	7,352.41€	304.35€

5.2.2 Valuation of a Cross Currency Swap without FX Basis Risk

Let's assume that an EUR based financial institution issues an USD denominated fixed rate liability[72] with notional $N_\$$ (see *Figure 60*).

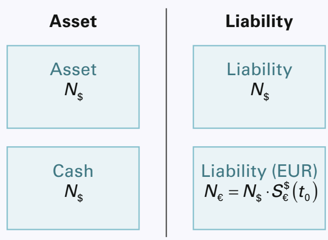
The financial institution does not want to exchange the USD liability into a EUR liability; therefore it enters into a fixed-to-float cross currency swap in order to hedge against foreign currency and interest rate risk.

FIGURE 60: Balance Sheet of the Financial Institution after the Issuance of USD Liability



At $t = t_0$ the cross currency swap requires the cash exchange of its notional; the notional will be fixed at $t = t_0, N_€ = N_\$ \cdot S_€^\$(t_0)$ and will be returned at $t = T$ with the same amount. So the financial institution locks in a fixed exchange rate $S_€^\$(t_0)$.

FIGURE 61: Balance Sheet of the Financial Institution with USD Liability and Cross Currency Swap ($t = t_0$)



Entering into the cross currency swap entails the following changes to the balance sheet[73]:

- adding a fixed rate asset denominated in USD for the contractually fixed repayment in USD of the CCS at maturity,
- adding a floating liability denominated in EUR for the contractually fixed payment in EUR of the CCS at maturity,
- exchange of cash to the amount of $N_\$$ in USD by an amount of $N_€ = N_\$ \cdot S_€^\$(t_0)$ in EUR,

portrayed in *Figure 61*.

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72 The argumentation works in the analogous way for a fixed asset e.g. a granted fixed rate loan.

73 In an IFRS balance sheet only a single amount would be recognized representing the fair value of the net amounts (including the exchange in cash) to be exchanged. However, the gross amounts are shown here for illustrative purposes.

The financial institution receives USD fixed coupon payment on the notional $N_\$$ and pays 3-month EURIBOR floating on $N_€ = N_\$ \cdot S_€^\$(t_0)$. *Figure 62* summarizes the payments of the cross currency swap.

The valuation of a cross currency swap can be stated in two equivalent ways:

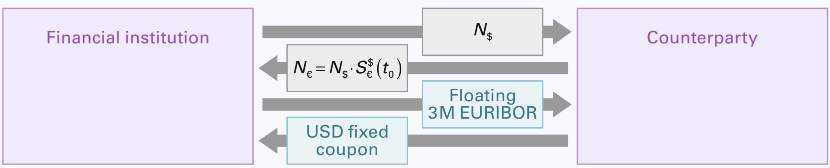
- in terms of decomposition into the fixed leg and the floating leg,
- in terms of forward exchange rates.

The cross currency swap can be decomposed into a fixed and a floating leg, so the present value at time $t = t_0$ is:

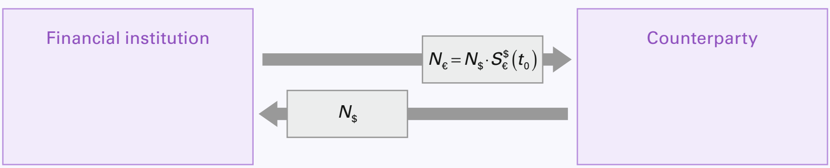
EQUATION 13: Valuation Formula of a Fixed-to-Float Cross Currency Swap

$$PV_{t_0}[\mathrm{CCS}] = S_€^\$(t_0)\left[\sum_{k=1}^{N} N_\$ \cdot c_\$^{\mathrm{CCS}}(t_0,T)\Delta(T_{k-1},T_k)\cdot B_\$^{3\mathrm{M}}(t_0,T_k) + N_\$ \cdot B_\$^{3\mathrm{M}}(t_0,T)\right] - \underbrace{PV_{t_0}[N_€\mathrm{Floater}]}_{=N_€}$$

FIGURE 62: Payment Schedule of a Cross Currency Swap



Return of the notional at maturity of the cross currency swap:



So far we have not specified the amount of the USD coupon $c_{\$}^{CCS}(t_0,T)$. At inception of the cross currency swap the present value equals zero, therefore:

$$0 = S_{€}^{\$}(t_0)\cdot\left[\begin{matrix}\sum_{k=1}^{N}\left(N_{\$}\cdot c_{\$}^{CCS}(t_0,T)\cdot\Delta(T_{k-1},T_k)\cdot B_{\$}^{3M}(t_0,T_k)\right)\\ +N_{\$}\cdot B_{\$}^{3M}(t_0,T)\end{matrix}\right]-N_{€}$$

$$\Rightarrow N_{€}-S_{€}^{\$}(t_0)N_{\$}\cdot B_{\$}^{3M}(t_0,T)$$
$$=S_{€}^{\$}(t_0)N_{\$}\cdot c_{\$}^{CCS}(t_0,T)\cdot\left[\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B_{\$}^{3M}(t_0,T_k)\right]$$

$$N_{€}\left[\frac{1-B_{\$}^{3M}(t_0,T)}{\sum_{k=1}^{T}\Delta(T_{k-1},T_k)\cdot B_{\$}^{3M}(t_0,T_k)}\right]=N_{\$}\cdot S_{€}^{\$}(t_0)c_{\$}^{CCS}(t_0,T)$$

$$\frac{N_{€}}{S_{€}^{\$}(t_0)}\left[\frac{1-B_{\$}^{3M}(t_0,T)}{\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B_{\$}^{3M}(t_0,T_k)}\right]=N_{\$}\cdot c_{\$}^{CCS}(t_0,T)$$

$$N_{\$}\cdot c_{\$}^{3M}(t_0,T)=N_{\$}\cdot c_{\$}^{CCS}(t_0,T)$$

$$c_{\$}^{3M}(t_0,T)=c_{\$}^{CCS}(t_0,T).$$

According to the equation above the USD coupon payment in a cross currency swap without FX basis risk is equal to the 3-month USD LIBOR interest rate swap rate. Please note that the valuation of a cross currency swap involves the recognition of cash payments in contrast to the valuation of plain vanilla interest rate swaps. The cross currency swap is written in terms of EUR zero coupon swaps, since the EUR represents the home currency.

For the purpose of stating the cross currency swap in terms of FX forward rates, the CCS is virtually decomposed into a fixed-to-fixed CCS with matching payments dates and a single currency interest rate swap in EUR, provided the fixed leg compensates the EUR of the fixed-to-fixed CCS. Considering the coupon payment of the fixed-to-fixed CCS at time $t = T_1$, where $c_{€,f}^{3M}(t_0,T)$ denotes the fixed rate of the EUR leg that equals the swap rate of the single currency interest swap in EUR, where the payment dates coincide the USD leg of the CCS.

$$F_{€,1}^{\$,CCS}\left[f_€^\$(T_1,T_1),T_1\right]$$
$$= N_\$ \cdot c_\$^{3M}(t_0,T)\cdot\Delta(t_0,T_1)\cdot S_€^\$(T_1) - c_{€,f}^{3M}(t_0,T)\cdot\Delta(t_0,T_1)\cdot N_€$$

$\Rightarrow$ using the equations in connection with the FX forward contract above:

$$F_{€,1}^{\$,CCS}\left[f_€^\$(t_0,T_1),t_0\right]$$
$$= PV_{t_0}\left[N_\$ \cdot c_\$^{3M}(t_0,T)\cdot\Delta(t_0,T_1)\cdot S_€^\$(T_1) - c_{€,f}^{3M}(t_0,T)\cdot\Delta(t_0,T_1)\cdot N_€\right]$$

$$= PV_{t_0}\left[N_\$ \cdot c_\$^{3M}(t_0,T)\cdot\Delta(t_0,T_1)\cdot S_€^\$(T_1)\right]$$
$$-c_{€,f}^{3M}(t_0,T)\cdot\Delta(t_0,T_1)\cdot B_€(t_0,T_1)\cdot N_€$$

$$= N_\$ \cdot c_\$^{3M}(t_0,T)\cdot\Delta(t_0,T_1)\cdot f_€^\$(t_0,T_1)\cdot B_€(t_0,T_1)$$
$$-c_{€,f}^{3M}(t_0,T)\cdot\Delta(t_0,T_1)\cdot B_€(t_0,T_1)\cdot N_€ .$$

Repeating the argument for every $t = T_1, T_2,\ldots, T_N = T$ and inserting the single currency EUR swap, the following formula for the cross currency swap is derived:

$$PV_{t_0}[\text{CCS}] = \sum_{k=1}^{N} F_{€,k}^{\$,CCS}\left[f_€^\$(t_0,T_k),t_0\right] + IRS_€(t_0,T)$$

$$= N_\$ \cdot c_\$^{3M}(t_0,T)\sum_{k=1}^{N} f_€^\$(t_0,T_k)\cdot\Delta(T_{k-1},T_k)\cdot B_€(t_0,T_k)$$
$$+f_€^\$(t_0,T_N)\cdot N_\$\cdot B_€(t_0,T_N)$$
$$-c_{€,f}^{3M}(t_0,T)\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B_€(t_0,T_k)\cdot N_€ - N_€\cdot B_€(t_0,T_N)$$
$$+c_{€,f}^{3M}(t_0,T)\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B_€(t_0,T_k)\cdot N_€ + N_€\cdot B_€(t_0,T_N)$$
$$-N_€$$

$$= N_\$ \cdot c_\$^{3M}(t_0,T)\sum_{k=1}^{N} f_€^\$(t_0,T_k)\cdot\Delta(T_{k-1},T_k)\cdot B_€(t_0,T_k)$$
$$+f_€^\$(t_0,T_N)\cdot N_\$\cdot B_€(t_0,T_N) - N_€ .$$

5.2.3 Fair Value Hedge Accounting Example of Cross Currency Swap without FX Basis Risk

This example covers a fixed rate liability denominated in USD that is hedged by a corresponding fixed-to-float USD to EUR cross currency swap (see *Table 32*).

The internal coupon is determined by the risk-equivalent bond method, and the hedging cost measurement method will be applied. Date of inception is the value date of liability and swap.

At inception t_0 = 01/23/20X9 the exchange rate of 1 EUR = 1.2795 USD and the discount factors shown in *Table 33* derived from USD vs. 3-month USD LIBOR swap rates for the indicated terms (in years) are given.

TABLE 32: Terms and Conditions of Hedging Relationship with a Cross Currency Swap

Terms and conditions	Fixed liability	CCS fixed leg	CCS float leg
Value date	01/23/20X9	01/23/20X9	01/23/20X9
Maturity	01/23/20X4	01/23/20X4	01/23/20X4
Fixed internal rate / tenor	2.3036 %	2.3036 %	3M
Notional	–100,000.00$	100,000.00$	–78,155.53€
Interest payment frequency	Semi-annually	Semi-annually	Quarterly
Day count convention	30/360	30/360	ACT/360

TABLE 33: Cross Currency Swap Example – Discount Factors at t_0

Years to maturity $T_i - t_0$	0.5	1	1.5	2	2.5
Discount factor $B_\$(t_0, T_i)$	0.992003	0.980924	0.974573	0.970315	0.958947
Years to maturity $T_i - t_0$	**3**	**3.5**	**4**	**4.5**	**5**
Discount factor $B_\$(t_0, T_i)$	0.945949	0.932932	0.918869	0.905236	0.890917

Using the terms and conditions given above the hedge fair value is calculated as follows:

$$HFV^{€}_{\text{liability}}(t_0)$$

$$= HFV^{\$}_{\text{liability}}(t_0) \cdot S^{\$}_{€}(t_0)$$

$$= -\left(\sum_{i=1}^{10} N_{\$} \cdot \frac{\text{internal coupon}}{2} \cdot B_{\$}(t_0, T_i) + N_{\$} \cdot B_{\$}(t_0, T_{10})\right) \cdot S^{\$}_{€}(t_0)$$

$$= \frac{\left(\sum_{i=1}^{10} \frac{-100,000.00\,\$ \cdot 2.3036\,\%}{2} \cdot B_{\$}(t_0, T_i) - 100,000.00\,\$ \cdot B_{\$}(t_0, T_{10})\right)}{(1.2795\,\$/€)}$$

$$= -78,155.53\,€\,.$$

Since the terms and conditions of the fixed side of the CCS are identical – only opposite sign of notional – the value of the fixed leg is the same – apart from the sign – as that of the hedged item. Thus this value can be used to determine the fair value of the swap, and only the floating side remains to be evaluated in order to determine the fair value of the swap. Since it is assumed that forwarding and discounting is performed on the same curve and the measurement is performed on reset days, the value of the floating leg equals the notional:

$$-N_{€} = -N_{\$} \cdot S^{\$}_{€}(t_0) = -100,000.00\,\$ \cdot \frac{1}{1.2795\,\$/€} = -78,155.53\,€\,.$$

Similarly, deriving the forward rates from the discount factors and discounting them yields the same result. Thus the fair value of the cross currency swap is 0 at inception:

$$FV_{\text{CCS}}(t_0) = \text{fixed side} + \text{floating side} = 78,155.53\,€ - 78,155.53\,€$$
$$= 0\,.$$

At t_1 = 07/23/20X9 we have the following market data: exchange rate 1 EUR = 1.4229 USD and the discount factors shown in *Table 34*.

TABLE 34: Cross Currency Swap Example – Discount Factors at t_1

Years to maturity $T_i - t_1$	**0.5**	**1**	**1.5**	**2**	**2.5**
Discount factor $B_\$(t_1, T_i)$	0.995262	0.985292	0.977969	0.970718	0.955851
Years to maturity $T_i - t_1$	**3**	**3.5**	**4**	**4.5**	
Discount factor $B_\$(t_1, T_i)$	0.938042	0.919859	0.899683	0.880227	

$$
\begin{aligned}
& HFV^{€}_{\text{liability}}(t_1) \\
& = HFV^{\$}_{\text{liability}}(t_1) \cdot S^{\$}_{€}(t_1) \\
& = -\left(\sum_{i=2}^{10} N_\$ \cdot \frac{\text{internal coupon}}{2} \cdot B_\$(t_1, T_i) + N_\$ \cdot B_\$(t_1, T_{10})\right) \cdot S^{\$}_{€}(t_1) \\
& = \frac{\left(\sum_{i=2}^{10} \frac{-100{,}000.00\,\$ \cdot 2.3036\,\%}{2} \cdot B_\$(t_1, T_i) - 100{,}000.00\,\$ \cdot B_\$(t_1, T_{10})\right)}{(1.4229\,\$/€)} \\
& = -68{,}760.48\,€ .
\end{aligned}
$$

Again the fixed leg of the CCS will have the same value with opposite sign and its floating leg again equals the notional $-N_€$ of the EUR leg resulting in:

$$
\begin{aligned}
FV_{\text{CCS}}(t_1) &= \text{fixed side} + \text{floating side} = 68{,}760.48\,€ - 78{,}155.53\,€ \\
&= -9{,}395.05\,€ .
\end{aligned}
$$

The total change of the hedge fair value equals:

$$
\begin{aligned}
\Delta HFV_{\text{liability}} &= HFV_{\text{liability}}(t_1) - HFV_{\text{liability}}(t_0) \\
&= -68{,}760.48\,€ - (-78{,}155.53\,€) \\
&= 9{,}395.05\,€ .
\end{aligned}
$$

This exactly offsets the changes of the CCS between t_0 and t_1 thus the hedge is 100 % effective.

On the other hand the changes of the hedge fair value can be decomposed into change due to interest rate movements and changes in the FX rate as follows:

$$\begin{aligned}
\Delta HFV_{\text{liability}} &= \Delta HFV_{\text{liability}}^{\text{interest}} - \Delta HFV_{\text{liability}}^{\text{FX}} \\
&= \left(HFV_{\text{liability}}^{\$}(t_1) - AC_{\text{liability}}^{\$}(t_1)\right) \cdot S_{€}^{\$}(t_1) \\
&\quad - \left(HFV_{\text{liability}}^{\$}(t_0) - AC_{\text{liability}}^{\$}(t_0)\right) \cdot S_{€}^{\$}(t_0) \\
&\quad + AC_{\text{liability}}^{\$}(t_0) \cdot \left(S_{€}^{\$}(t_1) - S_{€}^{\$}(t_0)\right) \\
&= -\left(97{,}839.29\,\$ - 100{,}000.00\,\$\right) / \left(1.4229\,\$/€\right) \\
&\quad - \left(-\left(100{,}000.00\,\$ - 100{,}000.00\,\$\right) / \left(1.2795\,\$/€\right)\right) \\
&\quad + \left(-100{,}000.00\,\$\right)\left(\frac{1}{1.4229\,\$/€} - \frac{1}{1.2795\,\$/€}\right) \\
&= 1{,}518.52\,€ + 7{,}876.52\,€.
\end{aligned}$$

Since there are no amortizations (premium/discount) related to the amortized cost book value, the booking entries of the hedge adjustment HA due as changes in fair value of the liability due to the hedged risk will coincide with the $\Delta HFV_{\text{liability}}$ above[74]:

$$\begin{aligned}
HA(t_1) &= \left(HFV_{\text{liability}}^{\$}(t_1) - AC_{\text{liability}}^{\$}(t_1)\right) \cdot S_{€}^{\$}(t_1) \\
&\quad - \left(HFV_{\text{liability}}^{\$}(t_0) - AC_{\text{liability}}^{\$}(t_0)\right) \cdot S_{€}^{\$}(t_0) \\
&\quad + AC_{\text{liability}}^{\$}(t_0) \cdot \left(S_{€}^{\$}(t_1) - S_{€}^{\$}(t_0)\right) \\
&= HFV_{\text{liability}}^{\$}(t_1) \cdot S_{€}^{\$}(t_1) - HFV_{\text{liability}}^{\$}(t_0) \cdot S_{€}^{\$}(t_0) \\
&= HFV_{\text{liability}}^{€}(t_1) - HFV_{\text{liability}}^{€}(t_0).
\end{aligned}$$

At t_2=01/23/20X0 we have the following market data: exchange rate 1 EUR = 1.4135 USD and the discount factors shown in *Table 35.*

74 With respect to the booking entries, there is no necessity to designate FX risk, since the cross currency swap requires the exchange in notional. These represent, as shown in *Figure 61,* recognized assets and liabilities and are subject to currency translation adjustments according to IAS 21.

TABLE 35: Cross Currency Swap Example – Discount Factors at t_2

Years to maturity $T_i - t_2$	0.5	1	1.5	2	2.5
Discount factor $B_\$(t_2,T_i)$	0.998102	0.991423	0.985127	0.977574	0.964527
Years to maturity $T_i - t_2$	**3**	**3.5**	**4**		
Discount factor $B_\$(t_2,T_i)$	0.948588	0.931658	0.912555		

$$
\begin{aligned}
&HFV^{€}_{\text{liability}}(t_2)\\
&= HFV^{\$}_{\text{liability}}(t_2)\cdot S^{\$}_{€}(t_2)\\
&= -\left(\sum_{i=3}^{10} N_\$\cdot\frac{\text{internal coupon}}{2}\cdot B_\$(t_2,T_i)+N_\$\cdot B_\$(t_2,T_{10})\right)\cdot S^{\$}_{€}(t_2)\\
&= \frac{\left(\sum_{i=3}^{10}\frac{-100{,}000.00\,\$\cdot 2.3036\,\%}{2}\cdot B_\$(t_2,T_i)-100{,}000.00\,\$\cdot B_\$(t_2,T_{10})\right)}{(1.4135\,\$/€)}\\
&= -70{,}842.11\,€\,.
\end{aligned}
$$

With the arguments above the fair value of the CCS is calculated to be

$$
\begin{aligned}
FV_{\text{CCS}}(t_2) &= \text{fixed side} + \text{floating side} = 70{,}842.11\,€ - 78{,}155.53\,€\\
&= -7{,}313.42\,€\,.
\end{aligned}
$$

Thus the change of the fair value of the CCS from t_1 to t_2 is given by

$$
\begin{aligned}
\Delta FV_{\text{CCS}}(t_1,t_2) &= FV_{\text{CCS}}(t_2) - FV_{\text{CCS}}(t_1)\\
&= -7{,}313.42\,€ - (-9{,}395.05\,€)\\
&= 2{,}081.62\,€\,.
\end{aligned}
$$

The total change of the hedge fair value equals:

$$
\begin{aligned}
\Delta HFV_{\text{liability}} &= HFV_{\text{liability}}(t_2) - HFV_{\text{liability}}(t_1)\\
&= -70{,}842.11\,€ - (-68{,}760.48\,€)\\
&= -2{,}081.62\,€\,.
\end{aligned}
$$

This exactly offsets the changes of the CCS between t_1 and t_2, thus the hedge is 100 % effective.

TABLE 36: Cross Currency Swap Example – Discount Factors at t_3

Years to maturity $T_i - t_2$	**0.5**	**1**	**1.5**	**2**	**2.5**
Discount factor $B_\$(t_2, T_i)$	0.99650	0.98924	0.98589	0.98395	0.97565
Years to maturity $T_i - t_2$	**3**	**3.5**			
Discount factor $B_\$(t_2, T_i)$	0.96560	0.95360			

The decomposition into change due to interest rate movements and changes in the FX rate reads as follows:

$$\begin{aligned}
\Delta HFV_{\text{liability}} &= \Delta HFV_{\text{liability}}^{\text{interest}} - \Delta HFV_{\text{liability}}^{\text{FX}} \\
&= \left(HFV_{\text{liability}}^{\$}(t_2) - AC_{\text{liability}}^{\$}(t_2)\right) \cdot S_{€}^{\$}(t_2) \\
&\quad - \left(HFV_{\text{liability}}^{\$}(t_1) - AC_{\text{liability}}^{\$}(t_1)\right) \cdot S_{€}^{\$}(t_1) \\
&\quad + AC_{\text{liability}}^{\$}(t_1) \cdot \left(S_{€}^{\$}(t_2) - S_{€}^{\$}(t_1)\right) \\
&= -(100{,}135.32\,\$ - 100{,}000.00\,\$)/(1.4135\,\$/€) \\
&\quad + (97{,}839.29\,\$ - 100{,}000.00\,\$)/(1.4229\,\$/€) \\
&\quad + (-100{,}000.00\,\$)\left(\frac{1}{1.4135\,\$/€} - \frac{1}{1.4229\,\$/€}\right) \\
&= -1{,}614.25\,€ - 467.36\,€.
\end{aligned}$$

Following the same argumentation as above, the hedge adjustment *HA* due as changes in fair value of the liability due to the hedged risk will coincide with $\Delta HFV_{\text{liability}}$.

At t_3=07/23/20X0 we have the following market data: exchange rate 1 EUR = 1.2897 USD and the discount factors shown in *Table 36*.

$$HFV_{\text{liability}}^{€}(t_3)$$

$$= HFV_{\text{liability}}^{\$}(t_3) \cdot S_{€}^{\$}(t_3)$$

$$= -\left(\sum_{i=4}^{10} N_\$ \cdot \frac{\text{internal coupon}}{2} \cdot B_\$(t_3, T_i) + N_\$ \cdot B_\$(t_3, T_{10})\right) \cdot S_{€}^{\$}(t_3)$$

$$= \frac{\left(\sum_{i=4}^{10} \frac{-100{,}000.00\,\$ \cdot 2.3036\,\%}{2} \cdot B_{\$}(t_3, T_i) - 100{,}000.00\,\$ \cdot B_{\$}(t_3, T_{10}) \right)}{(1.2897\,\$/€)}$$

$$= -80{,}057.95\,€.$$

The fair value of the CCS as the sum of the fixed leg being the opposite of the hedge fair value of the fixed liability and the floating leg, that again equals the notional in EUR, is calculated to be

$$\begin{aligned} FV_{\text{CCS}}(t_3) &= \text{fixed side} + \text{floating side} = 80{,}057.95\,€ - 78{,}155.53\,€ \\ &= 1{,}902.42\,€. \end{aligned}$$

Thus the change of the fair value of the CCS from t_2 to t_3 is given by

$$\begin{aligned} \Delta FV_{\text{CCS}}(t_2, t_3) &= FV_{\text{CCS}}(t_3) - FV_{\text{CCS}}(t_2) \\ &= 1{,}902.42\,€ - (-7{,}313.16\,€) \\ &= 9{,}215.84\,€. \end{aligned}$$

The total change of the hedge fair value equals:

$$\begin{aligned} \Delta HFV_{\text{liability}}(t_2, t_3) &= HFV_{\text{liability}}(t_3) - HFV_{\text{liability}}(t_2) \\ &= -80{,}057.95\,€ - (-70{,}842.11\,€) \\ &= -9{,}215.84\,€. \end{aligned}$$

This again exactly offsets the changes of the CCS between t_2 and t_3 thus the hedge is 100 % effective.

Since there is no amortization of the amortized costs the booking entries of the hedge adjustment will coincide with the $\Delta HFV_{\text{liability}}$ above.

On the other hand the changes of the hedge fair value can be decomposed into change due to interest rate movements and changes in the FX rate as follows:

$$\begin{aligned} \Delta HFV_{\text{liability}} &= \Delta HFV_{\text{liability}}^{\text{interest}} - \Delta HFV_{\text{liability}}^{\text{FX}} \\ &= \left(HFV_{\text{liability}}^{\$}(t_3) - AC_{\text{liability}}^{\$}(t_3) \right) \cdot S_{€}^{\$}(t_3) \\ &\quad - \left(HFV_{\text{liability}}^{\$}(t_2) - AC_{\text{liability}}^{\$}(t_2) \right) \cdot S_{€}^{\$}(t_2) \\ &\quad + AC_{\text{liability}}^{\$}(t_2) \cdot \left(S_{€}^{\$}(t_3) - S_{€}^{\$}(t_2) \right) \end{aligned}$$

TABLE 37: Cross Currency Swap Example – Calculation Results

Date	FV hedge item	FV changes hedged item	FV CCS	FV changes CCS	Effectiveness
t_0	–78,155.53€		0.00€		
t_1	–68,760.48€	9,395.05€	–9,395.05€	–9,395.05€	100%
t_2	–70,842.11€	–2,081.62€	–7,313.42€	2,081.62€	100%
t_3	–80,057.95€	–9,215.84€	1,902.42€	9,215.84€	100%
t_4	–76,067.33€	3,990.61€	–2,088.20€	–3,990.61€	100%

$$
\begin{aligned}
&= -\left(103{,}250.73\,\$ - 100{,}000.00\,\$\right)/\left(1.2897\,\$/€\right) \\
&\quad + \left(-100{,}135.32\,\$ - 100{,}000.00\,\$\right)/\left(1.4135\,\$/€\right) \\
&\quad - \left(-100{,}000.00\,\$\right)\left(\frac{1}{1.2897\,\$/€} - \frac{1}{1.4135\,\$/€}\right) \\
&= -2{,}424.80\,€ - 6{,}791.04\,€.
\end{aligned}
$$

Following the same argumentation as above, the hedge adjustment *HA* due as changes in fair value of the liability due to the hedged risk will coincide with $\Delta HFV_{\text{liability}}$.

Table 37 summarizes the calculations results.

5.3 Hedge Accounting of FX Risk with FX Basis Risk

5.3.1 Construction of the Discount Curves including FX Basis Risk

The above shows that an FX hedge and hedge accounting always include the exchange in cash payments and it had been assumed that the "price or premium" for exchanging cash is equal to zero. Derivative markets show that market participants charge a premium (positive or negative) for the exchange in cash; this is termed the FX basis and represented by the separately traded market instruments cross currency basis swaps.

In order to show the construction of discount curves in the presence of the FX basis the USD/EUR example from above is continued. By market convention the FX basis is shown on the EUR floating leg, so the USD currency represents the "numeraire". In the following the model economy is described and the floating sides of the interest rate swaps are written in terms of the forward rates. Similarly to the example above, the valuation of the interest rate swaps denominated in USD and the EUR cannot be modeled independently from each other. In the example – like above – it is assumed that the USD and the EUR are exposed to the same 3-month tenor:

EUR:

$$\overbrace{c_{€}^{3M}\left(t_0,T\right)\cdot\sum_{k=1}^{N}\Delta\left(T_{k-1},T_k\right)\cdot B_{€}^{3M}\left(t_0,T_k\right)}^{\text{PV of the fixed side of the 3M EURIBOR interest rate swap}}$$

$$\stackrel{!}{=}\underbrace{\delta\left(t_0,t_1\right)\cdot\underbrace{r_{€}^{3M}\left(t_0\right)}_{\substack{\text{3M EURIBOR}\\\text{spot rate}}}B_{€}^{3M}\left(t_0,t_1\right)+\sum_{j=2}^{4N}\delta\left(t_{j-1},t_j\right)\cdot f_{€}^{3M}\left(t_0,t_{j-1},t_j\right)\cdot B_{€}^{3M}\left(t_0,t_j\right)}_{\text{PV of the floating side of the 3M EURIBOR interest rate swap}}.$$

USD:

$$\overbrace{c_{\$}^{3M}\left(t_0,T\right)\cdot\sum_{k=1}^{N}\Delta\left(T_{k-1},T_k\right)\cdot B_{\$}^{3M}\left(t_0,T_k\right)}^{\text{PV of the fixed side of the 3M LIBOR interest rate swap}}$$

$$\stackrel{!}{=}\underbrace{\left(1-B_{\$}^{3M}\left(t_0,T\right)\right)}_{\substack{\text{PV of the floating side}\\\text{of the 3M LIBOR}\\\text{interest rate swap}}}$$

$$=\underbrace{\delta\left(t_0,t_1\right)\cdot r_{\$}^{3M}\left(t_0\right)B_{\$}^{3M}\left(t_0,t_1\right)+\sum_{j=2}^{4N}\delta\left(t_{j-1},t_j\right)\cdot f_{\$}^{3M}\left(t_0,t_{j-1},t_j\right)\cdot B_{\$}^{3M}\left(t_0,t_j\right)}_{\text{PV of the floating side of the 3M LIBOR interest rate swap}}.$$

USD/EUR basis spread $b(t_0, T)$ (float-to-float):

$$S_€^\$(t_0)\cdot N_\$\left[\begin{array}{l}\delta(t_0,t_1)\cdot r_\$^{3M}(t_0)B_\$^{3M}(t_0,t_1)\\ +\sum_{j=2}^{4N}\delta(t_{j-1},t_j)\cdot f_\$^{3M}(t_0,t_{j-1},t_j)\cdot B_\$^{3M}(t_0,t_j)+B_\$^{3M}(t_0,T)\end{array}\right]$$

$$=N_€\left[\begin{array}{l}\delta(t_0,t_1)\cdot r_€^{3M}(t_0)B_€^{3M}(t_0,t_1)\\ +\sum_{j=2}^{4N}\delta(t_{j-1},t_j)\cdot\left(f_€^{3M}(t_0,t_{j-1},t_j)+b(t_0,T)\right)\cdot B_€^{3M}(t_0,t_j)+B_€^{3M}(t_0,T)\end{array}\right].$$

EUR/USD exchange rate: $S_€^\$(t)$.

In order to simplify the notation the following definitions of the 3-month spot rates are used:

$$f_\$^{3M}(t_0,t_0,t_1):=r_\$^{3M}(t_0) \text{ and } f_€^{3M}(t_0,t_0,t_1):=r_€^{3M}(t_0).$$

Thus the following equilibrium conditions are valid:

EQUATION 14: Description of the Model Economy including the FX Basis

EUR:
$$\overbrace{c_€^{3M}(t_0,T)\cdot\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B_€^{3M}(t_0,T_k)}^{\text{PV of the fixed side of the 3M EURIBOR interest rate swap}} \stackrel{!}{=} \underbrace{\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot f_€^{3M}(t_0,t_{j-1},t_j)\cdot B_€^{3M}(t_0,t_j)}_{\text{PV of the floating side of the 3M EURIBOR interest rate swap}}$$

USD:
$$\overbrace{c_\$^{3M}(t_0,T)\cdot\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B_\$^{3M}(t_0,T_k)}^{\text{PV of the fixed side of the 3M LIBOR interest rate swap}} \stackrel{!}{=} \underbrace{\left(1-B_\$^{3M}(t_0,T)\right)}_{\text{PV of the floating side of the 3M LIBOR interest rate swap}}$$

$$= \underbrace{\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot f_\$^{3M}(t_0,t_{j-1},t_j)\cdot B_\$^{3M}(t_0,t_j)}_{\text{PV of the floating side of the 3M LIBOR interest rate swap}}$$

EUR/USD basis spread $b(t_0,T)$ (float-to-float)

$$S_€^\$(t_0)\cdot N_\$\left[\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot f_\$^{3M}(t_0,t_{j-1},t_j)\cdot B_\$^{3M}(t_0,t_j)+B_\$^{3M}(t_0,T)\right]$$

$$=N_€\left[\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot\left(f_€^{3M}(t_0,t_{j-1},t_j)+b(t_0,T)\right)\cdot B_€^{3M}(t_0,t_j)+B_€^{3M}(t_0,T)\right]$$

In *Equation 14* there are two unknown parameters: $B_{€}^{3M}(t_0,t_j)$ and $f_{€}^{3M}(t_0,t_{j-1},t_j)$, which are derived by bootstrapping algorithms in order to derive an arbitrage-free set of curves.

The cross currency basis swap is a traded float-to-float instrument that pays a regular (e.g. every 3-month) spread $b(t_0,T)$. It requires the exchange of cash (notional) at inception $t=t_0$ and a re-exchange of the notional at maturity $t=T$. The exchange in cash is defined by $N_{€}=N_{\$}\cdot S_{€}^{\$}(t_0)$. The present value of both floating sides can be stated as follows:

$$PV_{\$}^{1}[t_0,T]$$
$$=S_{€}^{\$}(t_0)\cdot N_{\$}\left[\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot f_{\$}^{3M}(t_0,t_{j-1},t_j)\cdot B_{\$}^{3M}(t_0,t_j)+B_{\$}^{3M}(t_0,T)\right],$$

$$PV_{€}^{2}[t_0,T]$$
$$=N_{€}\left[\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot\left(f_{€}^{3M}(t_0,t_{j-1},t_j)+b(t_0,T)\right)\cdot B_{€}^{3M}(t_0,t_j)+B_{€}^{3M}(t_0,T)\right].$$

where $b(t_0,T)$ denotes the 3-month USD LIBOR/3-month EURIBOR basis at t_0 with maturity T, which is added on the 3-month EURIBOR forward rates by market convention.

In order to derive the 3-month EURIBOR discount rate $B_{€}^{3M}(t_0,T)$ for every T the following steps are performed. By market convention the following assumptions hold:

- USD represents the "numeraire",
- For the 3-month USD LIBOR/3-month EURIBOR basis at t_0: $PV_{\$}^{1}[t_0,T]=PV_{€}^{2}[t_0,T]$.

Under these assumptions the 3-month EURIBOR discount rate $B_{€}^{3M}(t_0,T)$ can be derived, since:

$$\begin{aligned}
1 \underbrace{=}_{\substack{\text{USD}\\ \text{numeraire}}} & PV_{\$}^{1}[t_0,T] \\
= \; & PV_{€}^{2}[t_0,T] \\
= \; & \sum_{j=1}^{4N} \delta(t_{j-1},t_j)\cdot\left(f_{€}^{3M}(t_0,t_{j-1},t_j)+b(t_0,T)\right)\cdot B_{€}^{3M}(t_0,t_j)+B_{€}^{3M}(t_0,T)
\end{aligned}$$

$\Rightarrow$ using the definition of the EUR interest rate swap

$$\begin{aligned}
& B_{€}^{3M}(t_0,T) \\
&= 1-\sum_{j=1}^{4N} \delta(t_{j-1},t_j)\cdot\left(f_{€}^{3M}(t_0,t_{j-1},t_j)+b(t_0,T)\right)\cdot B_{€}^{3M}(t_0,t_j) \\
&= 1-\sum_{j=1}^{4N} \delta(t_{j-1},t_j)\cdot f_{€}^{3M}(t_0,t_{j-1},t_j)\cdot B_{€}^{3M}(t_0,t_j) \\
&\quad -\sum_{j=1}^{4N} \delta(t_{j-1},t_j)\cdot b(t_0,T)\cdot B_{€}^{3M}(t_0,t_j) \\
&= 1-c_{€}^{3M}(t_0,T)\cdot\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B_{€}^{3M}(t_0,T_k) \\
&\quad -b(t_0,T)\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot B_{€}^{3M}(t_0,t_j) \\
&= 1-c_{€}^{3M}(t_0,T)\cdot\sum_{k=1}^{N-1}\Delta(T_{k-1},T_k)\cdot B_{€}^{3M}(t_0,T_k) \\
&\quad -c_{€}^{3M}(t_0,T)\Delta(T_{N-1},T_N)B_{€}^{3M}(t_0,T)-b(t_0,T)\delta(t_{N-1},t_N)\cdot B_{€}^{3M}(t_0,T) \\
&\quad -b(t_0,T)\sum_{j=1}^{4N-1}\delta(t_{j-1},t_j)B_{€}^{3M}(t_0,t_j)
\end{aligned}$$

$$\begin{aligned}
\Rightarrow & B_{€}^{3M}(t_0,T)+c_{€}^{3M}(t_0,T)\Delta(T_{N-1},T_N)B_{€}^{3M}(t_0,T) \\
& +b(t_0,T)\delta(t_{N-1},t_N)\cdot B_{€}^{3M}(t_0,T) \\
& = 1-c_{€}^{3M}(t_0,T)\cdot\sum_{k=1}^{N-1}\Delta(T_{k-1},T_k)\cdot B_{€}^{3M}(t_0,T_k) \\
& \quad -b(t_0,T)\sum_{j=1}^{4N-1}\delta(t_{j-1},t_j)\cdot B_{€}^{3M}(t_0,t_j)
\end{aligned}$$

$$\Rightarrow B_{€}^{3M}(t_0,T)\cdot\left[1+c_{€}^{3M}(t_0,T)\Delta(T_{N-1},T_N)+b(t_0,T)\delta(t_{N-1},t_N)\right]$$

$$=1-c_{€}^{3M}(t_0,T)\cdot\sum_{k=1}^{N-1}\Delta(T_{k-1},T_k)\cdot B_{€}^{3M}(t_0,T_k)$$
$$-b(t_0,T)\sum_{j=1}^{4N-1}\delta(t_{j-1},t_j)\cdot B_{€}^{3M}(t_0,t_j)$$

$$B_{€}^{3M}(t_0,T)=\frac{\begin{pmatrix}1-c_{€}^{3M}(t_0,T)\cdot\sum_{k=1}^{N-1}\Delta(T_{k-1},T_k)\cdot B_{€}^{3M}(t_0,T_k)\\ -b(t_0,T)\sum_{j=1}^{4N-1}\delta(t_{j-1},t_j)\cdot B_{€}^{3M}(t_0,t_j)\end{pmatrix}}{\left[1+c_{€}^{3M}(t_0,T)\Delta(T_{N-1},T_N)+b(t_0,T)\delta(t_{N-1},t_N)\right]}.$$

The 3-month EURIBOR discount rate can be derived by the iterative steps outlined above for every T.

So far, the 3-month EURIBOR forward rate $f_{€}^{3M}(t_0,t_{j-1},t_j)$ has not been specified. In order to replicate the 3-month EURIBOR interest rate swap rates $c_{€}^{3M}(t_0,T)$, for every T an additional bootstrapping algorithm has to be performed, which is similar to the bootstrapping algorithm described in *Section 3.4.2.* As a result the floating leg of a 3-month EURIBOR interest rate swap is not equal to par anymore.

Summarizing the results:

- As described in *Section 3,* interest rate swaps, cross currency basis swaps, FX rates (spot) are traded on different and segmented markets.
- Incorporating the FX basis implies the creation of a valuation model using the absence of arbitrage principle and forming an integrated market for the exchange rate (spot), home and foreign interest rate swaps and the cross currency basis swap market.
- This model setup requires the performance of two (simultaneous) bootstrapping procedures in order to derive a consistent and arbitrage free set of discount curves.

TABLE 38: Implications of FX Basis Representations in Terms of Home and Foreign Currency

Currency	FX basis shown on the EUR leg (market convention)	FX basis shown on the USD leg (not market convention – but applied in practice)
EUR	– Discount curve and the EUR forwards **include** the FX basis as risk factor. – EUR floating side of an interest rate swap **does not equal par.**	– Discount curve and the EUR forwards **do not include** the FX basis as risk factor. – EUR floating side of an interest rate swap **equals par.**
USD	– Discount curve and the USD forwards **include no** FX basis as risk factor. – USD floating side of an interest rate swap **equals par.**	– Discount curve and the US forwards **include** the FX basis as risk factor. – USD floating side of an interest rate swap **does not equal par.**

- As will be shown below, all derivatives are priced consistently with the model setup defined above (refer to *Equation 14*). Therefore e.g. a fixed-to-float cross currency swap comprises the FX basis as a risk factor for valuation purposes. Entering into a fixed-to-float cross currency swap and applying the model setup of *Equation 14* does not imply entering into a separate cross currency basis swap additionally. Like in case of the tenor basis swaps, it is important to distinguish between separately traded float-to-float instruments and tenors considered as risk factors.
- The representation of these consistent discount curves depends on market conventions concerning the presentation of the FX basis. By market conventions the FX basis is quoted and taken into account for the foreign currency with respect to the USD. But this implies a spread (positive or negative) on the home currency discount curve unless the balance sheet preparer is based in the US. As a consequence the home currency discount curve entails the FX basis, and then the domestic banking business is exposed to FX basis risk. Therefore many banks have decided to incorporate the FX basis on the USD discount curve in order to obtain a home currency discount curve without the FX basis. In terms of the model setup above this does not represent an issue, since a similar derivation of a consistent set of discount curves can be derived by assuming e.g. that the EUR discount curve serves as numeraire. The bootstrapping algorithm above can then be applied in a

similar way, "only the roles of the USD and the EUR" are exchanged. *Table 38* summarizes this.

- Economically banks are required to take the FX basis into account, since otherwise their funding costs (funding loans or trading activities) and the corresponding transfer pricing mechanism within the treasury departments are inadequate.

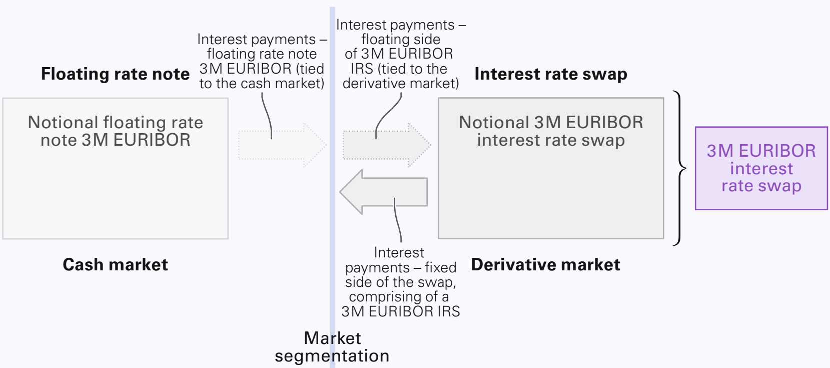
5.3.2 Example Cash Flow Hedge Accounting

According to *Table 38* there are two possibilities of the representation of a plain vanilla EUR interest rate swap. Based on this the question of the impact in case of cash flow hedge accounting according to IAS 39 arises. The following example is very similar to the example given in *Section 4.2.5*.

FX Basis Shown on the USD Leg

If the right-hand side of *Table 38* is considered, in the multi-curve setup the 3-month EURIBOR interest rate swap curve is chosen as "numeraire". In this case cash flow hedge according is performed similarly to the well-known "single-curve" case. In the example a 3-month EURIBOR floating rate note and a 3-month EURIBOR interest rate swap form the hedging relationship, which is shown in *Figure 63*.

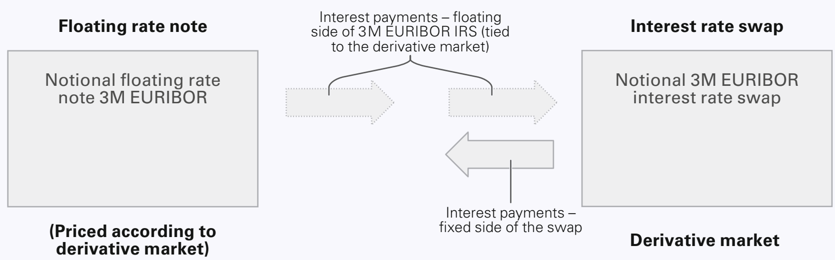
FIGURE 63: Hedge Relationship Consisting of a 3M EURIBOR Interest Rate Swap and a 3M EURIBOR Floating Rate Note



As shown in *Figure 63,* cash flow hedge accounting follows a similar valuation model as fair value hedge accounting:

- Cash and derivative market are different markets and follow different price discovery (pricing).
- Equating the cash flows of the floating rate note and the 3-month EURIBOR interest rate swap in a cash flow hedge accounting model in connection with the application of the hypothetical derivative method[75] constitutes a valuation model which relies on the interest rate swap market (derivative market). Given the prices of the interest rate swap market it is assumed in the cash flow hedge accounting model that the economic agent is indifferent between the income stream of the cash and the derivative market. This represents a strong assumption and leads to the elimination of basis risk between the cash and the derivative market (cash basis risk). In other words: the cash flow hedge accounting model according to IAS 39 forms an integrated market of the cash and derivative market.
- Since the 3-month EURIBOR interest rate swap has been chosen as "numeraire", the floating rate side of the 3-month EURIBOR interest rate swap resets to $1-B_{€}^{3M}(t,T)$ on every reset date $t=t_0,\ldots,T$.

FIGURE 64: Elimination of Basis Risk in a Hedge Relationship Consisting of a 3M EURIBOR Interest Rate Swap and a 3M EURIBOR Floating Rate Note



75 Only applicable for cash flow hedges.

The cash flow hedge accounting model and the elimination of the cash basis risk are summarized in *Figure 64*.

FX Basis Shown on the EUR Leg

Now the left-hand side of *Table 38* is considered. Since the USD is chosen as "numeraire", the representation of the cash flow hedge accounting model is not straightforward but follows the same economic rationale as before. In the multi-curve setup assuming USD as "numeraire", the 3-month EURIBOR interest rate swap is decomposed in t_0 as follows:

EQUATION 15: Decomposition of a 3M EURIBOR Interest Rate Swap

$$\overbrace{c_{€}^{3M}(t_0,T)\cdot\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B_{€}^{3M}(t_0,T_k)}^{\text{PV of the fixed side of the 3M EURIBOR interest rate swap}}-\underbrace{\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot f_{€}^{3M}(t_0,t_{j-1},t_j)\cdot B_{€}^{3M}(t_0,t_j)}_{\text{PV of the floating side of the 3M EURIBOR interest rate swap}}\overset{!}{=}0$$

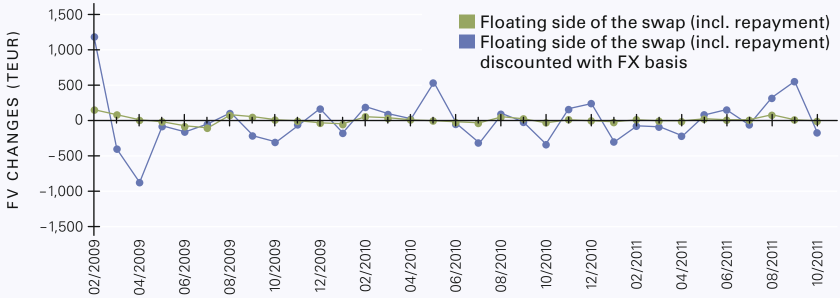
$$\begin{aligned}\Rightarrow 0 &= c_{€}^{3M}(t_0,T)\cdot\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B_{€}^{3M}(t_0,T_k)+\underbrace{B_{€}^{3M}(t_0,T)-B_{€}^{3M}(t_0,T)}_{=0}\\
&\quad-\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot\left(f_{€}^{3M}(t_0,t_{j-1},t_j)+\underbrace{b(t_0,T)-b(t_0,T)}_{=0}\right)\cdot B_{€}^{3M}(t_0,t_j)\\
&= c_{€}^{3M}(t_0,T)\cdot\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B_{€}^{3M}(t_0,T_k)+B_{€}^{3M}(t_0,T)\\
&\quad-\left[\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot\left(f_{€}^{3M}(t_0,t_{j-1},t_j)+b(t_0,T)\right)\cdot B_{€}^{3M}(t_0,t_j)+B_{€}^{3M}(t_0,T)\right]\\
&\quad+b(t_0,T)\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot B_{€}^{3M}(t_0,t_j)\\
&= \underbrace{c_{€}^{3M}(t_0,T)\cdot\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B_{€}^{3M}(t_0,T_k)+B_{€}^{3M}(t_0,T)-S_{€}^{\$}(t_0)\cdot N_{\$}\left[\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot f_{\$}^{3M}(t_0,t_{j-1},t_j)\cdot B_{\$}^{3M}(t_0,t_j)+B_{\$}^{3M}(t_0,T)\right]}_{\text{EUR/USD fixed-to-float cross currency swap}}\\
&\quad+b(t_0,T)\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot B_{€}^{3M}(t_0,t_j)\end{aligned}$$

where $1=N_{€}=S_{€}^{\$}(t_0)\cdot N_{\$}$ is assumed.

From the analysis above it follows that:

- In the multi-curve setup, if the USD interest rate swap is chosen as "numeraire", the floating rate side of the 3-month EURIBOR interest rate swap does not reset to $1 - B_{€}^{3M}(t,T)$ on every reset date $t = t_0, \ldots, T$. As will be shown below in *Equation 19,* the EUR floating leg resets on one minus the discount factor of maturity minus the current cross currency basis spread (FX basis spread) multiplied by the floating leg annuity. Therefore the dynamic of the floating rate side of the 3-month EURIBOR interest rate swap is different!
- The dynamics depend on the modeling of the multi-curve setup as described in the previous *Section 4;* the calibration using market quotes ensures the consistency of the multi-curve model with market quotes but results in different dynamics over time. This is illustrated in *Figure 65.*
- As shown above in the bootstrapping procedure in *Equation 15* and due to the discount curve, the 3-month EURIBOR interest rate swap is virtually decomposed into an EUR / USD fixed-to-float cross currency swap and the USD / EUR cross currency basis swap. This shows that the 3-month EURIBOR interest rate swap in this setup (l.h.s. of *Table 38*) depends on the FX basis. As a consequence, for the cash flow hedge accounting model under IAS 39 – due to the requirement of valuation of the hypothetical derivative according to market conventions – it is generally accepted to take into account this risk-equivalent decomposition of the 3-month EURIBOR interest rate swap in the USD / EUR fixed-to-float cross currency swap and the USD / EUR cross currency basis swap.
- Since the cash flow hedge accounting model according to IAS 39 represents a valuation model and follows the economic rationale above (reliance on the derivative market, absence of arbitrage, elimination of cash basis risk etc.), the cash flows of the hedged item are decomposed according to the decomposition of the 3-month EURIBOR interest rate swap. Continuing the example above *(Figure 63),* this is shown in *Figure 66.*

FIGURE 65: Different Dynamics of the Floating Side of an Interest Rate Swap Depending on the Choice of FX Basis Representation



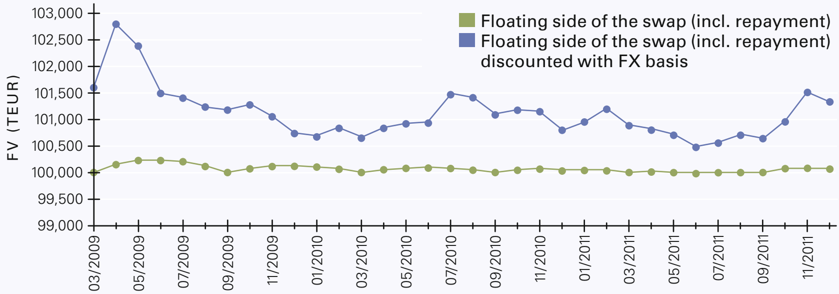
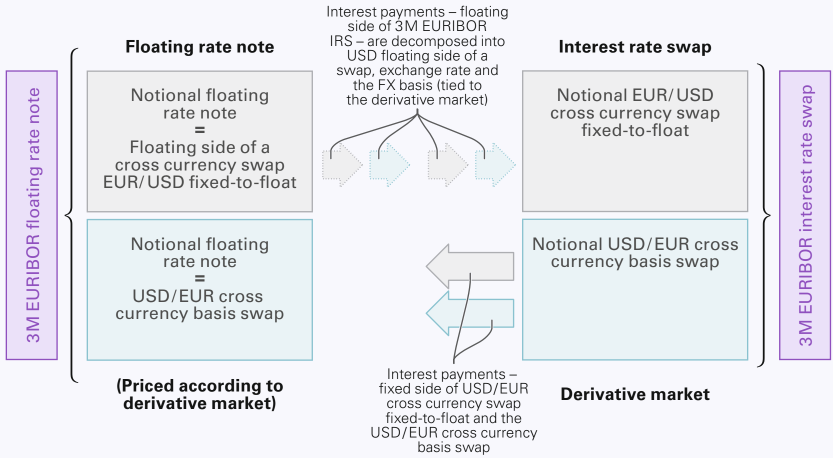


FIGURE 66: Risk-Equivalent Decomposition of a 3M EURIBOR Interest Rate Swap and a Multi-Curve Setup



- Under this setup, the cash payments of the 3-month EURIBOR floating rate note are considered as elements of the derivative market. By the chosen setup (cf. l.h.s. of *Table 18*) they are thus virtually decomposed into 3-month USD LIBOR floating payments, exchange rate and the cross currency basis spread. Accordingly the cash flow hedge accounting model under IAS 39 prescribes the hedging of variable cash flows (3-month EURIBOR), but in case of multi-curve models it is assumed in accordance with IAS 39.86(b) that this is equivalent to the variability of cash flows comprising the 3-month USD LIBOR and cross currency basis spread.
- Like the tenor basis swaps in *Section 4,* the FX basis plays an important role, since this is taken into account as an additional risk and valuation factor. In case of cash flow hedge accounting in accordance with IAS 39.86(b) all risk factors of a multi-curve setup are incorporated. As described in the sections before, taking into account tenor or FX basis risk does not mean that additional legal contracts are involved, the 3-month EURIBOR interest rate swap remains legally unchanged, as does its cash flow profile.
- As a consequence of IAS 39.86(b), KPMG Insights 7.7.630.30[76], 7.7.630.40[77], 7.7.630.50[78] and 7.7.640.10[79], both approaches of *Table 38* are applicable since they represent current market practise. The application and the impact of cash flow hedge accounting depend on the choice of the balance sheet preparer, since in a multi-curve setup the initial prices are unchanged, but the dynamics of both approaches lead to different interim P&L results (effective part, ineffective part). But over the total period (entire lifetime of the hedging relationship given no termination of the hedging relationship) the results of both approaches are identical.

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76 Change of discount rate in the hypothetical derivative without de-designation/re-designation.

77 Adjustment of the discount rate of a hypothetical derivative.

78 Hypothetical derivative method not available in a fair value hedging relationship.

79 Consideration of only the changes in fair value of the floating leg of the swap for effectiveness testing purposes when using the hypothetical derivative.

5.3.3 Cash and Carry Arbitrage Relationship – Interest Rate Parity including FX Basis Risk

In order to illustrate the effect of the FX basis on the FX forward rate the FX forward rates derived with and without FX basis are presented in *Figure 67*. The results in the figure are evaluated on the same date for different maturities based on sample data. The basis of the calculation formula remains unchanged and is presented by *Equation 10:*

$$S_{€}^{\$}(t_0)\cdot\frac{B_{\$}(t_0,T)}{B_{€}(t_0,T)} = f_{€}^{\$}(t_0,T),$$

with discount factors derived from a single-curve setup (without FX basis) and discount factors derived by means of the bootstrapping algorithm as described in *Section 5.3.1* taking the FX basis into account.

As *Figure 67* shows the FX forward rates including the FX basis are in accordance with the market quotes (composite quotes from Reuters). In *Figure 68* the corresponding (calculated) FX basis is depicted.

FIGURE 67: FX Forward Rates on one Reference Date for Different Maturities

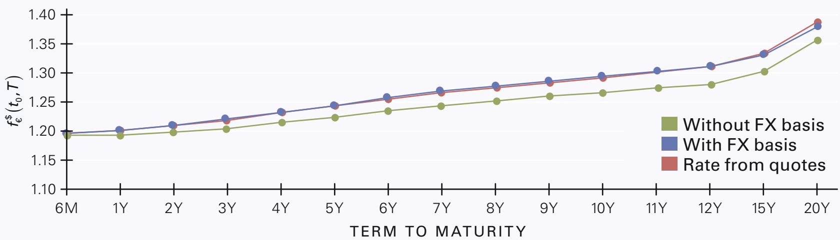


FIGURE 68: FX Basis on One Reference Date for Different Maturities

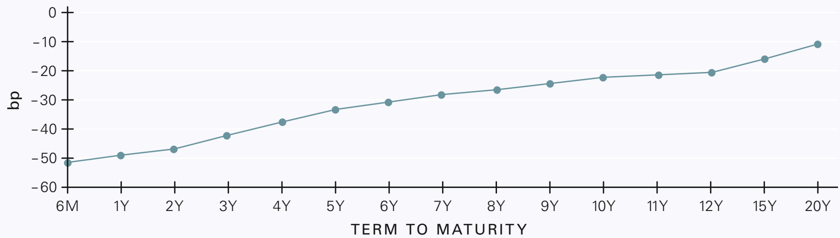
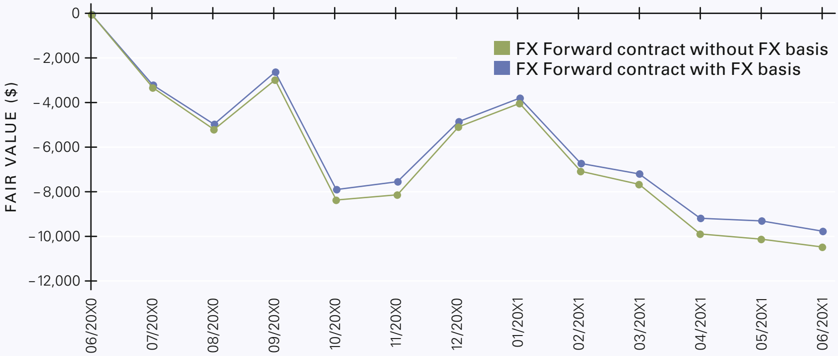


FIGURE 69: Evolution over Time of Sample FX Forward Contract



Since the FX basis is negative, the bootstrapped discount factors are greater than in the case without the FX basis and thus, according to the formula above, the FX forward rates are smaller. The formula also shows that the ratio of the two FX forward rates lies within the range of the discount factor for the given maturity and the basis spread as discount rate. If the FX basis were considered as an add on to the zero rates and continuous compounding were assumed this interpretation would be exact:

$$\frac{f^{\$}_{€,\text{FX basis}}(t_0,T)}{f^{\$}_{€}(t_0,T)} = S^{\$}_{€}(t_0)\cdot\frac{B_{\$}(t_0,T)}{B_{€,\text{FX basis}}(t_0,T)} \Bigg/ \left(S^{\$}_{€}(t_0)\cdot\frac{B_{\$}(t_0,T)}{B_{€}(t_0,T)}\right)$$

$$= \frac{B_{€}(t_0,T)}{B_{€,\text{FX basis}}(t_0,T)}$$

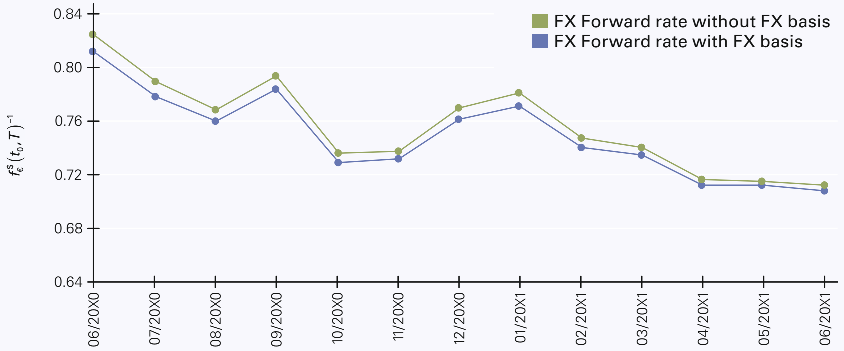
$$= \frac{\exp\left(-r_{T,\text{zero}}(T-t_0)\right)}{\exp\left(-\left(r_{T,\text{zero}}+b_{\text{zero}}(t_0,T)\right)(T-t_0)\right)}$$

$$= \exp\left(b_{\text{zero}}(t_0,T)(T-t_0)\right).$$

Using the following formula as derived in *Equation 11* to determine the fair value of an FX forward contract

$$F^{\$}_{€}\left[f^{\$}_{€}(t_0,T),t\right] = N_{\$}\cdot\left[f^{\$}_{€}(t,T)-f^{\$}_{€}(t_0,T)\right]\cdot B_{€}(t,T),$$

FIGURE 70: **Evolution over Time of Corresponding Inverse FX Forward Rate**



the evolution over time in a sample calculation can be presented as shown in *Figure 69,* where the corresponding FX forward rate evolves as shown in *Figure 70.*

As it can be seen from the formula and the sample calculations, the evolution of the FX forward contracts depends on the underlying market data.

5.3.4 Valuation of a Cross Currency Swap with FX Basis Risk

In the following the USD/EUR fixed-to-float cross currency swap example from above (refer to *Equation 13*) is continued.

$$PV_{t_0}[\text{CCS}]$$

$$= S_{€}^{\$}(t_0) N_{\$}\left[\sum_{k=1}^{T} c_{\$}^{\text{CCS}}(t_0,T)\Delta(T_{k-1},T_k)\cdot B_{\$}^{3M}(t_0,T_k) + B_{\$}^{3M}(t_0,T)\right]$$

$$-N_{€}\cdot PV_{t_0}[\text{Floater}]$$

$$\overset{!}{=} 0.$$

By definition, the present value of the cross currency swap is equal to zero at inception. According to the multi-curve setup including the FX basis *(Equation 14)* there are two ways of presenting the present value of a fixed-to-float cross currency swap:

- Adjustment due to the FX basis is taken into account in the coupon $c_{\$}^{\text{CCS}}(t_0,T)$.
- Adjustment due to the FX basis is taken into account on the EUR floating side $x(t_0,T)$.

Both representations are derived in the following:

In the first case the adjustment is taken in the coupon $c_{\$}^{\text{CCS}}(t_0,T)$, whereas the floating leg remains at forwarding on 3-month EURIBOR and is discounted with the factors obtained from the bootstrapping algorithm (multi-curve including the basis as described above), the FX adjustment is incorporated in the fixed coupon:

$$\begin{aligned}
&PV_{t_0}[\text{CCS}]\\
&= S_{€}^{\$}(t_0)N_{\$}\left[\sum_{k=1}^{N} c_{\$}^{\text{CCS}}(t_0,T)\Delta(T_{k-1},T_k)\cdot B_{\$}^{3M}(t_0,T_k)+B_{\$}^{3M}(t_0,T)\right]\\
&\quad -N_{€}\left[\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot f_{€}^{3M}(t_0,t_{j-1},t_j)\cdot B_{€}^{3M}(t_0,t_j)+B_{€}^{3M}(t_0,T)\right]\\
&\overset{!}{=} 0
\end{aligned}$$

$$\begin{aligned}
\Rightarrow\ & S_{€}^{\$}(t_0)N_{\$}\left[\sum_{k=1}^{N} c_{\$}^{\text{CCS}}(t_0,T)\Delta(T_{k-1},T_k)\cdot B_{\$}^{3M}(t_0,T_k)+B_{\$}^{3M}(t_0,T)\right]\\
&= N_{€}\left[\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot f_{€}^{3M}(t_0,t_{j-1},t_j)\cdot B_{€}^{3M}(t_0,t_j)+B_{€}^{3M}(t_0,T)\right]
\end{aligned}$$

$$\Rightarrow\ c_{\$}^{\text{CCS}}(t_0,T) = \frac{N_{€}\left[\begin{array}{l}\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot f_{€}^{3M}(t_0,t_{j-1},t_j)\cdot B_{€}^{3M}(t_0,t_j)\\ +B_{€}^{3M}(t_0,T)\end{array}\right]-S_{€}^{\$}(t_0)N_{\$}B_{\$}^{3M}(t_0,T)}{S_{€}^{\$}(t_0)N_{\$}\left[\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B_{\$}^{3M}(t_0,T_k)\right]}$$

using $S_{€}^{\$}(t_0)N_{\$} = N_{€}$:

$$c_{\$}^{CCS}(t_0,T) = \frac{\left[\begin{array}{l}\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot\left(f_{€}^{3M}(t_0,t_{j-1},t_j)+\underbrace{b(t_0,T)-b(t_0,T)}_{=0}\right)\cdot B_{€}^{3M}(t_0,t_j)\\+B_{€}^{3M}(t_0,T)-B_{\$}^{3M}(t_0,T)\end{array}\right]}{\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B_{\$}^{3M}(t_0,T_k)}$$

$$=\frac{\left[\begin{array}{l}\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot\left(f_{€}^{3M}(t_0,t_{j-1},t_j)+b(t_0,T)\right)\cdot B_{€}^{3M}(t_0,t_j)\\+B_{€}^{3M}(t_0,T)-b(t_0,T)\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot B_{€}^{3M}(t_0,t_j)-B_{\$}^{3M}(t_0,T)\end{array}\right]}{\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B_{\$}^{3M}(t_0,T_k)}$$

$$=\frac{1-B_{\$}^{3M}(t_0,T)-b(t_0,T)\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot B_{€}^{3M}(t_0,t_j)}{\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B_{\$}^{3M}(t_0,T_k)}$$

$$=c_{\$}^{3M}(t_0,T)-b(t_0,T)\cdot\frac{\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot B_{€}^{3M}(t_0,t_j)}{\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B_{\$}^{3M}(t_0,T_k)}.$$

Inserting this result into the present value formula for the fixed-to-float cross currency swap yields:

$$\begin{aligned}&PV_{t_0}[\text{CCS}]\\&=S_{€}^{\$}(t_0)N_{\$}\left[\sum_{k=1}^{N}c_{\$}^{CCS}(t_0,T)\Delta(T_{k-1},T_k)\cdot B_{\$}^{3M}(t_0,T_k)+B_{\$}^{3M}(t_0,T)\right]\\&\quad-N_{€}\left[\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot\left(f_{€}^{3M}(t_0,t_{j-1},t_j)\right)\cdot B_{€}^{3M}(t_0,t_j)+B_{€}^{3M}(t_0,T)\right].\end{aligned}$$

Using the definition from above,

$$c_{\$}^{CCS}\left(t_0,T\right)=c_{\$}^{3M}\left(t_0,T\right)-b\left(t_0,T\right)\cdot\frac{\sum_{j=1}^{4N}\delta\left(t_{j-1},t_j\right)\cdot B_{€}^{3M}\left(t_0,t_j\right)}{\sum_{k=1}^{N}\Delta\left(T_{k-1},T_k\right)\cdot B_{\$}^{3M}\left(t_0,T_k\right)}$$

EQUATION 16: Present Value of a Fixed-to-Float Cross Currency Swap (First Representation)

$$PV_{t_0}\left[CCS\right]=S_{€}^{\$}\left(t_0\right)N_{\$}\begin{bmatrix}\left[c_{\$}^{3M}\left(t_0,T\right)\sum_{k=1}^{N}\Delta\left(T_{k-1},T_k\right)\cdot B_{\$}^{3M}\left(t_0,T_k\right)+B_{\$}^{3M}\left(t_0,T\right)\right]\\-b\left(t_0,T\right)\sum_{j=1}^{4N}\delta\left(t_{j-1},t_j\right)\cdot B_{€}^{3M}\left(t_0,t_j\right)\end{bmatrix}$$
$$-N_{€}\left[\sum_{j=1}^{4N}\delta\left(t_{j-1},t_j\right)\cdot f_{€}^{3M}\left(t_0,t_{j-1},t_j\right)\cdot B_{€}^{3M}\left(t_0,t_j\right)+B_{€}^{3M}\left(t_0,T\right)\right]$$

Alternatively, assuming that the fixed coupon corresponds to the USD interest rate swap, the corresponding adjustment on the floating leg for the second representation is derived.

$$PV_{t_0}\left[CCS\right]$$
$$=S_{€}^{\$}\left(t_0\right)N_{\$}\left[\sum_{k=1}^{T}c_{\$}^{CCS}\left(t_0,T\right)\Delta\left(T_{k-1},T_k\right)\cdot B_{\$}^{3M}\left(t_0,T_k\right)+B_{\$}^{3M}\left(t_0,T\right)\right]$$
$$-N_{€}\cdot PV_0\left[\text{Floater}\right]\overset{!}{=}0$$
$$=S_{€}^{\$}\left(t_0\right)N_{\$}\left[\sum_{k=1}^{T}c_{\$}^{CCS}\left(t_0,T\right)\Delta\left(T_{k-1},T_k\right)\cdot B_{\$}^{3M}\left(t_0,T_k\right)+B_{\$}^{3M}\left(t_0,T\right)\right]$$
$$-N_{€}\begin{bmatrix}\sum_{j=1}^{4N}\delta\left(t_{j-1},t_j\right)\cdot\left(f_{€}^{3M}\left(t_0,t_{j-1},t_j\right)+x\left(t_0,T\right)\right)\cdot B_{€}^{3M}\left(t_0,t_j\right)\\+B_{€}^{3M}\left(t_0,T\right)\end{bmatrix},$$

expanding by zero and using definition of an USD IRS $c_{\$}^{3M}(t_0,T)$:

$$= S_{€}^{\$}(t_0) N_{\$}\left[\sum_{k=1}^{T} c_{\$}^{3M}(t_0,T)\Delta(T_{k-1},T_k)\cdot B_{\$}^{3M}(t_0,T_k) + B_{\$}^{3M}(t_0,T)\right]$$

$$-N_{€}\left[\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot\left(f_{€}^{3M}(t_0,t_{j-1},t_j)+x(t_0,T)\right)\cdot B_{€}^{3M}(t_0,t_j)+B_{€}^{3M}(t_0,T)\right]$$

$$-S_{€}^{\$}(t_0)\cdot N_{\$}\left[\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot f_{\$}^{3M}(t_0,t_{j-1},t_j)\cdot B_{\$}^{3M}(t_0,t_j)+B_{\$}^{3M}(t_0,T)\right]$$

$$+S_{€}^{\$}(t_0)\cdot N_{\$}\left[\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot f_{\$}^{3M}(t_0,t_{j-1},t_j)\cdot B_{\$}^{3M}(t_0,t_j)+B_{\$}^{3M}(t_0,T)\right]$$

$$= S_{€}^{\$}(t_0) N_{\$}\left[\sum_{k=1}^{T} c_{\$}^{3M}(t_0,T)\Delta(T_{k-1},T_k)\cdot B_{\$}^{3M}(t_0,T_k) + B_{\$}^{3M}(t_0,T)\right]$$

$$-S_{€}^{\$}(t_0)\cdot N_{\$}\left[\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot f_{\$}^{3M}(t_0,t_{j-1},t_j)\cdot B_{\$}^{3M}(t_0,t_j)+B_{\$}^{3M}(t_0,T)\right]$$

$$-N_{€}\left[\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot\left(f_{€}^{3M}(t_0,t_{j-1},t_j)+x(t_0,T)\right)\cdot B_{€}^{3M}(t_0,t_j)+B_{€}^{3M}(t_0,T)\right]$$

$$+S_{€}^{\$}(t_0)\cdot N_{\$}\left[\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot f_{\$}^{3M}(t_0,t_{j-1},t_j)\cdot B_{\$}^{3M}(t_0,t_j)+B_{\$}^{3M}(t_0,T)\right].$$

By definition the present value of the USD interest rate swap is zero (USD is numeraire), from this it follows that:

$$PV_{t_0}[\text{CCS}]$$

$$= -N_{€}\left[\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot\left(f_{€}^{3M}(t_0,t_{j-1},t_j)+x(t_0,T)\right)\cdot B_{€}^{3M}(t_0,t_j)+B_{€}^{3M}(t_0,T)\right]$$

$$+S_{€}^{\$}(t_0)\cdot N_{\$}\left[\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot f_{\$}^{3M}(t_0,t_{j-1},t_j)\cdot B_{\$}^{3M}(t_0,t_j)+B_{\$}^{3M}(t_0,T)\right]$$

$$\stackrel{!}{=} 0$$

$\Rightarrow\ x(t_0,T)=b(t_0,T)$ since this coincides with the definition of a cross currency basis swap.

Therefore the present value of a fixed-to-float cross currency swap can be presented as follows:

EQUATION 17: Present Value of a Fixed-to-Float Cross Currency Swap (Second Representation)

$$PV_{t_0}[CCS] = S_{€}^{\$}(t_0) N_{\$} \left[\sum_{k=1}^{T} c_{\$}^{3M}(t_0,T) \Delta(T_{k-1},T_k) \cdot B_{\$}^{3M}(t_0,T) + B_{\$}^{3M}(t_0,T) \right]$$
$$- N_{€} \left[\sum_{j=1}^{4N} \delta(t_{j-1},t_j) \cdot \left(f_{€}^{3M}(t_0,t_{j-1},t_j) + b(t_0,T) \right) \cdot B_{€}^{3M}(t_0,t_j) + B_{€}^{3M}(t_0,T) \right]$$

The comparison with *Equation 16* shows that at inception $t = t_0$ both approaches lead to the same valuation formula for a fixed-to-float CCS taking the FX basis into account, which gives the required initial value of 0 for the CCS. But the evolution over time is different: in the course of time the values will differ because the adjustments are tied to different discount factors and the fixed conversion factor is based on the market data at inception $t = t_0$.

In the second alternative the fixed-to-float cross currency swap can be presented (for valuation purposes) as USD on market interest rate swap and a float-to-float cross currency basis swap. In the first alternative – in comparison – it is decomposed into a fixed leg with an off-market USD interest rate swap coupon and a floating EUR leg. In this case the FX basis is taken into account in the coupon of the fixed side of the cross currency swap and is equal to the USD swap rate minus the FX basis multiplied by the ratio of the floating leg annuity in EUR and the fixed leg annuity in USD.

For technical purposes it may be useful to state the formula derived for the fixed rate of a CCS in the first alternative in a different way: as an expression of the basis spread and the ratio of annuities:

EQUATION 18: Adjustment Formula

$$b(t_0,T)\cdot\frac{\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot B_{€}^{3M}(t_0,t_j)}{\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B_{\$}^{3M}(t_0,T_k)}=c_{\$}^{3M}(t_0,T)-c_{\$}^{CCS}(t_0,T)$$

For the purpose of stating the cross currency swap in terms of FX forward rates, the same procedure as in *Section 5.2.3* is performed. But in this case the floating leg of the single currency interest rate swap in EUR is written in terms of different forwarding and discounting:

$$PV_{t_0}[\text{CCS}]$$

$$=\sum_{k=1}^{N}F_{€,k}^{\$,\text{CCS}}\left[f_{€}^{\$}(t_0,T_k),t_0\right]+IRS_{€}(t_0,T)$$

$$\begin{aligned}=&N_{\$}\cdot c_{\$}^{3M}(t_0,T)\sum_{k=1}^{N}f_{€}^{\$}(t_0,T_k)\cdot\Delta(T_{k-1},T_k)\cdot B_{€}(t_0,T_k)\\&+f_{€}^{\$}(t_0,T_N)\cdot N_{\$}\cdot B_{€}(t_0,T_N)-c_{€,f}^{3M}(t_0,T)\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B_{€}(t_0,T_k)\cdot N_{€}\\&-N_{€}\cdot B_{€}(t_0,T_N)+c_{€,f}^{3M}(t_0,T)\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B_{€}(t_0,T_k)\cdot N_{€}\\&+\sum_{j=1}^{4N}\delta(t_{j-1},t_j)f_{€}^{3M}(t_0,t_{j-1},t_j)\cdot B_{€}^{3M}(t_0,t_j)\end{aligned}$$

$$\begin{aligned}=&N_{\$}\cdot c_{\$}^{3M}(t_0,T)\sum_{k=1}^{N}f_{€}^{\$}(t_0,T_k)\cdot\Delta(T_{k-1},T_k)\cdot B_{€}(t_0,T_k)\\&+f_{€}^{\$}(t_0,T_N)\cdot N_{\$}\cdot B_{€}(t_0,T_N)-N_{€}\sum_{j=1}^{4N}\delta(t_{j-1},t_j)f_{€}^{3M}(t_0,t_{j-1},t_j)\cdot B_{€}^{3M}(t_0,t_j)\\&-N_{€}\cdot B_{€}(t_0,T_N).\end{aligned}$$

Mark-to-Market Feature

In order to reduce the market risk of the derivative, cross currency swaps often include a "mark-to-market (MtM) feature". According to this feature the inherent FX rate is regularly adjusted to the current market conditions. This is achieved by adjusting the notional on one side of a cross currency swap (e.g. the LIBOR flat leg of a cross currency basis swap on each fixing date), and the difference is paid as cash flow.

By this means the FX risk is reduced at each reset date in particular for long term contracts and especially if monotone development of the exchange rates occur. This feature should be taken into account in the valuation.

Recognition of FX Basis in the Discount Curve

When establishing a multi-curve framework, EUR based banks often prefer to have one discount curve for their functional currency EUR. For this purpose the banks recognize the FX basis in the valuation of cross currency products always in the discount curve of the foreign currency, which is in the case of USD contrary to the market convention as displayed in *Table 38.* Subsequently these effects – recognizing the FX basis on different legs of cross currency basis swaps – are analyzed.

Considering the example of a EUR/USD multi-curve setup according to market conventions as described in *Section 5.3.1,* the FX basis would be recognized on the EUR side, which is expressed in the following equilibrium condition (cf. *Equation 14*)[80]:

(M1)

$$\begin{aligned}
&S_{€}^{\$}(t_0)N_{\$}\\
&= S_{€}^{\$}(t_0)N_{\$}\left[\sum_{i=1}^{4N} f_{\$}^{3M}(t_0,t_{i-1},t_i)\delta_{\$}(t_{i-1},t_i)\cdot B_{\$}^{3M}(t_0,t_i)+B_{\$}^{3M}(t_0,T)\right]\\
&= N_{€}\left[\begin{aligned}&\sum_{j=1}^{4N}\delta_{€}(t_{j-1},t_j)\cdot\left(f_{€}^{FX/3M}(t_0,t_{j-1},t_j)+b(t_0,T)\right)\cdot B_{€}^{FX/3M}(t_0,t_j)\\ &+B_{€}^{FX/3M}(t_0,T)\end{aligned}\right].
\end{aligned}$$

80 For simplicity the EURIBOR/LIBOR discounting case is considered, but the arguments will be similar for the collateralized case described in *Section 6.*

Although this is the same condition as in the equilibrium conditions represented in *Equation 14,* the notation is slightly changed for EUR discount factors, e.g. $B_{€}^{FX/3M}(t_0,T)$ instead of $B_{€}^{3M}(t_0,T)$, in order to distinguish them in this subsection from the discount factors in the following approach (M2).

In the second approach (FX basis recognized in the USD discount curve) the following equilibrium condition would be assumed to hold.

(M2)

$$S_{€}^{\$}(t_0)N_{\$}\left[\sum_{i=1}^{4N}\begin{pmatrix} f_{\$}^{FX/3M}(t_0,t_{i-1},t_i) \\ -b(t_0,T)\end{pmatrix}\delta_{\$}(t_{i-1},t_i)\cdot B_{\$}^{FX/3M}(t_0,t_i)+B_{\$}^{FX/3M}(t_0,T)\right]$$

$$=N_{€}\left[\sum_{j=1}^{4N}\delta_{€}(t_{j-1},t_j)\cdot f_{€}^{3M}(t_0,t_{j-1},t_j)\cdot B_{€}^{3M}(t_0,t_j)+B_{€}^{3M}(t_0,T)\right]$$

$$=N_{€}.$$

In generalization of the previous notation for points in time, time periods and discount factors as e.g. introduced in *Sections 4.2.2* or *4.2.6,* the following abbreviations (for so called "annuities") will be used:

$$A_{\text{currency}}^{\text{curve}}(t,T)=\sum_{k=1}^{N}\Delta_{\$}(T_{k-1},T_k)\cdot B_{\text{currency}}^{\text{curve}}(t,T_k)$$

$$a_{\text{currency}}^{\text{curve}}(t,T)=\sum_{j=1}^{4N}\delta_{€}(t_{j-1},t_j)\cdot B_{\text{currency}}^{\text{curve}}(t,t_j),$$

where "curve" relates to the applied interest rate curve like 3-month EURIBOR or FED Funds and "currency" to the currency of the interest curve like "€" for EUR and "\$" for USD.

Starting from (M1) the following holds true using $S_{€}^{\$}(t_0)N_{\$} = N_{€}$ and the USD discounting as numeraire:

$$1=\sum_{j=1}^{4N}\delta_{€}(t_{j-1},t_j)\cdot\left(f_{€}^{\text{FX/3M}}(t_0,t_{j-1},t_j)+b(t_0,T)\right)\cdot B_{€}^{\text{FX/3M}}(t_0,t_j)$$
$$+B_{€}^{\text{FX/3M}}(t_0,T)$$
$$=c_{€}^{\text{3M}}(t_0,T)\cdot A_{€}^{\text{FX/3M}}(t_0,T)+b(t_0,T)\cdot\sum_{j=1}^{4N}\delta_{€}(t_{j-1},t_j)\cdot B_{€}^{\text{FX/3M}}(t_0,t_j)$$
$$+B_{€}^{\text{FX/3M}}(t_0,T)$$
$$=c_{€}^{\text{3M}}(t_0,T)\cdot A_{€}^{\text{FX/3M}}(t_0,T)\frac{A_{€}^{\text{3M}}(t_0,T)}{A_{€}^{\text{3M}}(t_0,T)}+b(t_0,T)\cdot a_{€}^{\text{FX/3M}}(t_0,T)$$
$$+B_{€}^{\text{FX/3M}}(t_0,T)\frac{B_{€}^{\text{3M}}(t_0,T)}{B_{€}^{\text{3M}}(t_0,T)}$$
$$=c_{€}^{\text{3M}}(t_0,T)\cdot A_{€}^{\text{3M}}(t_0,T)\frac{A_{€}^{\text{FX/3M}}(t_0,T)}{A_{€}^{\text{3M}}(t_0,T)}+b(t_0,T)\cdot a_{€}^{\text{FX/3M}}(t_0,T)$$
$$+B_{€}^{\text{3M}}(t_0,T)\frac{B_{€}^{\text{FX/3M}}(t_0,T)}{B_{€}^{\text{3M}}(t_0,T)}\ .$$

Exploiting the first line of (M1), i.e. forwarding equals discounting on the USD leg, and transferring the FX basis term on the other side yields:

$$\sum_{i=1}^{4N}f_{\$}^{\text{3M}}(t_0,t_{i-1},t_i)\delta_{\$}(t_{i-1},t_i)\cdot B_{\$}^{\text{3M}}(t_0,t_i)+B_{\$}^{\text{3M}}(t_0,T)$$
$$-b(t_0,T)\cdot a_{€}^{\text{FX/3M}}(t_0,T)$$
$$=c_{€}^{\text{3M}}(t_0,T)\cdot A_{€}^{\text{3M}}(t_0,T)\frac{A_{€}^{\text{FX/3M}}(t_0,T)}{A_{€}^{\text{3M}}(t_0,T)}+B_{€}^{\text{3M}}(t_0,T)\frac{B_{€}^{\text{FX/3M}}(t_0,T)}{B_{€}^{\text{3M}}(t_0,T)}$$

$$\Leftrightarrow\ c_{\$}^{\text{3M}}(t_0,T)A_{\$}^{\text{3M}}(t_0,T)+B_{\$}^{\text{3M}}(t_0,T)-b(t_0,T)\cdot a_{€}^{\text{FX/3M}}(t_0,T)$$
$$=c_{€}^{\text{3M}}(t_0,T)\cdot A_{€}^{\text{3M}}(t_0,T)\frac{A_{€}^{\text{FX/3M}}(t_0,T)}{A_{€}^{\text{3M}}(t_0,T)}+B_{€}^{\text{3M}}(t_0,T)\frac{B_{€}^{\text{FX/3M}}(t_0,T)}{B_{€}^{\text{3M}}(t_0,T)}$$

$$\Leftrightarrow c_{\$}^{3M}(t_0,T)A_{\$}^{FX/3M}(t_0,T)\frac{A_{\$}^{3M}(t_0,T)}{A_{\$}^{FX/3M}(t_0,T)}+B_{\$}^{FX/3M}(t_0,T)\frac{B_{\$}^{3M}(t_0,T)}{B_{\$}^{FX/3M}(t_0,T)}$$

$$-b(t_0,T)\cdot a_{\$}^{FX/3M}(t_0,T)\frac{a_{€}^{FX/3M}(t_0,T)}{a_{\$}^{FX/3M}(t_0,T)}$$

$$=c_{€}^{3M}(t_0,T)\cdot A_{€}^{3M}(t_0,T)\frac{A_{€}^{FX/3M}(t_0,T)}{A_{€}^{3M}(t_0,T)}+B_{€}^{3M}(t_0,T)\frac{B_{€}^{FX/3M}(t_0,T)}{B_{€}^{3M}(t_0,T)}\;.$$

The equilibrium condition of (M2) may be restated in the following way:

$$\sum_{i=1}^{4N}\left(f_{\$}^{FX/3M}(t_0,t_{i-1},t_j)-b(t_0,T)\right)\delta_{\$}(t_{i-1},t_i)\cdot B_{\$}^{FX/3M}(t_0,t_i)+B_{\$}^{FX/3M}(t_0,T)$$

$$=\sum_{j=1}^{4N}\delta_{€}(t_{j-1},t_j)\cdot f_{€}^{3M}(t_0,t_{j-1},t_j)\cdot B_{€}^{3M}(t_0,t_j)+B_{€}^{3M}(t_0,T)$$

$$=1$$

$$\Leftrightarrow c_{\$}^{3M}(t_0,T)A_{\$}^{FX/3M}(t_0,T)-b(t_0,T)a_{\$}^{FX/3M}(t_0,T)+B_{\$}^{FX/3M}(t_0,T)$$

$$=c_{€}^{3M}(t_0,T)A_{€}^{3M}(t_0,T)+B_{€}^{3M}(t_0,T).$$

The comparison of the coefficients of this equation and the equation derived from equilibrium condition (M1) shows that the difference between the two models is driven by the following ratios:

$$\frac{A_{\$}^{3M}(t_0,T)}{A_{\$}^{FX/3M}(t_0,T)},\frac{B_{\$}^{3M}(t_0,T)}{B_{\$}^{FX/3M}(t_0,T)},\frac{a_{€}^{FX/3M}(t_0,T)}{a_{\$}^{FX/3M}(t_0,T)},\frac{A_{€}^{FX/3M}(t_0,T)}{A_{€}^{3M}(t_0,T)},\frac{B_{€}^{FX/3M}(t_0,T)}{B_{€}^{3M}(t_0,T)}.$$

This can be regarded as a generalization of the formula derived in the course of the representation of a 6-month EURIBOR interest rate swap discounted with 3-month EURIBOR (refer to *Equation 5* in *Section 4.3.2;* similar representations will also be derived for collateralized derivatives in *Section 6.4*). The "ideal" case where all of these ratios equal 1 represents the situation where the FX basis is zero or no FX basis is taken into account on the USD or EUR leg. So, for an existing FX basis the consideration of the FX basis either on the EUR side or on the USD side results in a difference driven by ratios of discount factors or annuities. The impact on the valuation depends on their quantity.

TABLE 39:	Terms and Conditions of the Cross Currency Basis Swap	
Terms and conditions	**CCS EUR leg**	**CCS USD leg**
Value date	01/23/20X9	01/23/20X9
Maturity	01/23/20X4	01/23/20X4
Reference rate	3M EURIBOR	3M USD LIBOR
Interest payment frequency	Quarterly	Quarterly
Spread	–0.3375 bp	None
Notional	78,155.53€	–100,000.00$
Day count convention	ACT/360	ACT/360

In order to illustrate its impact, a sample calculation for the cross currency basis swap (see *Table 39*) is evaluated w.r.t. the different representations of the FX basis given by (M1) and (M2)[81]. In *Figure 71* and *Figure 72* the corresponding differences of dynamics (monthly changes) of both representations are illustrated in absolute and relative terms respectively.

Figure 72 shows that the two different representations lead to different results, which can be significant. Additionally it can be concluded that the ratios of annuities deviate from 1 depending on market conditions or the shape of the term structure. Similar results and figures can be derived for fixed-to-float cross currency swaps.

5.3.5 Hedge Accounting of FX Risk with FX Basis Risk

5.3.5.1 Fair Value Hedge Accounting with a Cross Currency Swap including the FX Basis Risk

In order to show the connection to hedge accounting the example from *Section 5.2.3* is continued. Without the FX basis risk the 3-month USD LIBOR interest rate swap rate is the relevant portion of cash flow for the hedged item and represents the hedged risk in connection with the actual FX spot rate.

81 Approximately the FX basis is simply added to the discount rates and no bootstrapping is performed in this example.

FIGURE 71: Differences in Monthly FV Changes for a Cross Currency Basis Swap Valued with FX Basis on the USD and the EUR Discount Curve Respectively

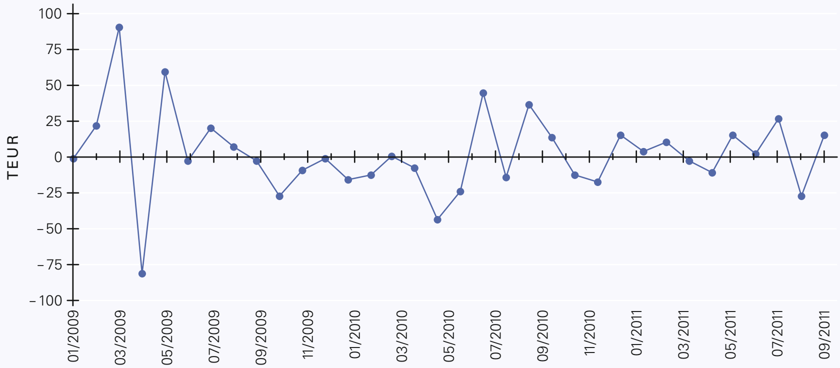
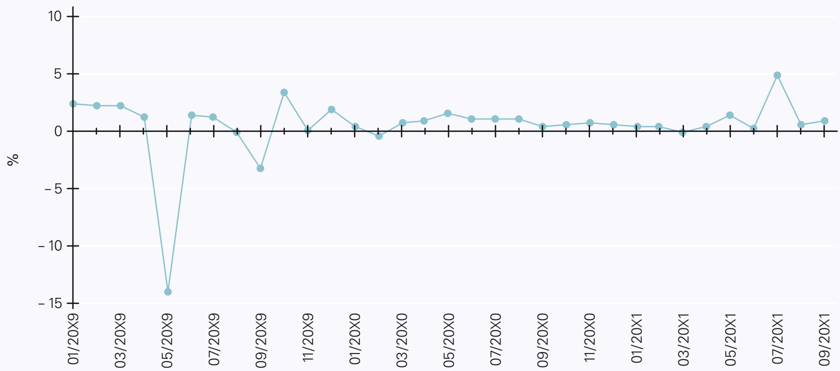


FIGURE 72: Relative Differences in Monthly FV Changes for a Cross Currency Basis Swap Valued with FX Basis on the USD and the EUR Discount Curve Respectively



With the FX basis risk the circumstances are more complicated. Starting point of the following analysis is the definition of the fixed-to-float cross currency swap in *Equation 16*. Let's analyze the changes in present value of the EUR floating leg at a subsequent reset date $t_4 = T_1$:

EQUATION 19: Reset of the EUR Floating Leg of a Cross Currency Swap

$$
\begin{aligned}
&-N_€\left[\sum_{j=5}^{4N}\delta(t_{j-1},t_j)\cdot\left(f_€^{3M}(t_4,t_{j-1},t_j)\right)\cdot B_€^{3M}(t_4,t_j)+B_€^{3M}(t_4,T)\right]\\
&=-N_€\left[\sum_{j=5}^{4N}\delta(t_{j-1},t_j)\cdot\left(f_€^{3M}(t_4,t_{j-1},t_j)+\underbrace{\left(b(t_4,T)-b(t_4,T)\right)}_{=0}\right)\cdot B_€^{3M}(t_4,t_j)+B_€^{3M}(t_4,T)\right]\\
&=-N_€\left[\sum_{j=5}^{4N}\delta(t_{j-1},t_j)\cdot\left(f_€^{3M}(t_4,t_{j-1},t_j)+b(t_4,T)\right)\cdot B_€^{3M}(t_4,t_j)+B_€^{3M}(t_4,T)\right]\\
&\quad+N_€ b(t_4,T)\left[\sum_{j=5}^{4N}\delta(t_{j-1},t_j)\cdot B_€^{3M}(t_4,t_j)\right]\\
&=-N_€\left[1-b(t_4,T)\left[\sum_{j=5}^{4N}\delta(t_{j-1},t_j)\cdot B_€^{3M}(t_4,t_j)\right]\right]\\
&=-N_€+N_\$\cdot S_€^\$(t_0)\cdot b(t_4,T)\left[\sum_{j=5}^{4N}\delta(t_{j-1},t_j)\cdot B_€^{3M}(t_4,t_j)\right]
\end{aligned}
$$

The results above show that the EUR floating leg resets to one minus the current cross currency basis spread multiplied by the floating leg annuity. The latter term causes ineffectiveness in terms of hedge accounting. The structure will be the same on each reset date: reset to $N_€$ plus a second term depending via $N_€ = N_\$ \cdot S_€^\$(t_0)$ on the initial spot rate and the current cross currency basis spread $b(t,T)$ for the remaining term and the corresponding annuity of the floating leg in EUR.

This result now defines the dynamic hedge accounting strategy:

In t_0 the hedged item is defined as fixed coupon liability with the following internal coupon:

$$c_{\text{int},t_0}^{\text{FX,3M}}\left(t_0,T\right)=c_{\$}^{\text{3M}}\left(t_0,T\right).$$

This corresponds to the USD discount curve used for the fixed leg of the CCS. But with the possibility to designate portions of cash flows, the internal coupon in t_0 can also be defined as follows:

$$c_{\text{int},t_0}^{\text{FX,3M}}\left(t_0,T\right)=c_{\$}^{\text{3M}}\left(t_0,T\right)-b\left(t_0,T\right)\cdot\frac{\sum_{j=1}^{4N}\delta\left(t_{j-1},t_j\right)\cdot B_{€}^{\text{3M}}\left(t_0,t_j\right)}{\sum_{k=1}^{N}\Delta\left(T_{k-1},T_k\right)\cdot B_{\$}^{\text{3M}}\left(t_0,T_k\right)}.$$

Using the latter representation, the initial internal coupon coincides with the fixed rate on the CCS.

In the single-curve case the FX basis is zero and so the rates for the single currency interest rate swap and the CCS also coincide.

In T_1 the cash flows of the hedged item are adjusted and defined as fixed coupon liability with the following coupon using the reset property of the EUR floating leg and the identity $N_{€}=N_{\$}\cdot S_{€}^{\$}\left(t_0\right)$ as derived above:

$$c_{\text{int},t_0}^{\text{FX,3M}}\left(T_1,T\right)=c_{\$}^{\text{3M}}\left(t_0,T\right)-\left[\begin{array}{l} b\left(t_0,T\right)\dfrac{\sum_{j=1}^{4N}\delta\left(t_{j-1},t_j\right)\cdot B_{€}^{\text{3M}}\left(t_0,t_j\right)}{\sum_{k=1}^{N}\Delta\left(T_{k-1},T_k\right)\cdot B_{\$}^{\text{3M}}\left(t_0,T_k\right)} \\ -b\left(t_4,T\right)\dfrac{S_{€}^{\$}\left(t_0\right)}{S_{€}^{\$}\left(T_1\right)}\dfrac{\sum_{j=5}^{4N}\delta\left(t_{j-1},t_j\right)\cdot B_{€}^{\text{3M}}\left(\text{t}_4,t_j\right)}{\sum_{k=2}^{N}\Delta\left(T_{k-1},T_k\right)\cdot B_{\$}^{\text{3M}}\left(T_1,T_k\right)}\end{array}\right]$$

$$=c_{\$}\left(t_0,T\right)+b\left(t_4,T\right)\frac{S_{€}^{\$}\left(t_0\right)}{S_{€}^{\$}\left(T_1\right)}\frac{\sum_{j=5}^{4N}\delta\left(t_{j-1},t_j\right)\cdot B_{€}^{\text{3M}}\left(t_4,t_j\right)}{\sum_{k=2}^{N}\Delta\left(T_{k-1},T_k\right)\cdot B_{\$}^{\text{3M}}\left(T_1,T_k\right)}.$$

In general terms this will lead to the following adjustments on a reset date *t* of the fixed leg:

EQUATION 20: Dynamically Adjusted Internal Coupon for the First Cross Currency Swap Representation (FX Basis Incorporated in Fixed Rate) for EURIBOR / LIBOR Discounting

$$c_{\text{int},t_0}^{\text{FX,3M}}(t,T) = c_{\$}^{\text{3M}}(t_0,T) - \left[\begin{array}{l} b(t_0,T)\dfrac{\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot B_{€}^{\text{3M}}(t_0,t_j)}{\sum_{k=1}^{N}\Delta(T_{k-1},T_k)\cdot B_{\$}^{\text{3M}}(t_0,T_k)} \\ -b(t,T)\dfrac{S_{€}^{\$}(t_0)}{S_{€}^{\$}(t)}\dfrac{\sum_{j>t}^{4N}\delta(t_{j-1},t_j)\cdot B_{€}^{\text{3M}}(t,t_j)}{\sum_{k>t}^{N}\Delta(T_{k-1},T_k)\cdot B_{\$}^{\text{3M}}(t,T_k)} \end{array} \right]$$

$$c_{\text{int},t_0}^{\text{FX,3M}}(t,T) = c_{\text{int},t_0}^{\text{FX,3M}}(t_0,T) + b(t,T)\frac{S_{€}^{\$}(t_0)}{S_{€}^{\$}(t)}\frac{\sum_{j>t}^{4N}\delta(t_{j-1},t_j)\cdot B_{€}^{\text{3M}}(t,t_j)}{\sum_{k>t}^{N}\Delta(T_{k-1},T_k)\cdot B_{\$}^{\text{3M}}(t,T_k)}$$

Therefore the initial coupon is dynamically adjusted by the current FX basis converted with the historical currency spot rate. Please note that the USD discount rate is not adjusted by the FX basis by assuming the market convention for the multi-curve setup so that the restrictions concerning "sub-LIBOR" restrictions under IAS 39 are avoided.

In order to show the effect of the definition of the cash flow in T_1, it is inserted in the present value formula of the hedged item:

$$HFV(T_1) := S_{€}^{\$}(T_1)N_{\$}\left[c_{\text{int},t_0}^{\text{FX,3M}}(T_1,T)\sum_{k=2}^{N}\Delta(T_{k-1},T_k)\cdot B_{\$}^{\text{3M}}(T_1,T_k) + B_{\$}^{\text{3M}}(T_1,T) \right]$$

$$= S_{€}^{\$}(T_1)N_{\$}\left[\begin{array}{l} c_{\text{int},t_0}^{\text{FX,3M}}(t_0,T)\sum_{k=2}^{N}\Delta(T_{k-1},T_k)\cdot B_{\$}^{\text{3M}}(T_1,T_k) + B_{\$}^{\text{3M}}(T_1,T) \\ +b(T_1,T)\dfrac{S_{€}^{\$}(t_0)}{S_{€}^{\$}(T_1)}\sum_{k=2}^{N}\Delta(T_{k-1},T_k)\cdot B_{\$}^{\text{3M}}(T_1,T_k) \\ \cdot\left(\sum_{j=5}^{4N}\delta(t_{j-1},t_j)\cdot B_{€}^{\text{3M}}(T_1,t_j)\right) \Big/ \left(\sum_{k=2}^{N}\Delta(T_{k-1},T_k)\cdot B_{\$}^{\text{3M}}(T_1,T_k)\right) \end{array} \right]$$

$$
= S_{€}^{\$}(T_1) N_{\$} \left[\begin{array}{l} c_{\text{int},t_0}^{\text{FX,3M}}(t_0,T) \sum_{k=2}^{N} \Delta(T_{k-1},T_k) \cdot B_{\$}^{3M}(T_1,T_k) + B_{\$}^{3M}(T_1,T) \\ +b(T_1,T) \dfrac{S_{€}^{\$}(t_0)}{S_{€}^{\$}(T_1)} \sum_{j=5}^{4N} \delta(t_{j-1},t_j) \cdot B_{€}^{3M}(T_1,t_j) \end{array} \right]
$$

$$
= S_{€}^{\$}(T_1) N_{\$} \left[c_{\text{int},t_0}^{\text{FX,3M}}(t_0,T) \sum_{k=2}^{N} \Delta(T_{k-1},T_k) \cdot B_{\$}^{3M}(T_1,T_k) + B_{\$}^{3M}(T_1,T) \right]
$$

$$
\underbrace{+b(T_1,T) \overbrace{S_{€}^{\$}(t_0) N_{\$}}^{=N_{€}} \sum_{j=5}^{4N} \delta(t_{j-1},t_j) \cdot B_{€}^{3M}(T_1,t_j)}_{\text{Reset amount floating side of the EUR leg}}.
$$

If the FX basis is taken into account on the EUR floating side, at the reset date $t_4 = T_1$:

$$
-N_{€} \left[\sum_{j=5}^{4N} \delta(t_{j-1},t_j) \cdot \begin{pmatrix} f_{€}^{3M}(t_4,t_{j-1},t_j) \\ +b(t_0,T) \end{pmatrix} \cdot B_{€}^{3M}(t_4,t_j) + B_{€}^{3M}(t_4,T) \right]
$$

$$
= -N_{€} \left[\begin{array}{l} \sum_{j=5}^{4N} \delta(t_{j-1},t_j) \cdot b(t_0,T) \cdot B_{€}^{3M}(t_4,t_j) + B_{€}^{3M}(t_4,T) \\ + \sum_{j=5}^{4N} \delta(t_{j-1},t_j) \cdot \begin{pmatrix} f_{€}^{3M}(t_4,t_{j-1},t_j) \\ + \underbrace{\left(b(t_4,T) - b(t_4,T)\right)}_{=0} \end{pmatrix} \cdot B_{€}^{3M}(t_4,t_j) \end{array} \right]
$$

$$
= -N_{€} \left[\begin{array}{l} b(t_0,T) \cdot \sum_{j=5}^{4N} \delta(t_{j-1},t_j) \cdot B_{€}^{3M}(t_4,t_j) \\ -b(t_4,T) \cdot \sum_{j=5}^{4N} \delta(t_{j-1},t_j) \cdot B_{€}^{3M}(t_4,t_j) \\ \underbrace{+ \sum_{j=5}^{4N} \delta(t_{j-1},t_j) \cdot \begin{pmatrix} f_{€}^{3M}(t_4,t_{j-1},t_j) \\ +b(t_4,T) \end{pmatrix} \cdot B_{€}^{3M}(t_4,t_j) + B_{€}^{3M}(t_4,T)}_{=1} \end{array} \right]
$$

$$
= -N_{€} \left[\left(b(t_0,T) - b(t_4,T)\right) \cdot \sum_{j=5}^{4N} \delta(t_{j-1},t_j) \cdot B_{€}^{3M}(t_4,t_j) + 1 \right]
$$

$$
= -N_{\$} \cdot S_{€}^{\$}(t_0) \left(b(t_0,T) - b(t_4,T)\right) \cdot \sum_{j=5}^{4N} \delta(t_{j-1},t_j) \cdot B_{€}^{3M}(t_4,t_j) - N_{€}.
$$

This result now defines the dynamic hedge accounting strategy:

In t_0 the hedged item is defined as fix coupon liability with the USD swap rate as coupon:

$$c_{\text{int},t_0}^{\text{FX,3M}}\left(t_0,T\right)=c_{\$}^{3\text{M}}\left(t_0,T\right).$$

Alternatively the initial definition

$$c_{\text{int},t_0}^{\text{FX,3M}}\left(t_0,T\right)=c_{\$}^{3\text{M}}\left(t_0,T\right)-b\left(t_0,T\right)\cdot\frac{\sum_{j=1}^{4N}\delta\left(t_{j-1},t_j\right)\cdot B_{€}^{3\text{M}}\left(t_0,t_j\right)}{\sum_{k=1}^{N}\Delta\left(T_{k-1},T_k\right)\cdot B_{\$}^{3\text{M}}\left(t_0,T_k\right)}$$

using the same rationale and consequences as above could be chosen.

In T_1 the cash flows of the hedged item are adjusted and defined as fix coupon liability with the following coupon using the reset property of the EUR floating leg and the identity $N_{€}=N_{\$}\cdot S_{€}^{\$}\left(t_0\right)$ as derived above:

$$c_{\text{int},t_0}^{\text{FX,3M}}\left(T_1,T\right)$$

$$=c_{\$}^{3\text{M}}\left(t_0,T\right)-\left(b\left(t_0,T\right)-b\left(T_1,T\right)\right)\cdot\frac{S_{€}^{\$}\left(t_0\right)}{S_{€}^{\$}\left(T_1\right)}\frac{\sum_{j=5}^{4N}\delta\left(t_{j-1},t_j\right)\cdot B_{€}^{3\text{M}}\left(T_1,t_j\right)}{\sum_{k=2}^{N}\Delta\left(T_{k-1},T_k\right)\cdot B_{\$}^{3\text{M}}\left(T_1,T_k\right)}.$$

Consequently for arbitrary t:

EQUATION 21: Dynamically Adjusted Internal Coupon for the Second Cross Currency Swap Representation (FX Basis as Constant Spread on the Floating Side) EURIBOR / LIBOR Discounting

$$c_{\text{int},t_0}^{\text{FX,3M}}\left(t,T\right)=c_{\$}^{3\text{M}}\left(t_0,T\right)-\left(b\left(t_0,T\right)-b\left(t,T\right)\right)\cdot\frac{S_{€}^{\$}\left(t_0\right)}{S_{€}^{\$}\left(t\right)}\frac{\sum_{j>t}^{4N}\delta\left(t_{j-1},t_j\right)\cdot B_{€}^{3\text{M}}\left(t,t_j\right)}{\sum_{k>t}^{N}\Delta\left(T_{k-1},T_k\right)\cdot B_{\$}^{3\text{M}}\left(t,T_k\right)}$$

This definition of the coupon is utilized to determine the fair changes of the hedged item.

TABLE 40: Example for a USD / EUR Fixed-to-Float Cross Currency Hedging Relationship

Terms and conditions	Fixed liability	CCS fixed leg	CCS float leg
Value date	01/23/20X9	01/23/20X9	01/23/20X9
Maturity	01/23/20X4	01/23/20X4	01/23/20X4
Fixed internal rate/tenor	2.3036%	2.3036%	3M
Interest payment frequency	Semi-annually	Semi-annually	Quarterly
Notional	–100,000.00$	100,000.00$	–78,155.53€
Day count convention	30/360	30/360	ACT/360

For illustration in the following the hedge accounting example of *Section 5.2.3* is considered in the multi-curve setup using the dynamical adjustment method as described above (see *Table 40*).

The internal coupon is determined by the risk-equivalent bond/loan method, and the hedging cost measurement method will be applied. Date of inception is the value date of liability and swap.

In order to avoid calculation repetitions only the relevant changes from the case without FX basis are presented first for the measurement of the hedging instrument (CCS) and then for the hedged item including the dynamical adjustment.

Since the adjustments of the FX basis are considered on the EUR side, i.e. the EUR discount curve is bootstrapped as described in *Section 5.3.1* and following the first alternative of valuation for CCS of *Section 5.3.4* the valuation of the fixed leg differs only in fixed coupon, whereas for the floating leg different curves for forwarding (3-month EURIBOR) and discounting (3-month EURIBOR including the USD FX basis) are used.

At inception t_0=01/23/20X9 the exchange rate of 1 EUR = 1.2795 USD and the discount factors derived from USD vs. 3-month USD LIBOR swap rates for the indicated terms (in years) shown in *Table 41* are given.

The given market data in EUR are shown in *Table 42*.

TABLE 41: Example for a USD / EUR Fixed-to-Float Cross Currency Hedging Relationship – Discount Factors at t_0

Years to maturity $T_i - t_0$	0.5	1	1.5	2	2.5
Discount factor $B_\$(t_0, T_i)$	0.992003	0.980924	0.974573	0.970315	0.958947
Years to maturity $T_i - t_0$	**3**	**3.5**	**4**	**4.5**	**5**
Discount factor $B_\$(t_0, T_i)$	0.945949	0.932932	0.918869	0.905236	0.890917

TABLE 42: Example for a USD / EUR Fixed-to-Float Cross Currency Hedging Relationship – Market Data

Years to maturity $t_j - t_0$	0.25	0.5	0.75	1	1.25	1.5	1.75
Discount factor $B_€(t_0, t_j)$	0.99587	0.99135	0.98586	0.98094	0.97696	0.97325	0.96979
Days per period $(t_j - t_{j-1})$	90	91	92	92	90	91	92
Forward rate $f_€^{3M}(t_0, t_{j-1}, t_j)$	2.17 %	2.38 %	2.40 %	2.45 %	2.02 %	2.17 %	1.75 %
Years to maturity $t_j - t_0$	**2**	**2.25**	**2.5**	**2.75**	**3**	**3.25**	**3.5**
Discount factor $B_€(t_0, t_j)$	0.96662	0.96079	0.95446	0.94768	0.94053	0.93331	0.92579
Days per period $(t_j - t_{j-1})$	92	90	91	92	92	91	91
Forward rate $f_€^{3M}(t_0, t_{j-1}, t_j)$	1.61 %	2.74 %	2.92 %	3.08 %	3.23 %	3.39 %	
Years to maturity $t_j - t_0$	**3.75**	**4**	**4.25**	**4.5**	**4.75**	**5**	
Discount factor $B_€(t_0, t_j)$	0.91787	0.90964	0.90205	0.89419	0.88604	0.87770	
Days per period $(t_j - t_{j-1})$	92	92	90	91	92	91	
Forward rate $f_€^{3M}(t_0, t_{j-1}, t_j)$	3.70 %	3.85 %	3.64 %	3.74 %	3.85 %	3.96 %	

Thus the value of the floating leg according to *Equation 16* is

$$N_{€}\left[\sum_{j=1}^{20}\delta\left(t_{j-1},t_j\right)\cdot f_{€}^{3M}\left(t_0,t_{j-1},t_j\right)\cdot B_{€}^{3M}\left(t_0,t_j\right)+B_{€}^{3M}\left(t_0,t_{20}\right)\right]$$

$$=-78{,}155.53\,€\cdot\left[\begin{array}{l}\frac{90}{360}\cdot 2.17\,\%\cdot 0.99587+\frac{91}{360}\cdot 2.38\,\%\cdot 0.99135+\ldots\\ +\frac{92}{360}\cdot 3.96\,\%\cdot 0.87770+0.87770\end{array}\right]$$

$$=-79{,}419.02\,€\,.$$

Also for the fixed leg the formula of *Equation 16* using the 3-month USD LIBOR swap rate and the FX basis can be used or, as the CCS should be zero at inception, the fact that also the fixed leg has to have this value with opposite sign.

$$S_{€}^{\$}\left(t_0\right)N_{\$}\left[\sum_{k=1}^{10}c_{\$}^{CCS}\left(t_0,T\right)\Delta\left(T_{k-1},T_k\right)\cdot B_{\$}^{3M}\left(t_0,T_k\right)+B_{\$}^{3M}\left(t_0,T_{10}\right)\right]$$
$$=79{,}419.02\,€\,.$$

This yields:

$$c_{\$}^{CCS}\left(t_0,T\right)=\left[\frac{79{,}419.02\,€}{S_{€}^{\$}\left(t_0\right)N_{\$}}-B_{\$}^{3M}\left(t_0,T_{10}\right)\right]\Big/\sum_{k=1}^{10}\Delta\left(T_{k-1},T_k\right)\cdot B_{\$}^{3M}\left(t_0,T_k\right)$$
$$=\left[\frac{79{,}419.02\,€}{78{,}155.53\,€}-B_{\$}^{3M}\left(t_0,T_{10}\right)\right]\Big/\sum_{k=1}^{10}\frac{B_{\$}^{3M}\left(t_0,T_k\right)}{2}$$
$$=2.64499\,\%\,.$$

Setting the fixed rate $c_{\$}^{CCS}\left(t_0,T\right)$ of the fair CCS to 2.64499% by definition results in a fair value of zero at inception t_0:

$$FV_{CCS}\left(t_0\right)=\text{fixed side}+\text{floating side}=79{,}419.02\,€-79{,}419.02\,€$$
$$=0\,.$$

But it should be noted that due to the fact that forwarding and discounting are not done on the same curve, the absolute value of each leg no longer coincides with that of the EUR notional $N_{€}=-78{,}155.53\,€$.

Using the market data for USD as in *Section 5.2.3* for the valuation of the fixed leg with the fixed rate $c_{\$}^{CCS}\left(t_0,T\right)$ and corresponding market

TABLE 43: Fair Values and Fair Value Changes of the Cross Currency Swap

Date	Fixed leg	Floating leg	FV CCS	FV changes CCS
t_0	79,419.02€	–79,419.02€	0.00€	
t_1	69,782.94€	–79,085.10€	–9,302.16€	–9,302.16€
t_2	71,773.14€	–78,667.63€	–6,894.49€	2,407.68€
t_3	80,964.64€	–79,014.69€	1,949.95€	8,844.44€
t_4	76,811.48€	–78,852.89€	–2,041.40€	–3,991.35€

data for the EUR leg with forwarding and discounting on different curves as shown above leads to the fair values and resulting fair value changes of the CCS for the different dates as shown in *Table 43*.

It can be seen that in the multi-curve case the floating leg does not reset to par.

At inception t_0 the internal coupon for the hedged item can be chosen to be the fair USD interest rate swap rate $c_{\$}^{3M}(t_0,T)$ or the fixed rate $c_{\$}^{CCS}(t_0,T)$ of the fair CCS as defined above. With the market data and calculation stated in *Section 5.2.3* the fair value is calculated to be EUR 78,155.53 and EUR 79,419.02 respectively.

If the internal coupon is kept constant over the period from t_0 to t_1, the same cash flows (but the first one that falls due on that date) are valued with the corresponding market data at t_1 showing the "aging effect" of the contract. Since in the dynamic strategy at each report date the hedge will be de-designated and re-designated, this hedge fair value with the internal coupon corresponding to the previous period at the report date will be denoted by $HFV_{t_0}^{\$,D}(t_1,T)$. Depending on the choice of the initial coupon, this hedge fair value at de-designation will be EUR −68,760.48 and EUR −69,782.94 respectively. These will be relevant for the booking entries for the hedge adjustment as demonstrated in the booking example in *Section 4.3.3*.

Also at t_1 the hedge will be re-designated with the new dynamically adjusted coupon

$$c_{\text{int},t_0}^{\text{FX,3M}}(t_1,T) = c_{\text{int},t_0}^{\text{FX,3M}}(t_0,T) + b(t_1,T)\frac{S_€^\$(t_0)}{S_€^\$(t_1)}\frac{\sum_{j>t}^{4N}\delta(t_{j-1},t_j)\cdot B_€^{3M}(t_1,t_j)}{\sum_{k>t}^{N}\Delta(T_{k-1},T_k)\cdot B_\$^{3M}(t_1,T_k)}.$$

Using the adjustment formula of *Equation 18* this can be written as

$$c_{\text{int},t_0}^{\text{FX,3M}}(t_1,T) = c_\$^{\text{CCS}}(t_0,T) + \frac{S_€^\$(t_0)}{S_€^\$(t_1)}\left[c_\$^{3M}(t_1,T) - c_\$^{\text{CCS}}(t_1,T)\right].$$

With $S_€^\$(t_0) = 1/1.2795, S_€^\$(t_1) = 1/1.4229, c_\$^{3M}(t_1,T) = 2.81063\,\%$ and $c_\$^{\text{CCS}}(t_1,T) = 3.08973\%$ the adjusted internal coupon is calculated to be

$$\begin{aligned} &c_{\text{int},t_0}^{\text{FX,3M}}(t_1,T) \\ &= 2.64499\,\% + \frac{1/1.2795}{1/1.4229}\left[2.81063\,\% - 3.08973\,\%\right] \\ &= 2.33460\,\%. \end{aligned}$$

The corresponding hedge fair value for the hedged item with adjusted internal coupon evaluated at t_1 will be denoted by $HFV_{t_0}^{\$,R}(t_1,T)$. Since the definition of the internal coupon at t_1 depends only on the way the general CCS for the hedging is defined, and is independent of the initial internal coupon the unique value is calculated to be EUR −68,853.37. This will be used to determine the discretization effect for the period from t_0 to t_1 recognized as part of the hedge adjustment (cf. the booking example in *Section 4.3.3*). Furthermore the value will serve as reference for a fair value hedge adjustment of the subsequent period from t_1 to t_2. Moreover this hedge fair value coincides with the value utilized for effectiveness measurement as presented in *Table 45.* The argument for the latter is that $HFV_{t_0}^{\$,R}(t_1,T)$ includes the correction of the market valuation for the previous period from t_0 to t_1 at the end of the period at t_1 which is only relevant for this period. Since t_1 is the reset date for the hedged item and both legs of the hedging CCS and no amortization of the book value is assumed by construction, the changes of hedge item and hedging instrument – as given in *Table 43* – exactly offset in the case when the USD swap rate is chosen as the initial coupon:

$$HFV_{t_0}^{\$,R}(t_1,T) - HFV_{t_0}^{\$,R}(t_0,T) = -68{,}853.37\,€ - (-78{,}155.53\,€)$$
$$= 9{,}302.16\,€$$

giving an effectiveness of 100%. Choosing the fair rate of the CCS as the starting value of the internal coupon of the hedged item the change is

$$HFV_{t_0}^{\$,R}(t_1,T) - HFV_{t_0}^{\$,R}(t_0,T) = -68{,}853.37\,€ - (-79{,}419.02\,€)$$
$$= 10{,}565.65\,€$$

giving an ineffectiveness of $10{,}565.65 / 9{,}302.16 = 113.58\,\%$.

Given the market data at each reporting date the dynamically adjusted internal coupon of the hedged item and the hedge fair values $HFV_{t_0}^{\$,D}(t_i,T)$ (in column denoted by HFV_D – relevant for the booking entries as shown in the booking example of *Section 4.3.3*) and $HFV_{t_0}^{\$,R}(t_i,T)$ (in column denoted by HFV_R – relevant for the determination of the discretization effect recognized as part of the hedge adjustment for the previous period as shown in the booking example of *Section 4.3.3,* as reference value for the subsequent period as well as for effectiveness testing) are calculated as shown in *Table 44.*

In conjunction with the fair value changes of the hedging instrument given in *Table 43* the results for effectiveness measurement shown in *Table 45* are obtained.

In this ideal case where the terms and conditions of hedged item and hedging instrument perfectly match and the reporting dates are chosen to be reset dates/interest payment dates for the hedged item as well as for both legs of the hedging instrument, 100% effectiveness is obtained. In the general case ineffectiveness will arise, as in the single-curve case, from the changes of the floating leg between reset dates and differences in the terms and conditions of hedging instrument and hedged item. Additionally in the multi-curve case with dynamical adjustment also the difference of the interest periods of fixed and floating leg of the hedging instrument will give rise to ineffectiveness[82].

TABLE 44: Dynamically Adjusted Coupon and Hedge Fair Values for the Hedged Item

Date	Fair interest rate	Fair CCS rate	FX spot rate	Adjusted internal coupon	HFV_D	HFV_R
t_0	2.3036%	2.6450%	1.2795	2.3036%/ 2.6450%	–78,155.53€/ –79,419.02€	–78,155.53€/ –79,419.02€
t_1	2.8106%	3.0897%	1.4229	2.3346%	–68,760.48€/ –69,768.94€	–68,853.37€
t_2	2.2685%	2.4385%	1.4135	2.4572%	–70,842.11€	–71,261.04€
t_3	1.3545%	1.6755%	1.2897	2.3215%	–80,057.95€	–80,105.48€
t_4	1.3363%	1.6391%	1.3521	2.3251%	–76,067.33€	–76,114.12€

TABLE 45: Example for a USD / EUR Fixed-to-Float Cross Currency Hedging Relationship

Date	Changes effectiveness measurement $HFV^R(t_j) - HFV^R(t_{j-1})$	Changes hedging instrument	Effectiveness
t_0			
t_1	9,302.16€/10,565.65€	–9,302.16€	100%/ 113.58%
t_2	–2,407.68€	2,407.68€	100.00%
t_3	–8,844.44€	8,844.44€	100.00%
t_4	3,991.35€	–3,991.35€	100.00%

As in the case of the hedge in one currency with different tenors (refer to *Section 4.3.3*) there are differences between booking entries for the hedge adjustment and the fair values used in the techniques for effectiveness testing[83]. This results from the dynamic adjustment feature based on the discrete adjustment of the internal coupon for the hedged item. Similarly to the booking entries derived for the single currency case in *Section 4.3.3,* the discretization effect will be taken into account in the booking entries of the example above. In this case the hedging instrument is given by a fixed-to-float cross currency swap involving FX risk.

82 For reasons of simplicity the impact of counterparty credit risk on the hedging instrument is neglected in this context.

83 E.g. due to amortizations, installment, impairment (not part of the hedged risk) or the consideration of clean and dirty fair values.

Subsequently to the de-designation of the hedge at each reporting date the booked hedge adjustments will be amortized over the remaining lifetime of the hedged item as it was demonstrated in the booking example in *Section 4.3.3.*

Assuming the MtM feature described at the end of the previous *Section 5.3.4* also for fixed-to-float cross currency swaps, in order to deal with this feature in context with hedge accounting the reset can be regarded as a virtual termination of the original cross currency swap and the start of a new one with same term to maturity and fixed rate but notionals according to the current FX spot rate. If the adjustment is recognized on the hedged currency leg, an adjustment of the hedge ratio might be necessary.

For illustration purposes at which point the MtM feature would enter into the formula it is assumed that the valuation of a cross currency swap with this feature may be approximated by that of a constant notional cross currency swap. In order to derive the internal coupon for a hedging relationship with a fixed-to-float cross currency swap including an MtM feature, the above argumentation is used. Following a similar reasoning as in the calculation of the constant notional cross currency swap (observing that the initial spot exchange rate in this case enters into the formula in the nominator of the ratio of spot exchange rates involved) the formula of the dynamically adjusted internal coupon takes the following form if the MtM feature is included:

$$c_{\text{int},t_0}^{\text{FX,3M}}(t,T) = c_{\$}^{3\text{M}}(t_0,T) - \left[\frac{S_{€}^{\$}\left(t^{\text{adj}}\right)}{S_{€}^{\$}(t)}\right]\left[\left(b(t_0,T)-b(t,T)\right)\frac{\sum_{j>t}^{4N}\delta\left(t_{j-1},t_j\right)\cdot B_{€}^{3\text{M}}\left(t,t_j\right)}{A_{\$}^{3\text{M}}(t,T)}\right].$$

Here $S_{€}^{\$}\left(t^{\text{adj}}\right)$ denotes the FX spot rate of the current or last adjustment date of the notional. Thus on the adjustment dates the ratio of the FX spot rates equals one. The terms involving the cross currency basis spread remain untouched since this is related with the expectation of interest rates in both currencies as from the beginning of the swap, and the fixed coupon of the hedging instrument remains the same when the notional is adjusted. For a more sophisticated derivation of the internal coupon in this case also the valuation of the MtM feature should be taken into account.

5.3.5.2 Cash Flow Hedge Accounting with an FX Contract including the FX Basis Risk

Considering the example of an FX forward contract of *Section 5.2.1* including the FX basis (in the EUR zero rates) first the corresponding FX forward rates are determined as shown in *Table 46*.

The general approach of *Section 5.2.1* remains the same, only the fair value of the FX forward contract changes as carried out in *Section 5.3.1* and the discounting of the spot component changes according to the changes in the market data (see *Table 47*).

TABLE 46: Example of an FX Forward Contract – FX Forward Rates

Date	Days to maturity	Inverse exchange rate $\left(S_{€}^{\$}\right)^{-1}$ USD for 1 EUR	EUR zero rate incl. FX basis date – maturity	USD zero rate date – maturity	Inverse FX forward rate $\left(f_{€}^{\$}\right)^{-1}$ date – maturity
t_0 = 06/08/20X0	365	1.1942	1.2225%	1.1966%	1.193894
t_1 = 12/08/20X0	182	1.3200	1.2185%	0.4584%	1.315028
T = 06/08/20X1	0	1.4608	1.0650%	0.1265%	1.460800

TABLE 47: Example of an FX Forward Contract – Market Data

Date	FX Forward contract FV	FX Forward contract FV changes	Spot component	Changes spot component	OCI (spot)	P&L (interest)
t_0 = 06/08/20X0	0.00€	0.00€	0.00€	0.00€	0.00€	0.00€
t_1 = 12/08/20X0	–7,668.26€	–7,668.26€	–7,931.63€	–7,931.63€	7,931.63€	–263.37€
T = 06/08/20X1	–15,303.89€	–7,635.63€	–15,282.43€	–7,350.80€	7,350.80€	284.84€

5.4 Other Hedge Accounting Approaches to Avoid P&L Volatility from FX Basis

In the sections above, hedge accounting in presence of the FX basis was portrayed. Since the P&L volatility resulting from the FX basis despite sound economic hedging relationships in foreign currency causes persisting headaches for treasury departments of banks, the question arises if the model described above is the "only" workable solution to this. In the following two approaches are briefly described with illustrations, so for this purpose a mathematical description is not considered.

The two "hedge accounting" approaches considered are the following:

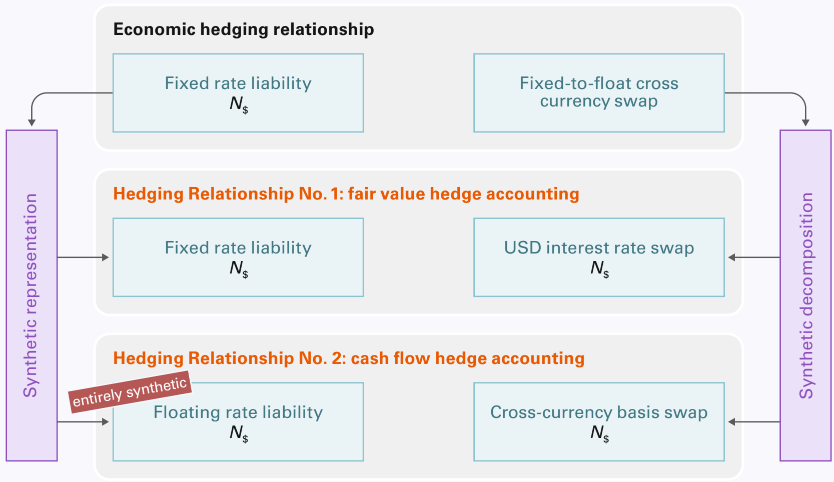
- In a fixed-to-float cross currency hedge the fixed-to-float cross currency swap is decomposed into an interest rate and an FX basis "component". The interest rate component is designated into a fair value hedge, while the FX basis is designated into a cash flow hedge.
- A stand-alone cross currency basis swap is designated into a cash flow hedge relationship.

The first hedge accounting model is schematically illustrated in *Figure 73* and described in the following:

The cross currency swap is decomposed synthetically into a USD interest rate swap and a cross currency basis swap. In order to meet the hedge accounting requirements, the fixed rate liability is correspondingly synthetically represented twice:

- The first hedge relationship contains fixed rate liability (including notional) cash flows and a plain vanilla USD interest rate swap. This fair value hedge corresponds to the usually applied fair value hedge; the hedged risk is the USD benchmark curve.
- The second hedge relationship contains the FX basis float-to-float derivative (including the exchange of notional cash flows) and a

FIGURE 73: Synthetic Decomposition of a Cross Currency Swap and Synthetic Representation of a Fixed Rate Liability for FX Hedge Accounting



synthetically created floating rate liability. Since this model intends to designate a cash flow hedge, it requires designating the variability in the FX basis (difference between USD LIBOR and EURIBOR) and the FX risk. For the sake of simplicity the decomposition of interest and FX basis cash flows is omitted.

As a result the economic hedging relationship is subdivided for accounting purposes into two separate hedge accounting relationships. If a consistent set of discount curves is applied, like e.g. *Equation 14,* the decomposition of the cross currency swap is economically consistent with the absence of arbitrage principle. Otherwise "off-market" valued instruments are created. Although hedge effectiveness can be easily achieved, this approach is questionable in view of IAS 39. According to IAS 39.74 the possibilities of a synthetic decomposition of hedging instruments are clearly stated. These rules refer to options and forward instruments, so not necessarily to one legal cross currency swap. Additionally according to IAS 39.76/IG F.1.12 permits an entity to designate a derivative simultaneously as a hedging instrument in a cash flow and fair value hedge. But this is not done in the approach above since the derivative is entirely decomposed. This issue can be circumvented by entering into a USD interest rate swap and a cross currency basis swap separately. IAS 39.77 permits the designation of two or more derivatives in a hedge accounting relationship. But then the issue of the synthetic representation of the fixed rate liability remains. There is not much guidance in IFRS on this particular subject, but IAS 39 IG C.1 does not allow to identify a floating rate instrument in a fixed rate instrument and to designate a synthetic hedged item. Neither is it acceptable to designate a synthetic instrument as hedged item. Therefore it is questionable whether the synthetic representation of the fixed rate liability as a floating rate liability meets the criteria of IAS 39 hedge accounting requirements and is in our view not permitted. Furthermore the derivation of the booking entries requires care; since the USD floating rate liability is entirely synthetic it does not represent a recognized liability, so it cannot trigger a currency translation adjustment according to IAS 21.

The following approach deals with a hedge accounting model which is applicable for stand-alone cross currency basis swaps. According to KPMG Insights 7.7.380.40 a cash flow hedge accounting model is applicable to float-to-float basis swaps if the hedged item is represented by an asset and a liability. In this case the hedged risk is represented by the variability of the differences between the floating rates of an asset and a liability (basis) as well as an FX risk[84]. This hedge accounting model is illustrated in *Figure 74.*

The major steps for this hedge accounting are summarized as follows:

- **Hedged items:** Combination of a group of (at least one) USD floating loans (assets) and (at least one) floating EUR funding. The tenors of both coincide with the tenor of the pay and receive floating side of the cross currency basis swap respectively.

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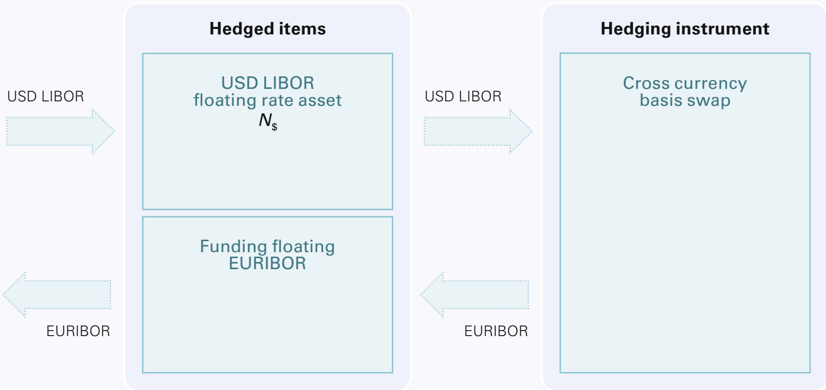
84 With respect to the booking entries, there is no necessity to designate FX risk, since the cross currency basis swap requires the exchange in notional. These represent, as shown in *Figure 61,* recognized assets and liabilities and are subject to currency translation adjustments according to IAS 21.

FIGURE 74: Cash Flow Hedge Accounting Approach for a Stand-Alone Cross Currency Basis Swap



- **Homogeneity** with respect to the variability of floating rates and FX risk has to be shown.
- **Effectiveness testing** is performed by two hypothetical derivatives: for both the EUR funding and the USD floating loan a hypothetical derivative (float-to-fixed interest rate swap and a float-to-fixed cross currency swap respectively) is constructed and compared to the cross currency basis swap that may be virtually decomposed correspondingly.
- According to the cash flow hedge accounting model the **booking entries** are determined by utilizing the "lower of test" and the effective part is recognized in OCI, while the ineffective part is recognized in P&L.

The outlined hedge accounting model unfortunately faces some practical limitations:

- Often tenors of floating rate loans are 1-month, while the liquid cross currency basis swap contains a 3-month tenor, the difference in tenor giving rise to ineffectiveness.

- The major types of floating rate loans are not represented by "plain vanilla" floating rate loans but by revolving credit facilities, so because of the uncertainty of drawings, interest cash flows, callable features or other embedded options, the hedged item has to be factored into a cash flow hedge of a "highly probable forecast transaction". Then "expectations" of drawings and cash flows have to be estimated, which usually requires a large data sample and furthermore reduces the amount subject to hedge accounting. As a consequence only part of the cross currency basis swap can be designated into the cash flow hedge accounting model and a reduction of P&L volatility is limited.

5.5 Interim Result

When applying FX hedging the question arises whether the FX basis meets the requirements of IAS 39.AG99F and AG110 ("separately identifiable and reliably measurable" and "hedged risk"). The answer is "yes" as a consequence of the economic rationale of multi-curve models, but from the hedge accounting perspective (effectiveness testing) the designation of FX basis risk as hedged risk is of limited relevance to the hedge accounting model. The positive answer is based on the following: According to market conventions and to economic hedging activities it is clear that the FX basis is a liquidly traded derivative, affects the P&L and is taken into account in economic hedging activities, since it represents a relevant risk factor in a bank's funding position. Moreover the FX basis is generally considered when banks grant loans or issue debt so it plays an integral part in treasury activities. From a more technical point of view the FX basis shares the same fate like any other "tenor basis swap" or risk factor "added" on the discount curve. As shown in *Section 3* the explanatory power of fair values evaluated with the discount curve derived from the derivative market with respect to cash market prices (e.g. bonds) is poor – apart from accidental statistical coincidence. So consequently it is impossible to demonstrate empirically that "tenors" systematically affect cash prices.

But this property is not of relevance and even if the requirements of hedge accounting are met, this does not ensure effective hedges in terms of IAS 39 for the FX basis. This results from the fact that benchmark curve hedging in case of the fair value hedge accounting model under IAS 39 is a single risk valuation factor model. As shown above, the USD discount curve cannot be modeled independently from the EUR discount curve. The application of the fair value hedge accounting model requires the definition of "one" single discount factor – whether USD or EUR – and also defines the hedged risk (3-month USD LIBOR or 3-month EURIBOR). The situation does not change if FX risk is added to the "hedged risk". So once defined, the discount factor only represents "one" risk factor in a multi-curve setup and the other remaining risk factors are neglected. The only way to take into account the remaining risk factors – like the FX basis – is to adjust the cash flows as described above. The practical outcome of all this is that the hedge will be defined in such a way that ineffectiveness is minimized.

In comparison the application of cash flow hedge accounting is easy. But the economic underpinnings of the cash flow hedge accounting model are similar to those of fair value hedge accounting. With respect to the role of tenor basis swaps as risk factors it is worth to note that, given the requirement to value hypothetical derivatives under IAS 39 according to market conventions (KPMG Insights 7.7.630.30), highly effectiveness of the hedging relationship is preserved, provided that the terms and conditions of hedged item and hedging instrument are sufficiently similar. By the required application of market conventions to evaluate the fair value of derivatives it is generally accepted that the fair value of the hypothetical derivative is synthetically decomposed into its economic risk factors – presuming the absence of arbitrage. As shown in connection with the bootstrapping algorithm, this results in a recovery of market quotes, but in different dynamics and consequently in different P & L effects.

SECTION SIX

6

Collateralized Derivative Pricing and Hedge Accounting according to IAS 39

6.1 Introduction – Collateralization and Multi-Curve Models

As described before, the re-assessment of risks by market participants involved in financial instruments transactions is a fact and leads to the implementation of multi-curve models. Additionally changes in market conventions and institutional changes within financial markets accelerate the implementation of multi-curve models. Clearing houses[85] such as SwapClear, (LCH.Clearnet)[86], Eurex Clearing AG (Deutsche Börse AG)[87] require the utilization of an overnight index for valuation purposes, e.g. EONIA. The increasing involvement of clearing houses and central counterparties in derivative transactions, in order to eliminate

85 For a description of the economics of clearing houses refer to Pirrong, C. (2011), "The Economics of Central Clearing: Theory and Practice", in: *ISDA Discussion Paper Series No. 1.*

86 cf. e.g. Whittall, C. (2010b) "LCH.Clearnet re-values $218 trillion swap portfolio using OIS", in: *Risk Magazine,* June 2010

87 cf. Eurex Clearing (2012)

counterparty risk, pushes financial markets towards standardization concerning discount curves derived from OIS rates.

This development is closely related to the treatment of collateralizations in derivative transactions, since clearing houses require daily collateral postings ("margins") and corresponding interest payments on cash collaterals. In consideration of daily exchanges of collateral postings, an overnight index to determine the interest on the collateral postings is considered adequate. This has an immediate consequence on the valuation of collateralized derivatives since, in order to avoid arbitrage opportunities, the applied discount curve has to be chosen according to the evaluation of interest payments on cash collateral.

These changes in the market environment are accompanied by modifications of the legal framework of the derivative business between two counterparties acting under an ISDA Master Agreement (2002), which represents the market standard for derivative transactions supplemented by a CSA. Accordingly, in such a CSA the evaluation of the interest associated with cash collateral postings of derivative transaction is changed to require the utilization of an overnight index, e.g. EONIA. Currently cash collateral is commonly eligible and posted in selected reference currencies (e.g. USD, EUR, GBP, JPY)[88]. According to this specific feature commonly used under the terms and conditions in the relevant framework documents for derivatives issued by ISDA (e.g. ISDA Master Agreement (2002), ISDA Credit Derivative Definitions (2003) and CSA), the cross currency basis spread (FX basis spread) cannot be neglected in connection with collateral postings, since the cash collateral can be referenced to a different (foreign) currency than the derivative transaction. Accordingly, the FX basis enters into the discount curve for valuation purposes. This feature of different trade and cash collateral currencies is currently under debate[89], since ISDA plans

88 See ISDA (2010B).

89 For a description refer to Sawyer, N. and Vaghela, V. (2012), "Standard CSA: the dollar dominance dispute", in: *Risk Magazine,* January 2012.

to introduce a new standardized CSA towards an alignment ("silo approach").[90] But this plan does not facilitate the situation for cross currency products which by definition deal with two currencies or deals traded in minor currencies with collateral postings in a reference currency, and thus for the valuation the FX basis has to be taken into account. Additional legal changes under the ISDA Master Agreement (2002) are the rules concerning "disputes" and "close outs" of derivative transactions, since these also require the utilization of overnight indices in order to determine the close out amount.

The features described represent the market standard for derivative transactions only in the interbank market ("collateralized derivative transactions"). Corporates also use the ISDA documentation for derivatives as standard, but not the CSA ("uncollateralized derivative transactions") due to liquidity requirements of collateral postings, which are considered unfavourable for corporates because of their liquidity constraints. Consequently overnight indices are not applied as discount rates for those derivative transactions and therefore e.g. the EURIBOR or LIBOR rates are applied. As a result of changes in market conventions, discount rates for derivative transactions become counterparty specific and yield to a segmentation of derivative markets. It is also important to note that evaluating interest on a cash collateral using an overnight index does not mean e.g. that a financial institution (bank) is able to (fully) fund on overnight index basis!

With respect to these developments within financial markets, valuation models have to be modified in order to reflect the increased number of risk and counterparty specific factors. Additionally discount curves cannot be derived from market data (e.g. swap rates) without taking into account different tenors. For example (in the interbank market) EURIBOR or LIBOR discount rates cannot be derived independently

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90 See ISDA (2010B)

from overnight index rates, so a “pure” EURIBOR or LIBOR curve ceased to exist. According to these circumstances financial institutions started to implement “multi-curve” models for derivative pricing in order to take into account the tenor dependence and the collateralization. Within these valuation models forwarding and discounting is performed on different curves.

In the following a multi-curve model for collateralized derivatives and its implications to hedge accounting are analyzed. The section is structured as follows:

1. A brief description of the interaction of funding, changes in interest evaluation of cash collaterals, performance measurement of hedges and VaR evaluations is provided.
2. Definition and derivation of a consistent and arbitrage free setup of discount and forward curves involving several risk factors like tenor and cross currency basis spreads for collateralized derivatives.
3. Implications of the multi-curve model for collateralized derivatives to hedge accounting according to IAS 39.

6.2 Performance Measurement of Economic Hedges

Performance measurement of banking activities is a complex and vast area in economic modeling. In the following we only consider a simple example in order to show the differences and relationships between performance measurement, funding and interest payments on collaterals of derivatives. The explanations commence in a single-curve model setup in order to reduce complexity (for additional explanations please refer to *Section 4*).

FIGURE 75: Economic Hedge Relationship Using a 5-Year 3M EURIBOR Interest Rate Swap and a Bond / Loan

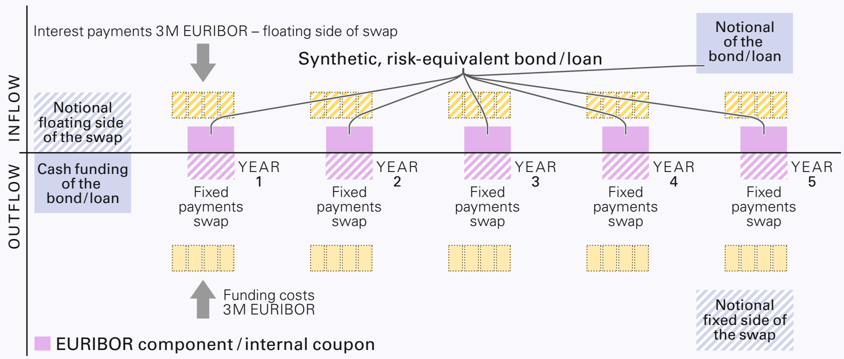
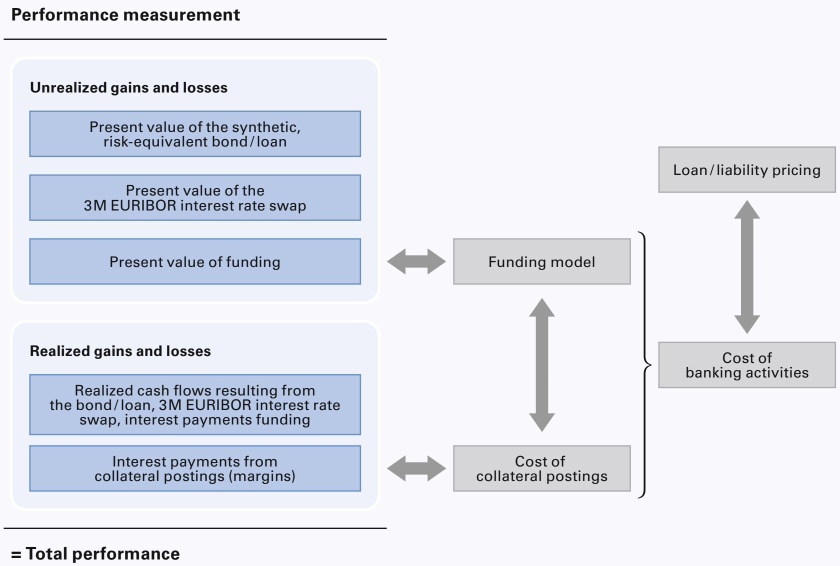


FIGURE 76: Components of a Performance Measurement



In *Figure 75* an economic hedging relationship is portrayed. As represented in this figure, the cash flows of the 5-year 3-month EURIBOR interest rate swap entirely offset the cash flows of the cash funding and the synthetic and risk-equivalent loan / bond. Consequently a net profit or performance of zero is expected. This conclusion rests on two assumptions:

- The performance of collateral postings in connection with the interest rate swap (derivative) and the corresponding funding / replacement costs are not considered.
- Cash funding is carried out on 3-month EURIBOR.

In *Figure 76* the components of performance measurement models are illustrated. Generally these models distinguish between realized and unrealized gains / losses. Realized gains / losses refer to interest payments of the 3-month EURIBOR interest rate swap, funding etc.

Unrealized gains / losses refer to present value evaluations of the financial instruments included in the performance measurement. It is important to note that the bond / loan is already decomposed into an interest rate risk bearing component (= the synthetic, risk-equivalent bond / loan), credit and other margin components. Therefore commonly performance measurement includes various components of "internal pricings" or "transfer prices" in order to reflect the decomposition of risk positions ("division of labor") and organizational responsibilities (credit and interest treasury departments) within a financial institution.

As represented in *Figure 76,* the interest of collateral postings (margins) affects the total performance. Therefore changes in interest evaluations of collateral postings (margins) impact directly the total performance and the interest or trading result according to IFRS.

Additionally there is an indirect effect on a financial institutions funding and the overall cost position. The existence of this effect is clear but its extent depends on the individual financial institutions' funding model. There are two major types of funding models:

- Overnight funding typically based on EONIA or EURIBOR ("short term lending") in order to fund positions (also termed "cost of carry"). This funding model is widely applied for trading activities and is usually performed by internal deals.[91] Alternatively repos can be used for short term funding.
- Models of the entire capital structure of the financial institution ("long term financing") including equity, hybrid capital, other debt financings, savings deposits etc. Such models require additional economic and econometric modeling. This funding model is usually applied in treasury departments.

Irrespective of the funding model cash collateral postings (margins) resulting from derivative transactions have to be funded on an overnight basis resulting in costs for the financial institution. Consequently there is also an indirect effect on loan and bond pricing since the total hedging costs alter in case of changes of interest payments on cash collateral. Similar economic reasoning holds in presence of FX basis risk.

In the following the change in interest payments resulting from the changes in CSA is illustrated.

6.3 Performance Measurement and the CSA Effect

If two parties enter into a derivative contract under an ISDA Master Agreement (2002) and a CSA determining cash as eligible collateral, cash collateral is posted if the hedging derivative has a present value different from zero. The counterparty with negative present value of the

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91 A discussion of internal deals is beyond the scope of the article.

derivative posts cash collateral and receives interest payments from the counterparty. According to the ISDA "Market Review of OTC Derivative Bilateral Collateralization Practices (2.0)" from 2010 the interest rate being paid on the collateral is agreed according to contractual features of the CSA. These features include: the interest period, the accrued (daily) interest, threshold or minimum transfer amounts. Typically cash collateral is being (re)called on a daily basis and generally referenced to the rate index, which is represented by the overnight funding rate for the relevant currency, e.g. Federal Funds H-15 for USD, EONIA for EUR and Sterling Overnight Interbank Average Rate (SONIA) for GBP. This gives a variety of possible terms and conditions to determine the interest amount of the collateral, but as reported in the ISDA "Market Review of OTC Derivative Bilateral Collateralization Practices", generally the simple (rather than the compounded) overnight (ON) funding rate for the applicable currency is used accrued daily but typically with a monthly payment period. This should be seen in context with the possible daily calls on collateral, the role of central counterparties and the plans on regulations of the OTC market or the development of a Standard CSA. In its Margin Survey 2011 ISDA reports that on average almost 70% (80% for large dealers) of all OTC derivatives are collateralized and even 79% (88% for large dealers) for the subset of all fixed income derivatives. 80% of the collateral has been posted as cash and at least for large dealers 61% of the portfolio reconciliation is done on a daily basis (31% overall).

In order to illustrate the impact on different interest rate evaluations for cash collateral postings, the following simplifying assumptions are made:

- Collateral is posted as cash of the same currency.
- There is no threshold, minimum transfer amount, rounding amount or other optional features in the CSA and, neglecting transactions costs, the posting of collateral mainly has an effect on the related interest payments.
- The posted collateral is approximated to be constant between monthly reporting dates.

The example in *Figure 75* is continued and the fixed rate bond/loan with 5-year maturity has a notional of EUR 100,000,000. The hedging instrument is a 5-year 3-month EURIBOR interest rate swap with matching terms and conditions. Cash funding is based on 3-month EURIBOR. Since a single-curve model is assumed – with the 3-month EURIBOR interest rate swap curve as benchmark curve – the EURIBOR component (internal coupon) of the hedged item coincides with the 5-year EURIBOR interest rate swap rate constructing the synthetic and risk-equivalent loan/bond.

FIGURE 77: FV Changes of a 3M EURIBOR Interest Rate Swap Based on 3M EURIBOR Discounting

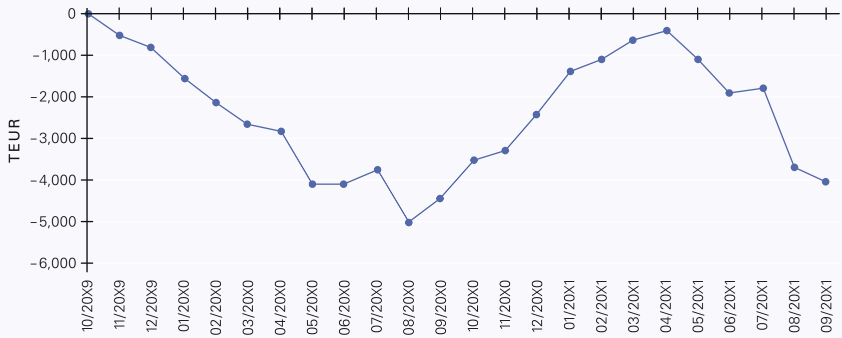
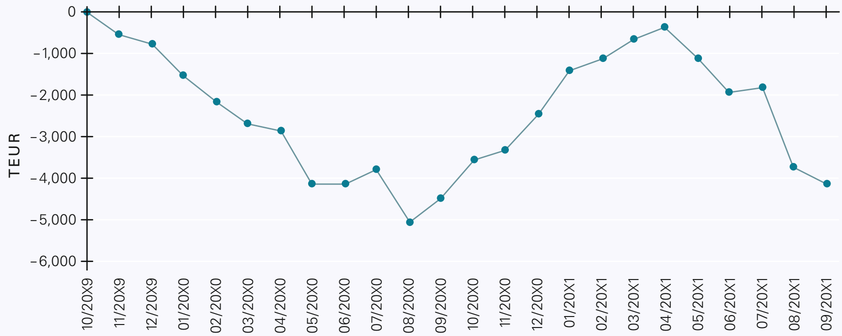


FIGURE 78: FV Changes of a 3M EURIBOR Interest Rate Swap Based on EONIA Discounting



Figures 77 and *78* present the monthly fair value changes of the 3-month EURIBOR interest rate swap discounted with 3-month EURIBOR and EONIA discount curve.

The figures show that the fair value of the payer swap is negative and therefore, according to the CSA, cash collateral and interest payments are exchanged. This observation holds true for the case of discounting on the 3-month EURIBOR curve in the single-curve approach as well as for collateralized trades commonly using OIS discounting on the EONIA curve. Thus the argument is qualitatively the same and the example may be continued with the swap discounted on OIS.

In *Table 48* the amount of interest payments for cash collateral postings referenced to the 3-month EURIBOR are compared to those references to (simple) EONIA interest rates. As a funding model, the 3-month EURIBOR cash funding is assumed.

TABLE 48: Comparison of Interest Payments on Cash Collateral Postings

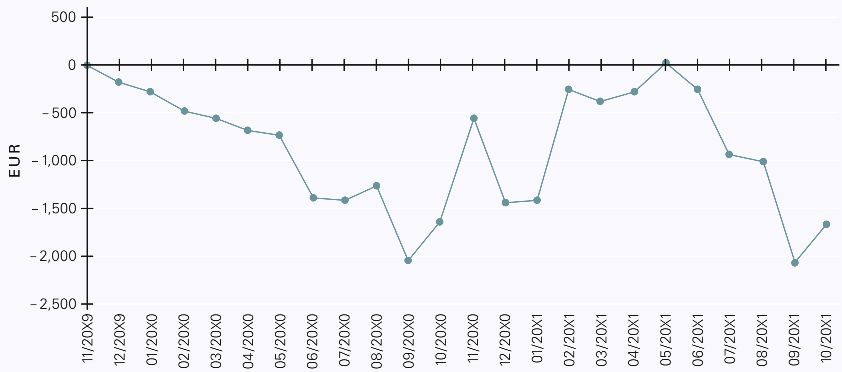
Date	Swap fair value (OIS discounting)	Interest payments on cash collateral postings		Funding model: 3M EURIBOR	Resulting impact	
		3M EURIBOR	EONIA		3M EURIBOR	EONIA
0	0					
11/23/20X9	–534,837	0.00	0.00	0.00	0.00	0.00
12/23/20X9	–798,713	318.67	155.99	–318.67	0.00	–162.68
…						
06/23/20X0	–4,135,106	2,467.02	1,075.55	–2,467.02	0.00	–1,391.47
07/23/20X0	–3,773,971	2,546.54	1,130.26	–2,546.54	0.00	–1,416.27
08/23/20X0	–5,050,954	2,876.08	1,624.90	–2,876.08	0.00	–1,251.18
09/23/20X0	–4,482,826	3,875.34	1,839.81	–3,875.34	0.00	–2,035.53
10/23/20X0	–3,560,211	3,279.93	1,639.97	–3,279.93	0.00	–1,639.97
…						

As it becomes apparent from *Table 48,* if the interest evaluation of the cash collateral postings differs from the interest of the funding model, there is an immediate effect on the total performance (P&L). When assuming a 3-month EURIBOR cash funding, impact of change in interest evaluations can be regarded as "tenor basis spread" difference between the EONIA interest rate and the 3-month EURIBOR. This is also illustrated in *Figure 79.*

Additionally it can be observed that the impact on differences in interest evaluations of cash collaterals and funding is negative. This can be regarded as an increase in funding costs and therefore affecting the funding model of the entire financial institution.

As mentioned above, in practice for funding a different model than 3-month EURIBOR cash funding, e.g. funding by repos or capital, structure models may be applied. A different funding model would change the values for the funding in *Table 48,* but an overall effect of changes due to different interest evaluations of cash collateral remains.

FIGURE 79: Difference of Interest Payments on Cash Collateral at 3M EURIBOR and EONIA Rate



6.4 Initial Valuation Effects Resulting from Changes in Discount Curves

With the change-over from discounting on e.g. 3-month EURIBOR to OIS discounting as the market consensus for the correct pricing of collateralized trades, financial institutions have to cope with an initial valuation effect when first adopting the new valuation method for existing collateralized trades. In the following this initial valuation effect is analyzed.

Firstly in general the EONIA rate is less than a EURIBOR or LIBOR rate for the same period (refer to *Section 3*):

$$r_{\text{EONIA}}(t_0,t) \leq r_{\text{EURIBOR}}(t_0,t) \text{ resp. } \frac{1}{r_{\text{EURIBOR}}(t_0,t)} \leq \frac{1}{r_{\text{EONIA}}(t_0,t)}.$$

Thus, assuming discounting on (homogeneous) discount curves, the discount factors derived from EONIA curves will be greater than those of the EURIBOR or LIBOR curve for the same time period:

$$B_{\text{EONIA}}(t_0,t) \geq B_{\text{EURIBOR}}(t_0,t).$$

In the following the example of a 5-year 3-month EURIBOR interest rate swap as in the previous *Section 6.3* is continued, assuming that the payer swap has a negative present value (fair value) in case of 3-month EURIBOR discounting:

$$PV_{\text{EURIBOR}}^{\text{payer swap}}(t) \leq 0.$$

In order to analyze the effect of changing discount curves the following assumptions are made:

- The valuation of 3-month EURIBOR interest rate swap (value date t_0) at the reset date T_k is considered.

- The 3-month EURIBOR discount curve can be modeled independently from the EONIA discount curve and the following equilibrium conditions hold:

$$c^{3M}(T_k,T)\cdot A^{3M}(T_k,T)=\Lambda^{3M}(T_k,T)=1-B^{3M}(T_k,T)$$

$$c^{3M}(T_k,T)\cdot A^{EONIA}(T_k,T)=\Lambda^{3M/EONIA}(T_k,T).$$

Using the definitions above the following properties can be derived:

$$\underbrace{c^{3M}(t_0,T)\cdot A^{3M}(T_k,T)-\Lambda^{3M}(T_k,T)}_{\text{PV swap EURIBOR discounting}}$$

$$=c^{3M}(t_0,T)\cdot A^{EONIA}(T_k,T)\cdot\frac{A^{3M}(T_k,T)}{A^{EONIA}(T_k,T)}-c^{3M}(T_k,T)\cdot A^{3M}(T_k,T)$$

$$=\left[c^{3M}(t_0,T)\cdot A^{EONIA}(T_k,T)-c^{3M}(T_k,T)\cdot A^{EONIA}(T_k,T)\right]\cdot\frac{A^{3M}(T_k,T)}{A^{EONIA}(T_k,T)}$$

$$=\left[\underbrace{c^{3M}(t_0,T)\cdot A^{EONIA}(T_k,T)-\Lambda^{3M/EONIA}(T_k,T)}_{\text{PV swap EONIA discounting}}\right]\cdot\frac{A^{3M}(T_k,T)}{A^{EONIA}(T_k,T)}.$$

Thus:

$$PV_{\text{EURIBOR disc}}^{\text{payer swap}}(T_k)=\left(PV_{\text{EONIA disc}}^{\text{payer swap}}(T_k)\cdot\left[\frac{A^{3M}(T_k,T)}{A^{EONIA}(T_k,T)}\right]\right)$$

$$\Rightarrow\quad PV_{\text{EONIA disc}}^{\text{payer swap}}(T_k)\leq PV_{\text{EURIBOR disc}}^{\text{payer swap}}(T_k)\qquad\leq\qquad 0$$

$$\begin{pmatrix}PV_{\text{EONIA disc}}^{\text{payer swap}}(T_k)\\-PV_{\text{EURIBOR disc}}^{\text{payer swap}}(T_k)\end{pmatrix}=\left(PV_{\text{EONIA disc}}^{\text{payer swap}}(T_k)\cdot\left[1-\frac{A^{3M}(T_k,T)}{A^{EONIA}(T_k,T)}\right]\right)$$

$$\begin{pmatrix}PV_{\text{EONIA disc}}^{\text{payer swap}}(T_k)\\-PV_{\text{EURIBOR disc}}^{\text{payer swap}}(T_k)\end{pmatrix}=\left(PV_{\text{EURIBOR disc}}^{\text{payer swap}}(T_k)\cdot\left[\frac{A^{EONIA}(T_k,T)}{A^{3M}(T_k,T)}-1\right]\right)$$

$$\left(PV_{\text{EURIBOR disc}}^{\text{payer swap}}(T_k)-PV_{\text{EONIA disc}}^{\text{payer swap}}(T_k)\right)\geq 0.$$

If the present value of the interest rate swap is negative, then the cash collateral has to be posted and interest based on EONIA is received. On the assumption of 3-month EURIBOR cash funding the overall impact is negative in comparison to the EURIBOR case (funding and collateral interest on 3-month EURIBOR). Consequently it is plausible that the present value of the interest rate swap using EONIA discounting is less than for EURIBOR discounting.

Therefore the initial valuation effect from changing discount curves can be regarded as the present value difference of different interest payments on future cash collaterals.

Considering the case of just one interest period to maturity the formula above is reduced to:

$$\underbrace{\left[c^{3M}(t_0,T)\cdot A^{EONIA}(T_k,T)-\Lambda^{3M/EONIA}(T_k,T)\right]}_{\text{PV of swap OIS discounting}}\left[1-\frac{A^{3M}(T_k,T)}{A^{EONIA}(T_k,T)}\right]$$

$$=\left[c^{3M}(t_0,T)\cdot A^{EONIA}(T_k,T)-\Lambda^{3M/EONIA}(T_k,T)\right]\left[1-\frac{B^{3M}(T_k,T)}{B^{EONIA}(T_k,T)}\right]$$

$$=\left[c^{3M}(t_0,T)\cdot A^{EONIA}(T_k,T)-\Lambda^{3M/EONIA}(T_k,T)\right]\left[1-\frac{1+\Delta r^{EONIA}(T_k,T)}{1+\Delta r^{3M}(T_k,T)}\right]$$

$$=\left[c^{3M}(t_0,T)\cdot A^{EONIA}(T_k,T)-\Lambda^{3M/EONIA}(T_k,T)\right]$$
$$\cdot\Delta\left[\frac{r^{3M}(T_k,T)-r^{EONIA}(T_k,T)}{1+\Delta r^{3M}(T_k,T)}\right]$$

$$=\underbrace{\left[c^{3M}(t_0,T)\cdot A^{EONIA}(T_k,T)-\Lambda^{3M/EONIA}(T_k,T)\right]}_{\text{PV of swap = collateral amount}}$$
$$\underbrace{\cdot\Delta}_{\substack{\text{Interest}\\\text{period}}}\underbrace{\left[r^{3M}(T_k,T)-r^{EONIA}(T_k,T)\right]}_{\text{Difference of interest rates}}\cdot\underbrace{B^{3M}(T_k,T)}_{\text{Discount factor}}.$$

This approximation can be used to illustrate that the change in discounting is of the range of the discounted interest difference (3-month EURIBOR and EONIA for the given interest period) of the cash collateral which is represented by the present value of the 3-month EURIBOR interest rate swap discounted on EONIA (see *Table 49*).

As becomes apparent from *Table 49,* the (exact) initial valuation effect is EUR 49,772, while the approximated present value of future interest payments on cash collateral is EUR 34,529.

TABLE 49: Example of the Initial Valuation Effect from Changing Discount Curves and Present Value of Future Interest Payments on Cash Collateral

Date	Simulated fair value changes of the 3M EURIBOR interest rate swap (= cash collateral amount)	Present value of future interest payments on cash collateral
01/23/2012	–1,664,874	
04/23/2012	–1,427,119	–8,332
07/23/2012	–1,178,954	–7,148
10/23/2012	–917,709	–5,941
01/23/2012	–829,683	–4,603
04/23/2012	–574,036	–4,047
07/23/2012	–305,497	–2,821
10/23/2012	–23,835	–1,513
01/23/2012	0	–124
	Sum	–34,529
	Initial valuation effect	–49,772

6.5 Risk Factors and Their Effect on Value at Risk Evaluations

To round off the picture, the effect on Value at Risk evaluations is briefly portrayed, a full treatment of Value at Risk evaluations in presence of multi-curve models respectively multiple risk factor models being beyond scope.

If market participants re-assess risk considerations in financial markets, then this does not only affect valuation, but also classical risk management tools like Value at Risk. But in comparison to pricing financial instruments in a multi-curve setup, risk management tools require the estimate of inherent risks of a portfolio of financial assets. Therefore "accuracy" or appropriateness of the choice of several risk factors instead of only one single risk factor in Value at Risk evaluations depends on the individual portfolio of financial instruments and materiality considerations.

The following example considers a 3-month EURIBOR interest rate swap with 9 years time-to-maturity. Please keep in mind that the change of the discount curve from EURIBOR to EONIA results in the decomposition of the 3-month EURIBOR interest rate swap into an EONIA interest rate swap and a 3-month EURIBOR/EONIA tenor basis swap. Consequently different discount curves imply different relevant risk factors for the evaluation of Value at Risk, as shown in *Table 50.*

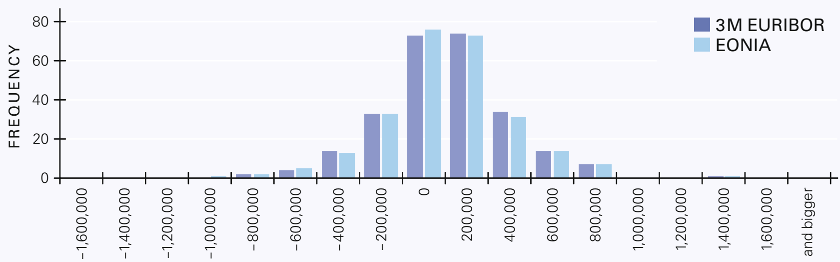
TABLE 50: Comparison of Relevant Risk Factors in a Single- and Multi-Curve Model Setup for Value at Risk Evaluations

Risk factors	Single-curve model	Multi-curve model
3M EURIBOR discount curve	√	–
EONIA discount curve	–	√
3M EURIBOR/EONIA basis swap spread	–	√

TABLE 51: **FV Changes of a 3M EURIBOR Interest Rate Swap Using Different Discount Curves**

	3M EURIBOR discounting			EONIA discounting		
Class	**Frequency**	**Rel. frequency**	**Cum. rel. frequency**	**Frequency**	**Rel. frequency**	**Cum. rel. frequency**
–1,200,000	0	0.00%	0.00%	0	0.0%	0.00%
–1,000,000	0	0.00%	0.00%	1	0.4%	0.39%
–800,000	2	0.78%	0.78%	2	0.8%	**1.17%**
–600,000	4	1.56%	**2.34%**	5	2.0%	3.13%
–400,000	14	5.47%	7.81%	13	5.1%	8.20%
–200,000	33	12.89%	20.70%	33	12.9%	21.09%
0	73	28.52%	49.22%	76	29.7%	50.78%
200,000	74	28.91%	78.13%	73	28.5%	79.30%
400,000	34	13.28%	91.41%	31	12.1%	91.41%
600,000	14	5.47%	96.88%	14	5.5%	96.88%
800,000	7	2.73%	99.61%	7	2.7%	99.61%
1,000,000	0	0.00%	99.61%	0	0.0%	99.61%
1,200,000	0	0.00%	99.61%	0	0.0%	99.61%
1,400,000	1	0.39%	100.00%	1	0.4%	100.00%
1,600,000	0	0.00%	100.00%	0	0.0%	100.00%
>1,600,000	0	0.00%	100.00%	0	0.0%	100.00%
Total	**256**	**100.00%**		**256**	**100.0%**	

FIGURE 80: **Histogram of Fair Value Changes of a 3M EURIBOR Interest Rate Swap Using Different Discount Curves**



In order to illustrate the impact of different discount curves, the differences in present values (fair values) of the 3-month EURIBOR interest rate swap discounted with 3-month EURIBOR and EONIA is simulated by historical simulation. Using a history of approximately 250 daily changes in the market curves (EURIBOR, EONIA), different present values for the 3-month EURIBOR interest rate swap are evaluated. The results are shown in *Table 51.*

The figures reveal some slight changes in the empirical distribution of fair value changes of the 3-month interest rate swap if the discount curve is changed. In case of 3-month discounting the VaR with 99% confidence level is equal to TEUR 600, while in case of EONIA discounting the VaR equals TEUR 800. The results show an increase in risk due to different discounting.

The corresponding histogram from *Table 51* is shown in *Figure 80.*

6.6 Re-Assessment of Market Risk in Financial Markets and Economic Hedging

The re-assessment of market risk in the course of the financial market crisis by market participants is an economic fact. Despite the increase in the number of risk factors, the economic rationale behind the determination of risk factors for valuation and the assessment of financial market risk is similar. Throughout the remainder of this section the simplifying assumptions on collateralization similar as in *Section 6.3,*

- collateral is posted as cash of the same currency,
- there is no threshold, minimum transfer amount, rounding amount or other optional features in the CSA,
- transactions costs are neglected,

are made, unless explicitly stated otherwise.

Let's take the example a one risk factor economy. When assuming that financial market participants consider the interest rate risk as the only relevant risk factor, financial market participants need to determine the "interest rate risk". Interest rate risk is an unobservable risk; you cannot go into the market and buy 8% interest rate or "observe" changes in interest rates. In order to determine interest rate risk, a "yard stick" or "benchmark" is required.

Interest rates are always tied to traded financial instruments and cannot be "observed" or "determined" independently. Deciding on a "yard stick" or "benchmark" firstly requires a decision on a set of traded financial instruments in order to derive prices and corresponding risk factors (= changes in prices of the benchmark). But as an immediate consequence, the determination of risk factors itself is a model and represents an approximation of reality. With respect to "interest rate risk" there are a number of possible financial instruments to be considered: interest rate swaps, government bonds, corporate bonds, repos etc. Market participants prevalently consider the derivative market as the most liquid, reliable source of prices and as a means of deriving risk factors. This inevitably implies that the "benchmark" is tied to market conditions/conventions (e.g. credit and counterparty risk), price discovery (e.g. supply and demand) and the legal framework of the set of financial instruments utilized to derive the benchmark and cannot be separated. In case of derivatives, which are described in *Section 3,* e.g. the legal framework is illustrated on the basis of the ISDA documentation for derivatives including collateralization according to CSA in particular in the case of the interbank market etc.[92] In *Section 4* the connection between the economic rationale and hedge accounting according to IAS 39 in a single-curve model is shown. Additionally, there is an important conclusion if interest rate derivatives are chosen as the best estimate of interest rate risk (market risk): In addition derivative prices are also considered as the best estimate of prices to price all financial instruments. The valuation of financial instruments using interest rate derivative prices ("fair value") is termed "pricing of financial instruments according to their hedging costs".

If market participants regard more than one risk factor as relevant, then the example of *Section 4* can be extended to more risk factors without violating the economic rationale described in the example above. Consequently multi-curve models incorporating various risk factors can be seen as generalization of single-curve (single risk factor) models.

Market participants in financial markets are exposed to various types of market price risks, e.g.: interest rate risk, tenor basis spread risk and FX risk. Continuing the economic rationale from above, market participants have to determine the "yard stick" or "benchmark" for all these types of risk. Like above, all the types of risk mentioned are

92 Further frameworks are e.g. the German master agreement for financial transactions *(Rahmenvertrag für Finanztermingeschäfte)* with its annexes and supplements issued by the Association of German Banks *(Bundesverband Deutscher Banken)* and the European Master Agreement for financial transactions with its annexes and supplements issued by the Banking Federation of the European Union. The legal mechanism of these master agreements is similar to the ISDA Master Agreement (2002).

unobservable risks – the only exception being the FX spot rate that is directly observable. Now market participants have to choose amongst traded financial instruments in order to assign a set of traded financial instruments for each type of risk. In *Figure 81* an example of a set of risk factors regarded as relevant for financial market participants is provided, assuming only two currencies, USD and EUR.

The assignment of types of risk factors to a corresponding set of traded financial instruments is presented in *Table 52.*

Utilizing *Table 52, Figure 81* can be “re-written” in terms of financial instruments (see *Figure 82*).

There are several important properties and features in context with *Figure 82:*

- It can be observed that market participants consider traded derivatives as the best estimate of risk and prices in financial markets.
- In a multiple risk factor model economy the risk factors cannot be modeled independently. Simultaneous modeling of risk factors itself is a model.
- A prominent example is FX risk. The USD/EUR spot exchange rate is a cash price and observable, but it includes the exchange of cash (converting EUR cash into USD cash or vice versa). But as soon as the re-exchange in cash takes place at some future point in time, interest rate risk is present. This is the reason why there is an interrelationship between three types of risk, FX, exchange in cash (liquidity) and interest rate risk. Therefore FX risk is in most cases unobservable, because it is tied to several risk factors (like interest rate risk), and a model is needed to separate all types of risks. This feature is also termed “overlay risk” and is present in both: single- or multi-factor models.
- The selection of risk factors, like in *Figures 81* and *82,* depends not only on the assessment by market participants, but also on the individual preferences of market participants (balance sheet pre-

FIGURE 81 **Risk Factor Model in a Multi-Risk Economy of Financial Markets Relevant for Interest Rate and FX Risk**

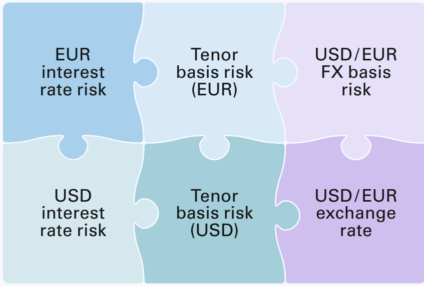
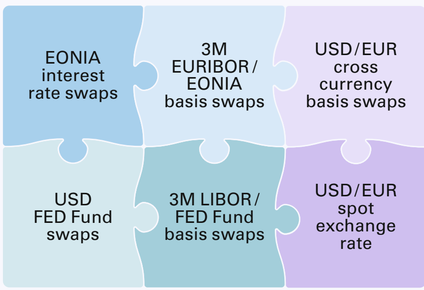


TABLE 52: **Assignment of Types of Financial Market Risk to Corresponding Sets of Traded Financial Instruments**

Types of financial market risk	Sets of traded financial instruments
EUR interest rate risk	EONIA interest rate swaps
Tenor basis risk (EUR)	3M EURIBOR/EONIA basis swaps
USD/EUR FX basis risk	USD/EUR cross currency basis swaps
USD interest rate risk	FED Funds interest rate swaps
Tenor basis risk (USD)	3M USD LIBOR/FED Funds basis swaps
USD/EUR exchange rate risk	USD/EUR spot exchange rate

FIGURE 82 **Financial Instruments Representing Various Types of Risk Factors in a Multi-Risk Economy Relevant to Interest Rate and FX Risk**



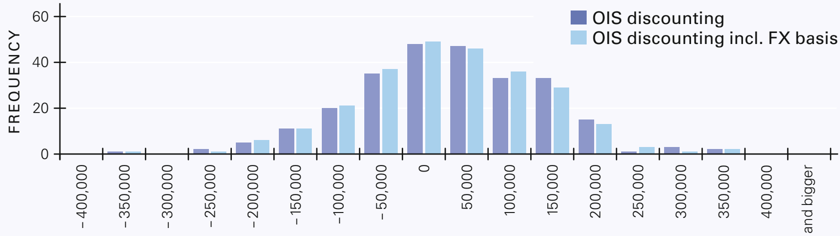
parers). For example: the 6-month EURIBOR/EONIA basis swaps are not included in *Table 52,* because e.g. this type of risk is not relevant for the specific market participant using this model. Otherwise the list above has to be augmented by this supplementary risk factor.

- The list of market risk factors as mentioned above also represents valuation factors.
- Like in the single risk factor model, since derivative prices are considered as the best estimate of prices to price all financial instruments, these will be priced ("fair valued") according to their hedging costs. As shown in *Section 4,* this can be described by the absence of arbitrage principle.

The analysis above illustrates the financial economics of multi-curve models and indicates their practical relevance: Pricing models incorporating a set of risk factors require consistent curve setup, loan and transfer pricing (treasury departments). For example: granting loans in foreign currency and neglecting the FX basis in loan pricing can result in an immediate and significant economic loss.

To illustrate the impact of the FX basis incorporated in a multi-curve setup in more detail, *Figure 83* shows the empirical VaR distribution of a financial instrument denominated in EUR, in one case discounted with the plain EONIA discount curve and in the other with the FX basis adjusted EONIA curve.

FIGURE 83: Value at Risk Evaluation Using OIS Discounting and OIS Discounting including FX Basis



Similar as in *Section 6.5* the calculated values show that the VaR with 99 % confidence level is equal to TEUR 250 for (pure) EONIA discounting, whereas in case of EONIA discounting including the (USD) FX basis the VaR equals TEUR 200. This can also be seen in *Figure 83.*

6.7 Multi-Curve Model Economy for Collateralized Derivatives

6.7.1 Equilibrium Conditions and Derivation of the Model Setup

Continuing with the set of financial market risk factors outlined in *Figure 82,* in the following the corresponding mathematical description of the multi-curve setup for collateralized derivatives is provided, which is very similar to the model setups analyzed in the previous sections.

Defining the abbreviations:

$$A_{€}^{\text{EONIA}}(t_0,T) := \sum_{k=t_1}^{N} B_{€}^{\text{EONIA}}(t_0,T_k)\cdot\Delta(T_{k-1},T_k)$$

$$A_{\$}^{\text{FED}}(t_0,T) := \sum_{k=1}^{N} B_{\$}^{\text{FED}}(t_0,T_k)\cdot\Delta(T_{k-1},T_k)$$

the equilibrium conditions in t_0 for the set of derivatives are given as follows:

EQUATION 22: Equilibrium Conditions in a Multi-Curve Setup

EONIA interest rate swap (forwarding = discounting)

$$c_{€}^{\text{EONIA}}(t_0,T)\cdot A_{€}^{\text{EONIA}}(t_0,T) = \Lambda_{€}^{\text{EONIA}}(t_0,T) = 1 - B_{€}^{\text{EONIA}}(t_0,T)$$
$$= \sum_{j=1}^{N}\delta(t_{j-1},t_j)\cdot f_{€}^{\text{EONIA}}(t_0,t_{j-1},t_j)\cdot B_{€}^{\text{EONIA}}(t_0,t_j)$$

EQUATION 22: Equilibrium Conditions in a Multi-Curve Setup *(continued)*

3-month EURIBOR interest rate swap (forwarding ≠ discounting)

$$c_€^{3M}(t_0,T)\cdot A_€^{EONIA}(t_0,T)=\Lambda_€^{3M/EONIA}(t_0,T)$$
$$=\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot f_€^{3M/EONIA}(t_0,t_{j-1},t_j)\cdot B_€^{EONIA}(t_0,t_j)$$

3-month EURIBOR/EONIA basis swap (forwarding ≠ discounting)

$$\left[c_€^{3M}(t_0,T)-c_€^{EONIA}(t_0,T)\right]\cdot A_€^{EONIA}(t_0,T) = \Lambda_€^{3M/EONIA}(t_0,T)-\Lambda_€^{EONIA}(t_0,T)$$
$$= \Lambda_€^{3M/EONIA}(t_0,T)-\left(1-B_€^{EONIA}(t_0,T)\right)$$

FED Funds interest rate swap (forwarding = discounting)

$$c_\$^{FED}(t_0,T)\cdot A_\$^{FED}(t_0,T)=\Lambda_\$^{FED}(t_0,T)=1-B_\$^{FED}(t_0,T)$$
$$=\sum_{j=1}^{N}\delta(t_{j-1},t_j)\cdot f_\$^{FED}(t_0,t_{j-1},t_j)\cdot B_\$^{FED}(t_0,t_j)$$

3-month USD LIBOR interest rate swap (forwarding ≠ discounting)[93]

$$c_\$^{3M}(t_0,T)\cdot A_\$^{FED}(t_0,T)=\Lambda_\$^{3M/FED}(t_0,T)$$
$$=\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot f_\$^{3M/FED}(t_0,t_{j-1},t_j)\cdot B_\$^{FED}(t_0,t_j)$$

3-month USD LIBOR/FED Funds basis swap (forwarding ≠ discounting)

$$\left[c_\$^{3M}(t_0,T)-c_\$^{FED}(t_0,T)\right]\cdot A_\$^{FED}(t_0,T) = \Lambda_\$^{3M/FED}(t_0,T)-\Lambda_\$^{FED}(t_0,T)$$
$$= \Lambda_\$^{3M/FED}(t_0,T)-\left(1-B_\$^{FED}(t_0,T)\right)$$

3-month USD LIBOR/3-month EURIBOR basis swap

$$S_€^\$(t_0)\cdot N_\$\begin{bmatrix}\delta(t_0,t_1)\cdot r_\$^{3M}(t_0)B_\$^{FED}(t_0,t_1)\\ +\sum_{j=2}^{4N}\delta(t_{j-1},t_j)\cdot f_\$^{3M/FED}(t_0,t_{j-1},t_j)\cdot B_\$^{FED}(t_0,t_j)+B_\$^{FED}(t_0,T)\end{bmatrix}$$
$$=N_€\begin{bmatrix}\delta(t_0,t_1)\cdot r_€^{3M}(t_0)B_€^{FX/EONIA}(t_0,t_1)\\ +\sum_{j=2}^{4N}\delta(t_{j-1},t_j)\cdot\left(f_€^{3M/EONIA}(t_0,t_{j-1},t_j)+b(t_0,T)\right)\cdot B_€^{FX/EONIA}(t_0,t_j)+B_€^{FX/EONIA}(t_0,T)\end{bmatrix}$$

EUR/USD exchange rate: $S_€^\$(t)$

93 Although market conventions for tenor basis swaps involving FED Funds might be different from those mentioned in *Section 3,* the following notation is used, assuming that market quotes are adequately converted.

The equilibrium conditions above mathematically describe the model economy in a multi-curve setup for collateralized derivatives. In order to solve the system of equilibrium conditions in *Equation 22,* bootstrapping algorithms similar to those described in *Section 5* are utilized, but some additional properties of/assumptions regarding the model economy are of importance:

- The solution of *Equation 22* immediately results in an integrated market of the interest rate swaps mentioned above in *Section 6.1* resolving market segmentation of (collateralized) derivative markets. As already shown in an analysis in *Section 4* for the case of quoted 3-month/6-month EURIBOR basis spreads and the difference of quoted 3-month and 6-month EURIBOR interest rate swap quotes *(Figure 48)* and in the case of USD FED Funds/ 3-month USD LIBOR spread vs. the difference of 3-month USD LIBOR swap rate and OIS rate in *Section 3 (Figure 32)*, the integrated market assumption of tenor and interest rate swaps is a good approximation of reality.
- Both the 3-month EURIBOR interest rate swaps and 3-month USD LIBOR interest rate swaps are collateralized interest rate swaps (interest rate derivatives with CSA).
- The EONIA and the FED Funds zero swap rates represent the relevant discount rates for all collateralized interest rate swap derivatives in the model economy. This is consistent with market conventions. Therefore EONIA and FED Funds interest swap rates are utilized to derive discount curves ($B_{€}^{\text{EONIA}}(t_0,t_j)$, $B_{\$}^{\text{FED}}(t_0,t_j), \forall j=1,\ldots,4N$). For both, the discount and forward curve are identical, similar to a single-curve model setup. The floating side of these swaps resets to par minus discounted repayment. It is assumed that EONIA and FED Funds interest rate swaps pay annually on the floating side (compounded).
- Since the EONIA and the FED Funds discount curves are used to discount the 3-month EURIBOR interest rate swaps and 3-month USD LIBOR interest rate swaps, respectively, the forward curves ($f_{€}^{\text{3M/EONIA}}(t_0,t_{j-1},t_j), f_{\$}^{\text{3M/FED}}(t_0,t_{j-1},t_j), \forall j=1,\ldots,4N$) have to

be derived using a bootstrapping algorithm similar to that described in *Section 4*. Consequently the floating sides of both interest rate swaps do not reset to par minus discounted repayment. For both interest rate swaps annual payments on the fixed side and quarterly payments on the floating side are assumed.

- Regarding the definition of the 3-month USD LIBOR/3-month EURIBOR basis swap there are two additional aspects. Firstly the cash collateral associated with the cross currency basis swap is posted in USD. Therefore the USD floating side of the cross currency basis swap is discounted by the FED Funds discount curve. Secondly – without additional assumption – the system of equations above cannot be solved since there are more unknowns than conditions. It is assumed that the forward curve derived from collateralized 3-month EURIBOR interest rate swaps ($f_{€}^{3M/EONIA}\left(t_0,t_{j-1},t_j\right)$, $\forall j=1,\ldots,4N$) show similar dynamic behaviour as the forward rates ($f_{€}^{3M/FX/EONIA}\left(t_0,t_{j-1},t_j\right),\forall j=1,\ldots,4N$) derived from 3-month EURIBOR interest rate swaps collateralized with USD used in the valuation of cross currency basis swaps; therefore in the superscript "FX" is omitted. As a consequence of this modeling assumption a further bootstrapping algorithm needs to be performed in order to derive the "FX basis adjusted" EONIA discount curve ($B_{€}^{FX/EONIA}\left(t_0,t_j\right),\forall j=1,\ldots,4N$) for the valuation of cross currency products, e.g. cross currency basis swaps. As a result, a consistent set for each currency is achieved, but for FX related discounting the "FX basis adjusted" EONIA discount curve is utilized.
- The solution of the *Equation 22* above requires "market prices" of derivatives as model input and – according to the model – these market prices are "recovered" after solving the system of equations above. This feature of the model is also termed "calibration to market prices".

In order to derive the "FX basis adjusted" EONIA discount curve ($B_{€}^{FX/EONIA}\left(t_0,t_j\right),\forall j=1,\ldots,4N$), the following bootstrapping algorithm is derived using the following assumptions given by market conventions:

- USD represents the "numeraire",
- For the 3-month USD LIBOR/3-month EURIBOR basis at t_0 $PV_\$^1[t_0,T]=PV_€^2[t_0,T]$, where:

$$PV_\$^1[t_0,T] = S_€^\$(t_0)\cdot N_\$\begin{bmatrix}\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot f_\$^{3M/FED}(t_0,t_{j-1},t_j)\cdot B_\$^{FED}(t_0,t_j)\\ +B_\$^{FED}(t_0,T)\end{bmatrix}$$

$$PV_€^1[t_0,T] = N_€\begin{bmatrix}\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot\left(f_€^{3M/EONIA}(t_0,t_{j-1},t_j)+b(t_0,T)\right)\cdot B_€^{FX/EONIA}(t_0,t_j)\\ +B_€^{FX/EONIA}(t_0,T)\end{bmatrix}.$$

Using $S_€^\$(t_0)N_\$ = N_€$ at t_0, the definition of interest rate swaps according to *Equation 22* and $PV_\$^1[t_0,T]=PV_€^2[t_0,T]$ yield:

$$\left[\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot f_\$^{3M/FED}(t_0,t_{j-1},t_j)\cdot B_\$^{FED}(t_0,t_j)+B_\$^{FED}(t_0,T)\right]$$
$$=\begin{bmatrix}\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot\left(f_€^{3M/EONIA}(t_0,t_{j-1},t_j)+b(t_0,T)\right)\cdot B_€^{FX/EONIA}(t_0,t_j)\\ +B_€^{FX/EONIA}(t_0,T)\end{bmatrix}$$

$$\Rightarrow c_\$^{3M}(t_0,T)\cdot A_\$^{FED}(t_0,T)+B_\$^{FED}(t_0,T)$$
$$=\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot\left(f_€^{3M/EONIA}(t_0,t_{j-1},t_j)+b(t_0,T)\right)\cdot B_€^{FX/EONIA}(t_0,t_j)$$
$$+B_€^{FX/EONIA}(t_0,T)$$

$$\Rightarrow c_\$^{3M}(t_0,T)\cdot A_\$^{FED}(t_0,T)+B_\$^{FED}(t_0,T)$$
$$=\sum_{j=1}^{4N-1}\delta(t_{j-1},t_j)\cdot\left(f_€^{3M/EONIA}(t_0,t_{j-1},t_j)+b(t_0,T)\right)\cdot B_€^{FX/EONIA}(t_0,t_j)$$
$$+B_€^{FX/EONIA}(t_0,T)$$
$$+\delta(t_{4N-1},t_{4N})\cdot\left(f_€^{3M/EONIA}(t_0,t_{4N-1},t_{4N})+b(t_0,T)\right)\cdot B_€^{FX/EONIA}(t_0,T)$$

$$\Rightarrow\ c_{\$}^{3M}(t_0,T)\cdot A_{\$}^{FED}(t_0,T)+B_{\$}^{FED}(t_0,T)$$
$$-\sum_{j=1}^{4N-1}\delta(t_{j-1},t_j)\cdot\left(f_{€}^{3M/EONIA}(t_0,t_{j-1},t_j)+b(t_0,T)\right)\cdot B_{€}^{FX/EONIA}(t_0,t_j)$$
$$=B_{€}^{FX/EONIA}(t_0,T)\left[1+\delta(t_{4N-1},t_{4N})\cdot\left(f_{€}^{3M/EONIA}(t_0,t_{4N-1},t_{4N})+b(t_0,T)\right)\right]$$

$$\Rightarrow\ B_{€}^{FX/EONIA}(t_0,T)$$
$$=\frac{\begin{bmatrix}\left[c_{\$}^{3M}(t_0,T)\cdot A_{\$}^{FED}(t_0,T)+B_{\$}^{FED}(t_0,T)\right]\\ -\sum_{j=1}^{4N-1}\delta(t_{j-1},t_j)\cdot\left(f_{€}^{3M/EONIA}(t_0,t_{j-1},t_j)+b(t_0,T)\right)\cdot B_{€}^{FX/EONIA}(t_0,t_j)\end{bmatrix}}{\left[1+\delta(t_{4N-1},t_{4N})\cdot\left(f_{€}^{3M/EONIA}(t_0,t_{4N-1},t_{4N})+b(t_0,T)\right)\right]}.$$

According to the definition of the "FX basis adjusted" EONIA discount curve, the formula above cannot be simplified by using 3-month EURIBOR interest rate swap rates. Furthermore the present value of the 3-month USD LIBOR side is not equal to par at t_0, since forwarding and discounting are done w.r.t. different curves.

Assuming the FX basis is taken into account on the EUR floating leg (market convention), the valuation formula of a fixed-to-float cross currency basis swap, $S_{€}^{\$}(t_0)N_{\$}=N_{€}$, is given by:

$$PV_{CCS}(t_0,T)$$
$$=S_{€}^{\$}(t_0)\cdot N_{\$}\left[c_{\$}(t_0,T)\cdot A_{\$}^{FED}(t_0,T)+B_{\$}^{FED}(t_0,T)\right]$$
$$-N_{€}\begin{bmatrix}\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot\left(f_{€}^{3M/EONIA}(t_0,t_{j-1},t_j)+b(t_0,T)\right)\cdot B_{€}^{FX/EONIA}(t_0,t_j)\\ +B_{€}^{FX/EONIA}(t_0,T)\end{bmatrix}$$
$$\overset{!}{=}0$$

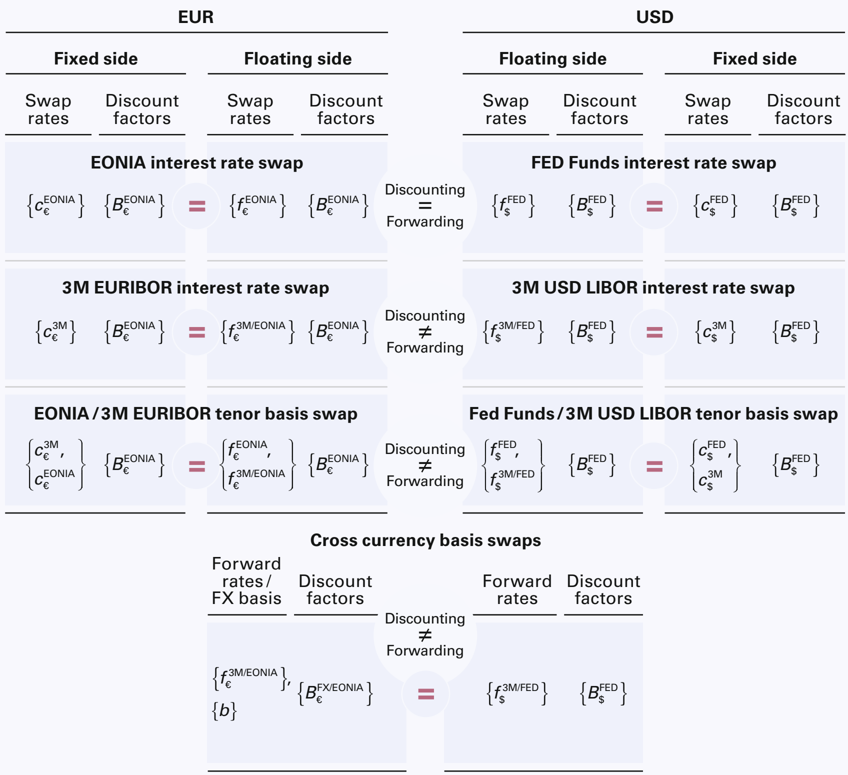
$$S_{€}^{\$}(t_0)\cdot N_{\$}\left[c_{\$}(t_0,T)\cdot A_{\$}^{FED}(t_0,T)+B_{\$}^{FED}(t_0,T)\right]$$
$$-N_{€}\left[\sum_{j=1}^{4N}\delta(t_{j-1},t_j)\cdot f_{\$}^{3M/FED}(t_0,t_{j-1},t_j)\cdot B_{\$}^{FED}(t_0,t_j)+B_{\$}^{FED}(t_0,T)\right]$$
$$=0$$

$$\Rightarrow c_{\$}(t_0,T) = c_{\$}^{3M}(t_0,T).$$

The coupon of a fixed-to-float cross currency basis swap coincides with the 3-month USD LIBOR swap rate.

Figure 84 summarizes the structure of the multi-curve model setup analyzed above. The "=" sign depicted in *Figure 84* represents the equilibrium conditions and market integration.

FIGURE 8: Model Structure of a Multi-Curve Model for Collateralized Derivatives



6.7.2 Comparison of Single- and Multi-Curve Setup and Risk Factor Considerations

Table 53 summarizes the facts concerning the multi-curve model setup described above and compares it with a single-curve model setup:

- Market data inputs are also driven by the individual circumstances of the balance sheet preparer, e.g. if there is no inventory of 6-month EURIBOR interest rate swaps, these will not be included in the equilibrium conditions; otherwise the equilibrium conditions in *Equation 22* need to be augmented. This also affects the considerations of risk factors because it depends on individual circumstances wether e.g. the 6-month EURIBOR/EONIA tenor basis swap needs to be taken into account as a risk factor.
- It is important to note the role of model output and risk factors. Not every model output coincides with risk factors and vice versa. The "FX basis adjusted" EONIA discount curve is not a risk factor, since it is derived from the EONIA, FED Funds discount curve and from the FX basis, which represent risk factors. Similar arguments hold for some forward rates, e.g. $f_{€}^{EONIA}$, $f_{\$}^{FED}$.
- Within collateralized multi-curve models, discount curves like EURIBOR or LIBOR do not exist anymore.
- Market conventions and modeling assumptions both affect multi-curve as well as single-curve model setups. In both cases the individual choice of financial instruments for deriving discount factors as well as the usage of different interpolation models yield different results of discount curves, which in particular in the case of EURIBOR/LIBOR are additionally influenced by assumptions concerning the modeling of the short term $< 1-2$ y (inclusion of FRA and/or futures etc.).
- As presented in *Table 53,* the modeling assumptions are to a certain extent individual to each balance sheet preparer, which results in different discount and forward curves. Consequently the fair values and value changes of hedged items and derivatives are different and not necessarily comparable between different balance sheet preparers.

The "bootstrapping algorithm" plays an important role in solving the set of equations outlined in *Equation 22* and also affects hedge accounting. In order to illustrate the effect of the bootstrapping algorithm, the set of equations in *Equation 22* is reduced to two equations (EONIA and 3-month EURIBOR interest rate swaps) and compared to a single-curve model setup consisting of 3-month EURIBOR interest rate swaps. In the single-curve model setup the forward rates can be directly inferred from the 3-month EURIBOR discount curve. Then the present value of the floating rate side of the interest rate swap equals par minus the discounted repayment. In the multi-curve model setup the fixed side and the floating side of the (collateralized) 3-month EURIBOR interest rate swap are discounted by EONIA. By market convention the bootstrapping algorithm assumes a present value of zero of the entire swap

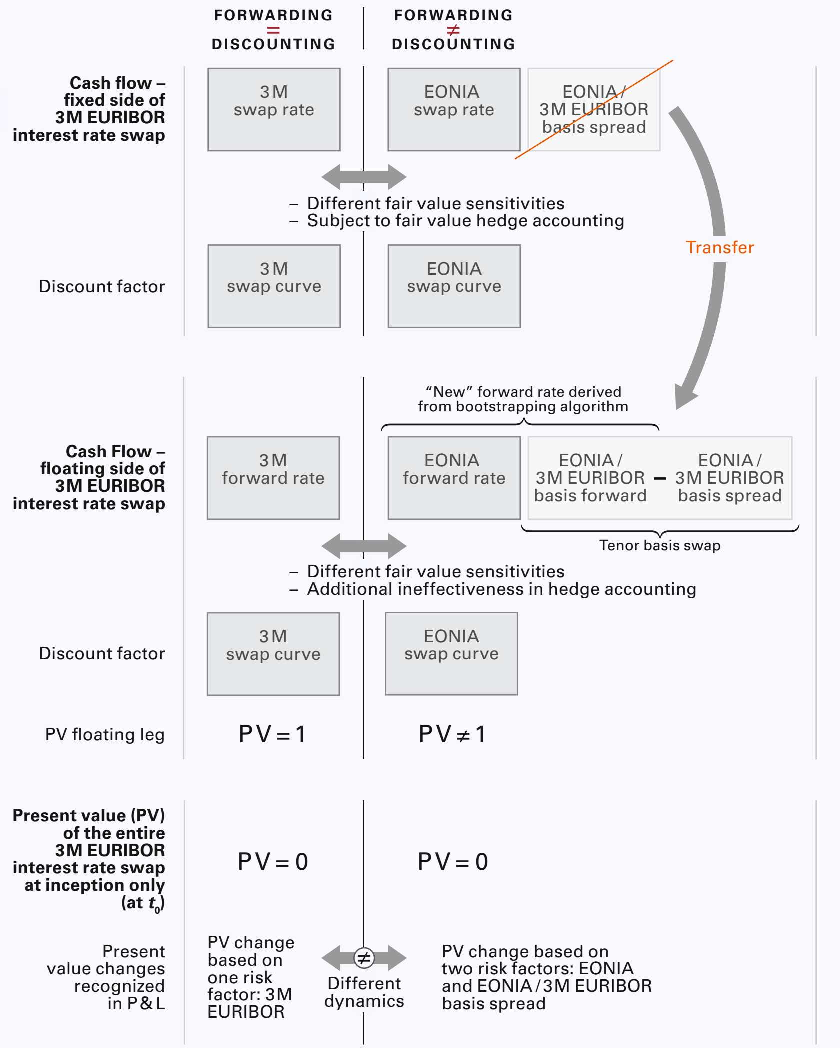
TABLE 53: Summary and Comparison of Single- and Multi-Curve Models

	Multi-curve setup	Single-curve setup
Market data input	– Swap rates: EONIA, FED Funds, 3M EURIBOR/LIBOR, cross currency basis swaps (tenor basis swaps). – Forward rate agreements, futures etc. depending on modeling assumptions for the short term <1–2 year(s). – USD/EUR exchange rate.	– 3M EURIBOR/LIBOR swap rates. – Forward rate agreements, futures etc. depending on modeling assumptions for the short term <1–2 year(s). – USD/EUR exchange rate.
Modeling assumptions/ market conventions	– Interpolation of discount curve (e.g. cubic spline). – Assumptions concerning modeling the short term of the forward curve <1–2 year(s). – Collateralization. – Day count conventions. – Modeling assumptions with respect to solutions of the bootstrapping algorithm (e.g. dynamic of (un-)collateralized forward rates). – Representation of the FX basis (on the USD or EUR discount curve). – Collateralization currency of cross currency basis swaps.	– Interpolation of discount curve (e.g. cubic spline). – Collateralization. – Day-count conventions. – Assumptions concerning modeling the short term of the interest rate curve <1–2 year(s).
Model output	– Discount curves: $B_{€}^{EONIA}$, $B_{\$}^{FED}$, $B_{€}^{FX/EONIA}$. – Forward rates: $f_{€}^{EONIA}$, $f_{\$}^{FED}$, $f_{€}^{3M/EONIA}$, $f_{\$}^{3M/FED}$.	– Separate discount curves: $B_{€}$, $B_{\$}$. – Separate forward rates: $f_{€}$ and/or $f_{\$}$.
Risk factors	– Discount curves: $B_{€}^{EONIA}$, $B_{\$}^{FED}$. – Tenor basis swaps (EONIA/OIS basis swaps, FX basis, etc.).	– Separate discount curves: $B_{€}$, $B_{\$}$.

at inception (t_0) – similar to the single-curve model setup – and then solves for 3-month EURIBOR forward rates, which are the only unknown parameter. Since the contractual cash flows of the 3-month EURIBOR interest rate swap (= 3-month EURIBOR interest rate swap rate) remain unaffected by changes in discount curves, there is a present value effect on the fixed side of the interest rate swap. This valuation effect can be associated with a 3-month EURIBOR/EONIA basis spread effect. When assuming a present value of zero for the entire interest rate swap, the present value of the fixed side of the interest rate swap equals the present value of the floating side of the interest rate swap. Consequently the 3-month EURIBOR/EONIA basis spread effect is "transferred" to the floating side of the interest rate swap, which results in a present value different from par minus the discounted repayment. Therefore the change of discount curve results in different present value sensitivities of the fixed leg and floating leg of the interest rate swap. When considering the fixed leg with respect to fair value hedge accounting – which will be shown below –, the "cash flow" included in the 3-month EURIBOR interest rate swap rate subject to fair value hedge accounting is associated with the EONIA swap rate. Accordingly the 3-month EURIBOR is decomposed into an EONIA interest rate swap and a 3-month EURIBOR/EONIA basis swap, which mainly impacts the floating rate side of the 3-month EURIBOR interest rate swap. This decomposition is synthetic and results from the equilibrium conditions as well as the risk factor considerations.

To sum it up, in comparison to the single-curve model setup, the present value and the present value changes of the entire swap stem from two risk factors: EONIA discount curve and 3-month EURIBOR/EONIA basis swap spread. Consequently the present values at inception are zero in both model setups, but the changes in present values over time are different. With respect to fair value hedge accounting, the "new" floating side of the 3-month EURIBOR interest rate swap consists of the floating side of an EONIA interest rate swap and a 3-month EURIBOR/EONIA basis swap, which results in additional hedge ineffectiveness represented by fair value changes of the 3-month

FIGURE 85: Illustrative Example of the Impact of the Change in Discount Curve



EURIBOR/EONIA basis swap spread. As illustrated in *Figure 85,* the fair value sensitivities on the fixed side of the swap are mainly driven by changes in EONIA, while the floating side is driven by the two risk factors EONIA and tenor basis spread.

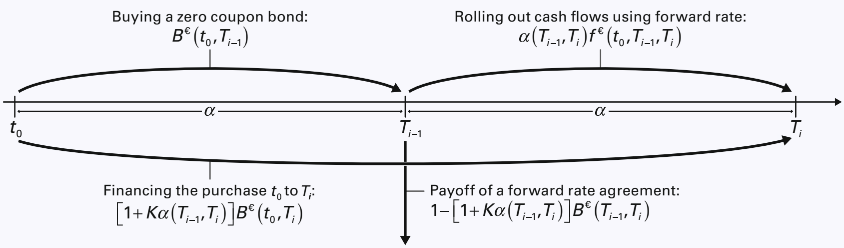
6.7.3 Analogy between Forward Rate Agreements and Tenor Basis Swaps in a Multi-Curve Setup

In the previous sections the forward rates have been determined by the bootstrapping algorithms which derive forward rates recursively from interest rate swap rates. Consequently there is no explicit expression for forward rates except when the discount curve coincides with the forward curve, i.e. the single-curve case. In the following, forward rates in a multi-curve setup will be analyzed by drawing the analogy between forward rate agreements and tenor basis swaps.[94]

6.7.3.1 Forward Rate Agreements in a Single-Curve Model Setup

The forward rate agreement is a collateralized financial contract under the ISDA Master Agreement (2002) with CSA, which is written on EURIBOR/LIBOR rates and requires a cash payment at maturity. The cash payment at maturity is based on the difference between the realized EURIBOR/LIBOR rates and the pre-specified forward rates.

FIGURE 86: Replication of the Payoff of a Forward Rate Agreement



94 Following Morini, M. (2009), Jarrow, R. A. and Turnbull, S. (1996), pp. 412–415.

It is assumed that the EURIBOR discount curve is the appropriate discount curve despite collateralization; this feature will be altered in the next subsection.

The derivation of the forward rate agreement rate K for the period (T_{i-1}, T_i) of a forward rate agreement maturing at T_{i-1} involves a replication argument and the application of the absence of arbitrage principle.

As depicted in *Figure 86,* there are three "transactions" involved in the derivation of the forward rate agreement rate: buying a zero coupon bond at t_0 with maturity T_{i-1}, investing the proceeds from T_{i-1} to T_i and the financing of the transaction by borrowing t_0 to T_i. The forward rate agreement involves two "legs" with different maturity (tenor).

In order to derive the replication strategy some notation is introduced:

EURIBOR rates:

$L(T_{i-1}, T_i)$ defines the EURIBOR rate at time T_{i-1} for the period (T_{i-1}, T_i) and $\alpha(T_{i-1}, T_i)$, the corresponding time fraction,

Discount rates:

$$B^{€}(T_{i-1}, T_i) = \frac{1}{\left[1 + \alpha(T_{i-1}, T_i) L(T_{i-1}, T_i)\right]}.$$

The definition of the discount rates above follows the definition from the previous section, using simple compounding for short term (less than 1 y) discount factors. As mentioned above, FRAs are utilized to model the short term.

The payoff of a $T_{i-1} \times T_i$ forward rate agreement FRA at time T_{i-1} (maturity) with notional 1 is expressed as follows:

$$FRA(t_0, T_{i-1}, T_i, K) = \frac{\left[\left(L(T_{i-1}, T_i) - K\right)\alpha(T_{i-1}, T_i)\right]}{\left[1 + \alpha(T_{i-1}, T_i) L(T_{i-1}, T_i)\right]},$$

where K represents the forward rate agreement rate and $\alpha(T_{i-1},T_i)$ the time fraction of the period (T_{i-1},T_i) according to the relevant day count convention. Rearranging the FRA yields the following expression:

$$FRA(t_0,T_{i-1},T_i,K)=\frac{\left[\overbrace{1-1}^{=0}+\left(L(T_{i-1},T_i)-K\right)\alpha(T_{i-1},T_i)\right]}{\left[1+\alpha(T_{i-1},T_i)L(T_{i-1},T_i)\right]}$$

$$=\frac{\left[1+L(T_{i-1},T_i)\alpha(T_{i-1},T_i)-1-K\alpha(T_{i-1},T_i)\right]}{\left[1+\alpha(T_{i-1},T_i)L(T_{i-1},T_i)\right]}$$

$$=1-\frac{\left[1+K\alpha(T_{i-1},T_i)\right]}{\left[1+\alpha(T_{i-1},T_i)L(T_{i-1},T_i)\right]}.$$

The replication strategy of the forward rate agreement can be stated as follows:

Buy a zero coupon bond with maturityT_{i-1} at t_0 and re-invest the proceeds (=1) at time T_{i-1} to T_i at EURIBOR rate $L(T_{i-1},T_i)$, in order to finance the purchase of the zero coupon bond borrow $\left[1+K\alpha(T_{i-1},T_i)\right]B^{€}(t_0,T_i)$ at the EURIBOR rate $L(t_0,T_i)$. The value of the portfolio $P(t_0,T_{i-1})$ at time t_0 comprising these financial instruments can be stated as follows:

$$P(t_0,T_{i-1})=B^{€}(t_0,T_{i-1})-\left[1+K\alpha(T_{i-1},T_i)\right]\cdot B^{€}(t_0,T_i)\overset{!}{=}0.$$

In order to avoid arbitrage, the forward rate agreement rate K will be set to a value which results in a portfolio value of zero at t_0. At time T_{i-1} the payoff of the portfolio $P(T_{i-1},T_{i-1})$ is equal to:

$$P(T_{i-1},T_{i-1})=1-\left[1+K\alpha(T_{i-1},T_i)\right]\cdot B^{€}(T_{i-1},T_i)$$

$$=1-\left[1+K\alpha(T_{i-1},T_i)\right]\cdot\frac{1}{\left[1+\alpha(T_{i-1},T_i)L(T_{i-1},T_i)\right]},$$

which coincides with the payoff of the forward rate agreement. Therefore the forward rate agreement rate K is:

$$B^{€}(t_0,T_{i-1}) - \left[1 + K\alpha(T_{i-1},T_i)\right] \cdot B^{€}(t_0,T_i) \overset{!}{=} 0$$

$$\Rightarrow K = \frac{1}{\alpha(T_{i-1},T_i)}\left(\frac{B^{€}(t_0,T_{i-1})}{B^{€}(t_0,T_i)} - 1\right) = f^{€}(t_0,T_{i-1},T_i).$$

The following results can be derived:

- In a single curve setup the forward rate agreement rate K is equal to the forward rate $f^{€}(t_0,T_{i-1},T_i)$.
- The forward rate agreement rate K is set at inception of the FRA, therefore the FRA is a bet on realized interest rates between T_{i-1} and T_i and the pre-specified FRA rate $f^{€}(t_0,T_{i-1},T_i)$. The payoff can be either positive or negative.
- The replication strategy involves three transactions which are dependent on the tenors: T_{i-1} and T_i. In a single-curve setup, where tenor basis spreads are assumed to be of minor importance, the impact of the different tenors associated with T_{i-1} and T_i and the corresponding transactions involved in the replication strategy is also of limited relevance.
- This property does not hold anymore if tenor basis spreads are of importance. Additionally forward rate agreements are contracted under the ISDA Master Agreement (2002) with CSA and therefore represent collateralized trades, the appropriate discount rate is represented by OIS. Both features will be considered in the following subsection.

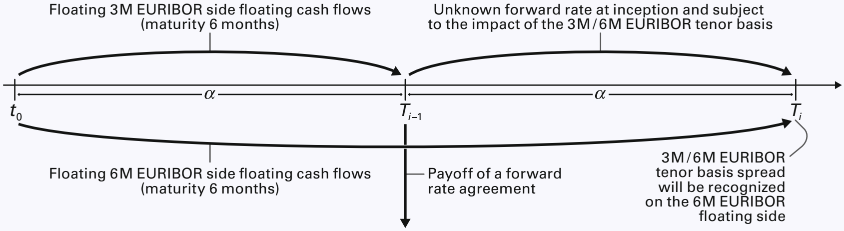
6.7.3.2 Forward Rate Agreements in a Multi-Curve Model Setup

In the following, explicit expressions for the forward rate are derived in order to contemplate the description of forward rates according to the bootstrapping algorithm used so far. This applies especially to cases where discounting and forwarding do not coincide. Following the analysis above, there are two important features of the FRA, which will be exploited in the subsequent analysis:

- The FRA rate derived according to the absence of arbitrage principle can be related to the forward rate.
- The replication argument in connection with the absence of arbitrage principle utilizes transactions of different tenors. The forward rate agreement consists of "two legs" with different tenors.
- In the following a portfolio of two interest rate swaps: 3-month EURIBOR interest rate swap and 6-month EURIBOR interest rate swap, both with a maturity of 6 months, is considered. This portfolio results in 3-month EURIBOR and 6-month EURIBOR variable cash flows including the tenor basis spread and can be considered as 3-month EURIBOR/6-month EURIBOR tenor basis swap where the tenor basis spread equals the difference of the fixed sides.

Therefore the basic idea is to express a tenor basis swap as a forward rate agreement and to evaluate the forward rate agreement rate K

FIGURE 87: Drawing the Analogy of a 3M/6M Tenor Basis Swap to a Forward Rate Agreement



which can be considered as an explicit expression of the forward rate. In order to facilitate the evaluations the term

$$\frac{1}{\left[1+\alpha\left(T_{i-1},T_i\right)L\left(T_{i-1},T_i\right)\right]}$$

in the payoff of the forward rate agreement at T_{i-1} is neglected; when utilizing the martingale calculus (change in measure), this approach can be justified[95]. Considering the 3-month/6-month EURIBOR tenor basis swap (USD convention: float-to-float instrument with constant basis spread on one leg), and assuming the tenor basis spread is recognized on the 6-month floating leg, the fair value of a 3-month/6-month EURIBOR tenor basis swap at $t_0 = 0$ can be represented by:

$$0 = Basis(0,2\alpha,Z) = E_0^{EONIA}\left[\overbrace{\alpha L(0,\alpha)+\alpha L(\alpha,2\alpha)}^{\text{3M EURIBOR floating leg}} - \underbrace{2\alpha\left(L(0,2\alpha)-Z\right)}_{\substack{\text{6M EURIBOR floating leg}\\ \text{including spread}}}\right],$$

using $\alpha\left(t_0,T_{i-1}\right)=\alpha\left(T_{i-1},T_i\right)=\alpha$ and denoting by E_0^{EONIA} the expectation operator with discount curve on EONIA at t_0, since collateralization is taken into account. Accordingly, when denoting by $EONIA\left(T_{i-1},T_i\right)$ the EONIA rate from T_{i-1} until T_i, the discount rates are defined as follows:

$$B_{€}^{EONIA}\left(T_{i-1},T_i\right)=\frac{1}{\left[1+\alpha\left(T_{i-1},T_i\right)EONIA\left(T_{i-1},T_i\right)\right]}.$$

This is used in rewriting the tenor basis swap as:

EQUATION 23: Derivation of the Forward Rate (EUR)

$$0 = Basis(0,2\alpha,Z) = E_0^{EONIA}\left[\overbrace{\alpha L(0,\alpha)+\alpha L(\alpha,2\alpha)}^{\text{3M LIBOR floating leg}} - \underbrace{2\alpha\left(L(0,2\alpha)-Z\right)}_{\substack{\text{6M LIBOR floating leg}\\ \text{including spread}}}\right]$$

95 Refer to Morini, M. (2009), pp. 17–18.

EQUATION 23: Derivation of the Forward Rate (EUR) *(continued)*

$$= E_0^{\text{EONIA}}\left[\alpha L(0,\alpha) + \alpha L(\alpha,2\alpha) + 2\alpha Z - 2\alpha L(0,2\alpha)\right]$$

$$= E_0^{\text{EONIA}}\left[\alpha L(\alpha,2\alpha)\right] - \alpha E_0^{\text{EONIA}}\left[-L(0,\alpha) - 2Z + 2L(0,2\alpha)\right]$$

$$= E_0^{\text{EONIA}}\left[\alpha L(\alpha,2\alpha)\right] - \alpha\begin{bmatrix} -B_{€}^{\text{EONIA}}(0,\alpha)\dfrac{1}{\alpha}\left(\dfrac{1}{B^{€}(0,\alpha)} - 1\right) - 2B_{€}^{\text{EONIA}}(0,2\alpha)Z \\ +2B_{€}^{\text{EONIA}}(0,2\alpha)\dfrac{1}{2\alpha}\left(\dfrac{1}{B^{€}(0,2\alpha)} - 1\right) \end{bmatrix}$$

$$= E_0^{\text{EONIA}}\left[\alpha L(\alpha,2\alpha)\right] - \alpha B_{€}^{\text{EONIA}}(0,2\alpha)\left[-\frac{B_{€}^{\text{EONIA}}(0,\alpha)}{B_{€}^{\text{EONIA}}(0,2\alpha)}\frac{1}{\alpha}\left(\frac{1}{B^{€}(0,\alpha)} - 1\right) - 2Z + \frac{1}{\alpha}\left(\frac{1}{B^{€}(0,2\alpha)} - 1\right)\right]$$

$$= E_0^{\text{EONIA}}\left[\alpha L(\alpha,2\alpha)\right] - \alpha B_{€}^{\text{EONIA}}(0,2\alpha)\frac{1}{\alpha}\left[-\frac{B_{€}^{\text{EONIA}}(0,\alpha)}{B_{€}^{\text{EONIA}}(0,2\alpha)}\left(\frac{1}{B^{€}(0,\alpha)} - 1\right) - 2\alpha Z + \left(\frac{1}{B^{€}(0,2\alpha)} - 1\right)\right]$$

$$= E_0^{\text{EONIA}}\left[\alpha L(\alpha,2\alpha)\right] - \alpha B_{€}^{\text{EONIA}}(0,2\alpha)\frac{1}{\alpha}\begin{bmatrix} -\dfrac{1}{B^{€}(0,\alpha)}\left(\dfrac{B_{€}^{\text{EONIA}}(0,\alpha)}{B_{€}^{\text{EONIA}}(0,2\alpha)}\right) + \dfrac{B_{€}^{\text{EONIA}}(0,\alpha)}{B_{€}^{\text{EONIA}}(0,2\alpha)} \\ + \overbrace{\dfrac{B^{€}(0,\alpha)}{B^{€}(0,2\alpha)} - \dfrac{B^{€}(0,\alpha)}{B^{€}(0,2\alpha)}}^{=0} + \dfrac{1}{B^{€}(0,2\alpha)} - 1 - 2\alpha Z \end{bmatrix}$$

$$= E_0^{\text{EONIA}}\left[\alpha L(\alpha,2\alpha)\right] - \alpha B_{€}^{\text{EONIA}}(0,2\alpha)\frac{1}{\alpha}\begin{bmatrix} -\dfrac{1}{B^{€}(0,\alpha)}\left(\dfrac{B_{€}^{\text{EONIA}}(0,\alpha)}{B_{€}^{\text{EONIA}}(0,2\alpha)} - \dfrac{B^{€}(0,\alpha)}{B^{€}(0,2\alpha)}\right) \\ +\left(\dfrac{B_{€}^{\text{EONIA}}(0,\alpha)}{B_{€}^{\text{EONIA}}(0,2\alpha)} - \dfrac{B^{€}(0,\alpha)}{B^{€}(0,2\alpha)}\right) + \left(\dfrac{B^{€}(0,\alpha)}{B^{€}(0,2\alpha)} - 1\right) - 2\alpha Z \end{bmatrix}$$

EQUATION 23: Derivation of the Forward Rate (EUR) *(continued)*

$$=E_0^{EONIA}\left[\alpha L(\alpha,2\alpha)\right]$$

$$-\alpha B_{€}^{EONIA}(0,2\alpha)\frac{1}{\alpha}\left[\begin{array}{l}\frac{1}{B^{€}(0,\alpha)}\left(\frac{B^{€}(0,\alpha)}{B^{€}(0,2\alpha)}-\frac{B_{€}^{EONIA}(0,\alpha)}{B_{€}^{EONIA}(0,2\alpha)}\right)\\-\left(\frac{B^{€}(0,\alpha)}{B^{€}(0,2\alpha)}-\frac{B_{€}^{EONIA}(0,\alpha)}{B_{€}^{EONIA}(0,2\alpha)}\right)+\left(\frac{B^{€}(0,\alpha)}{B^{€}(0,2\alpha)}-1\right)-2\alpha Z\end{array}\right]$$

$$=E_0^{EONIA}\left[\alpha L(\alpha,2\alpha)\right]$$

$$-\alpha B_{€}^{EONIA}(0,2\alpha)\frac{1}{\alpha}\left[\begin{array}{l}\left(\frac{B^{€}(0,\alpha)}{B^{€}(0,2\alpha)}-1\right)\\+\left(\frac{1}{B^{€}(0,\alpha)}-1\right)\left(\frac{B^{€}(0,\alpha)}{B^{€}(0,2\alpha)}-\frac{B_{€}^{EONIA}(0,\alpha)}{B_{€}^{EONIA}(0,2\alpha)}\right)-2\alpha Z\end{array}\right]$$

$$=E_0^{EONIA}\left[\alpha L(\alpha,2\alpha)\right]$$

$$-\alpha B_{€}^{EONIA}(0,2\alpha)\cdot\left[\underbrace{f^{€}(t_0,\alpha,2\alpha)+\left(\frac{1}{B^{€}(0,\alpha)}-1\right)\left(\begin{array}{l}f^{€}(t_0,\alpha,2\alpha)\\-f_{€}^{EONIA}(t_0,\alpha,2\alpha)\end{array}\right)-2Z}_{:=\widetilde{K}(Z)}\right]$$

$$=E_0^{EONIA}\left[\alpha\left(L(\alpha,2\alpha)-\widetilde{K}(Z)\right)\right]$$

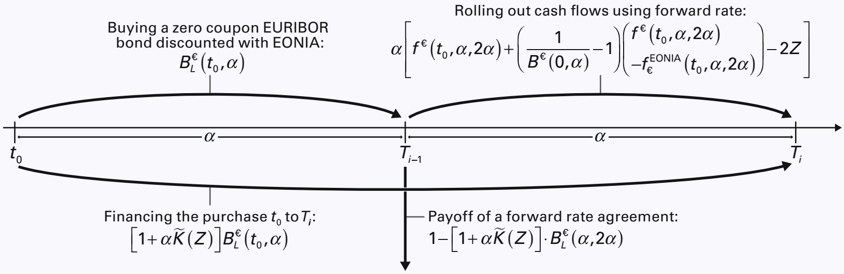
The derivation above shows that a tenor basis swap can be considered as forward rate agreement, whereas the forward rate agreement rate $\widetilde{K}(Z)$ depends on the tenor basis spread Z. Furthermore there is an explicit representation of the forward rate agreement rate:

EQUATION 24: Forward Rate Formula for EUR

$$\widetilde{K}(Z)=f^{€}(t_0,\alpha,2\alpha)+\left(\frac{1}{B^{€}(0,\alpha)}-1\right)\left(f^{€}(t_0,\alpha,2\alpha)-f_{€}^{EONIA}(t_0,\alpha,2\alpha)\right)-2Z$$

The forward rate agreement rate depends on the EURIBOR forward rate $f^{€}(t_0,\alpha,2\alpha)$ of the single-curve model ("forward rate before the financial crisis") and the difference between the EURIBOR forward

FIGURE 88: Replication Strategy of a Forward Rate Agreement in Presence of a 3M/6M Tenor Basis Risk



rate and the EONIA forward rate $f_{€}^{EONIA}(t_0,\alpha,2\alpha)$. The representation of the forward rate agreement rate also reveals the impact of cases in which forwarding and discounting do not coincide. The forwarding is performed according to $\widetilde{K}(Z)$ whereas discount curve is EONIA. *Figure 88* illustrates the corresponding replication strategy in presence of tenor basis risk and EONIA discounting; $B_L^{€}(t_0,\alpha)$ denotes a zero coupon EURIBOR bond discounted with EONIA.

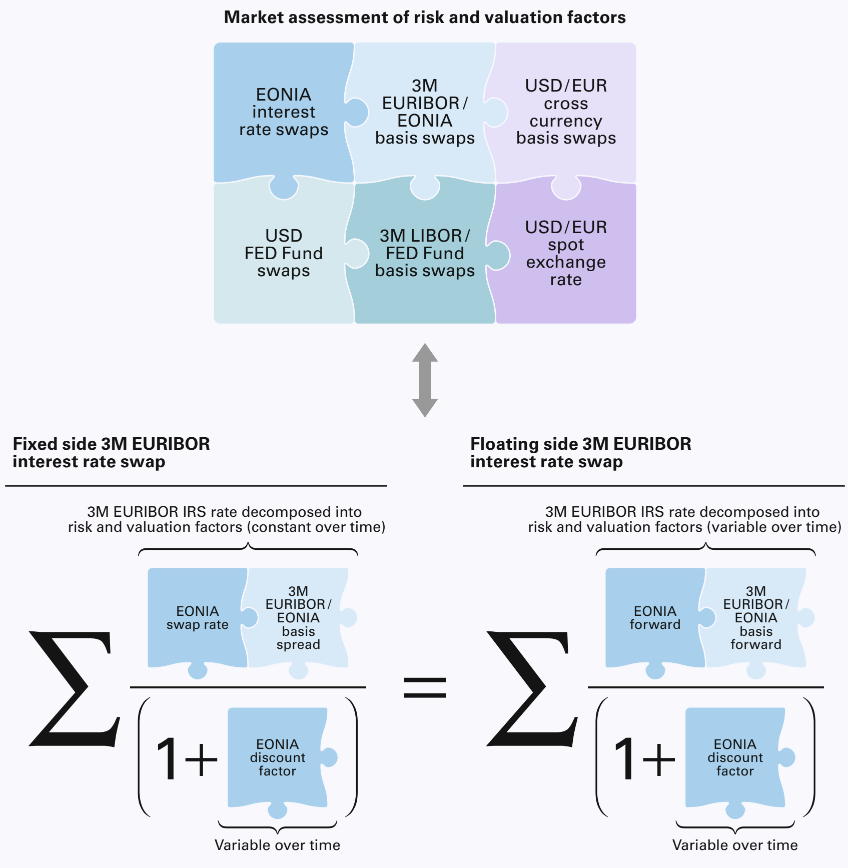
6.8 Hedge Accounting in the Multi-Curve Model Economy for Collateralized Derivatives

6.8.1 The Challenge of Aligning Hedge Accounting according to IAS 39 and Multi-Curve Models

The following section analyzes the impact of multi-curve models which reflect the markets' assessment of risk and valuation factors, to hedge accounting according to IAS 39. The description of the challenge to hedge accounting follows directly from *Figure 82:* How can various risk and valuation factors be incorporated into cash flow and fair value hedge accounting models?

Figure 89 illustrates the challenges for hedge accounting: since the markets' assessment with respect to risk and valuation factors has changed, the valuation of financial instruments has changed arccordingly. As outlined above in *Section 6.7.2,* in a multi-factor model a 3-month EURIBOR interest rate swap (discounted on EONIA) is decomposed into two risk and valuation factors. While the initial present value (fair value) does not change ($PV = 0$ at t_0) its dynamic in time (fair value changes) changes according to the EONIA discount rate and 3-month EURIBOR / EONIA swap spread.

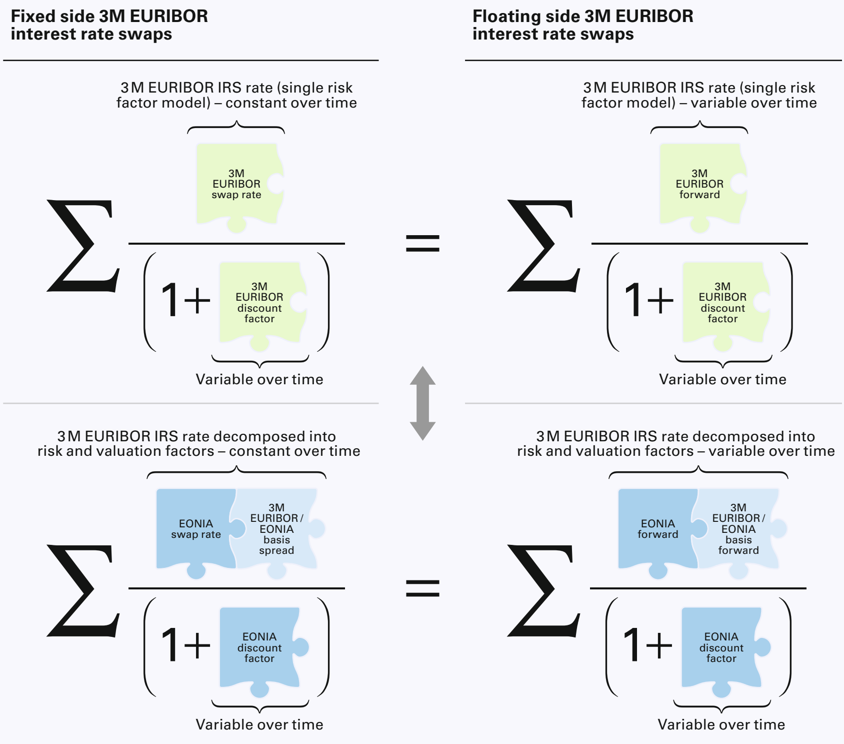
FIGURE 89 **Market Assessment of Risk Factors and Its Impact on Valuation of a 3M EURIBOR Interest Rate Swap**



Note that multi-curve and single-curve models cannot be compared easily, since e.g. in the multi-curve model the 3-month EURIBOR discount curve does not exist, but the 3-month EURIBOR interest rate swap rates do. Using the metaphorical language introduced above, the differences in derivative modeling can be illustrated as shown in *Figure 90* (single-curve model in the upper part, multi-curve model below).

The following section will analyze the impact on hedge accounting and show that both hedge accounting models – cash flow and fair value hedge according to IAS 39 – follow the same economic rationale.

FIGURE 90: Comparison of Single- and Multi-Curve Valuation Models for an Interest Rate Swap (at t_0)

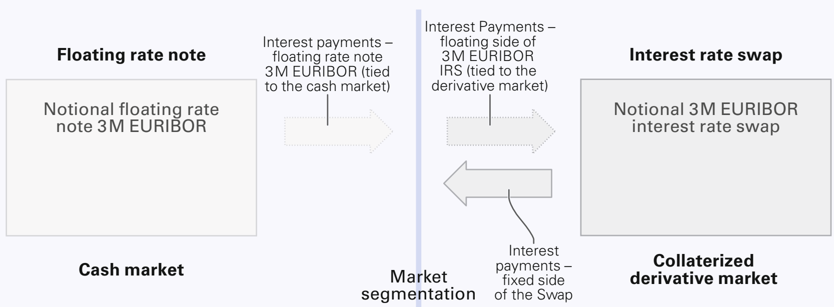


6.8.2 Cash Flow Hedge Accounting

In the following the impact of the re-assessment of financial market risk is illustrated by a commonly applied cash flow hedge accounting model. Let's consider a 3-month EURIBOR floating rate note (cash market) and a 3-month EURIBOR interest rate swap. Since both financial instruments are tied to the 3-month EURIBOR money market rate and therefore the cash flows in both financial instruments change correspondingly to changes in 3-month EURIBOR, it is assumed that the requirements according to IAS 39.86(b) are met ("variability of cash flow criteria") and cash flow hedge accounting can be applied.

It has to be acknowledged that IAS 39 does not provide explanations with respect to the economic reasoning behind this hedge accounting model. As is apparent from financial markets economics and legal contracts, cash and derivative markets are separate markets with different pricings, market participants and market conventions (illustrated by the straight line in *Figure 91*). How can it be justified that the "variability of cash flow criteria" is met and effectiveness testing is performed using the prices of derivative markets?

FIGURE 91: Example of Cash Flow Hedge Accounting according to IAS 39



Summarizing the facts from financial economics:

- Derivative markets and cash markets are distinct markets.
- Comparing (full) fair value changes of a 3-month EURIBOR floating rate note with fair value changes of the floating side of a corresponding 3-month EURIBOR interest rate swap shows low explanatory power. Therefore an empirical relationship between both prices and markets is not supported by analysis of real examples, as *Figure 12* in the executive summary *(Section 2)* shows. There a floating rate note is chosen with a rating comparable to AA^{-}, which corresponds to the credit rating of swap curves.
- If effectiveness testing were performed by comparing (full) fair value changes of the 3-month EURIBOR floating rate note with fair value changes of the floating side of the 3-month EURIBOR interest rate swap, effectiveness would not be achieved and therefore cash flow hedge accounting cannot be applied (see *Figure 12* in *Section 2* or *Figure 36, Figure 37* in *Section 3*).

As indicated in the previous sections, cash flow hedge accounting can be justified if an integrated market for cash and derivative instruments is assumed. Such an integrated market is inevitably linked to the following modeling assumption: Derivative prices are the only relevant source of prices and representatives of risk factors![96]

In *Figure 92* the cash flow profile of a cash flow hedge and the impact of multiple risk factors are shown. The cash flow profile indicates that the cash flow hedge accounting on its own, i.e. excluding funding in general, is not consistent with fair value based sensitivity interest rate risk management of financial institutions since it creates “unhedged” positions (payments of the fixed side of the 3-month EURIBOR interest rate swap). The cash flow hedge accounting model is applied in order to eliminate P & L volatility resulting from the accounting mismatch.

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96 For details on this argument refer also to Schubert, D. (2011).

Figure 92 shows two important features:

- Multiple factor models do not change the cash flows of the financial instruments involved.
- Multiple risk factors affect all relevant cash flows in terms of their risk assessment. Therefore it must be distinguished between the cash flow perspective and the risk/valuation perspective.

FIGURE 92: Transition from Cash Flow to Risk Factor and Valuation Perspective in a Cash Flow Hedge Accounting Model

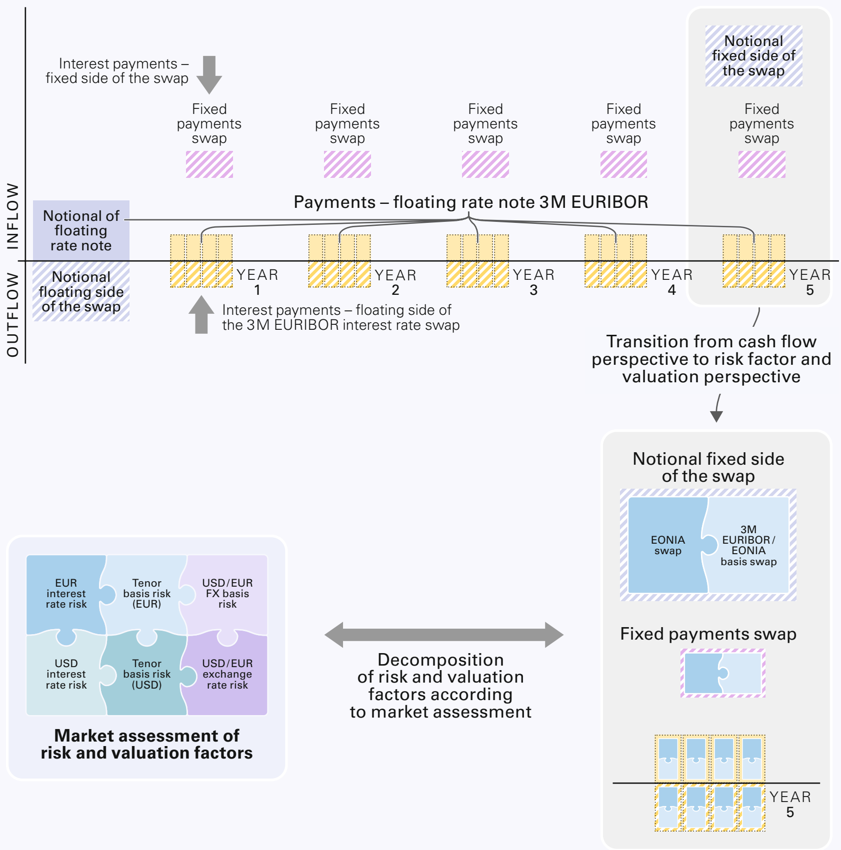


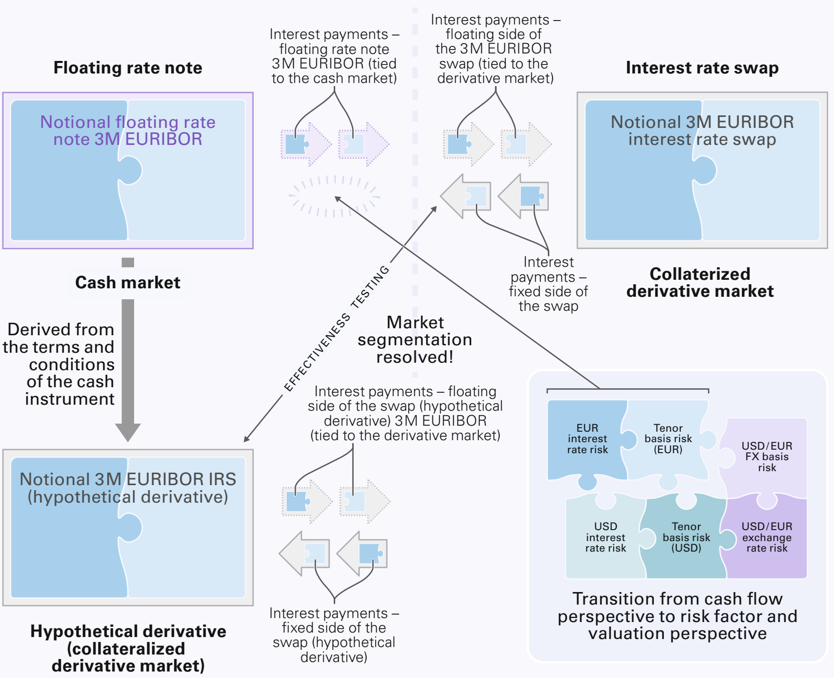
Figure 92 portrays the impact of the markets' assessment of risk and valuation factors on the cash flow profile of a cash flow hedge (a hedge accounting relationship consisting of a 3-month EURIBOR floating instrument and a 3-month EURIBOR interest rate swap discounted on EONIA) according to IAS 39. In this hedge accounting relationship EUR interest rate risk and tenor basis risk are present, since EONIA interest rate swap and 3-month EURIBOR / EONIA basis swap are utilized, taking the derivative market as the relevant source for risk and fair value measurement.

Following the illustration in *Figure 91,* the impact on the markets' assessment of risk factors in the cash flow hedge accounting model also resolves the market segmentation of cash and derivative markets. Additionally, due to the application of the hypothetical derivative method, the impact of the decomposition of derivatives defined by the multi-curve model *(Equation 22)* into its risk factors (refer to *Figure 82* and *Figure 84*) does not affect the effectiveness testing, since the fair value changes of the hypothetical and the real derivative – measured in the same way – are compared.

As described in the previous *Section 6.7,* the role of tenor basis swaps and the decomposition of derivatives into their risk components according to the multi-curve model setup are crucial. The virtual decomposition of (hypothetical) derivatives into risk factors in the cash flow hedge accounting model is generally accepted as a consequence of IAS 39.86(b), 88, KPMG Insights 7.7.630.30, 7.7.630.40, 7.7.630.50 and 7.7.640.10, since e.g. the valuation of the hypothetical derivative according to market convention is required and the decomposition does not affect the results of the effectiveness test. In addition it is noted that from the legal perspective the acceptance of cash collateral posting in a foreign currency according to a CSA under ISDA results in an acceptance of the FX basis, since the cash posted in foreign currency has to be funded and the funding must be taken into account for the determination of the fair value.

The cash flow hedge accounting model according to IAS 39 can be justified by the absence of arbitrage principle using the derivative prices as the only relevant price for cash and derivative markets as well as its corresponding risk decomposition. The impact of multi-curve models is limited: due to the application of the hypothetical derivative method no additional ineffectiveness is expected, provided that the terms and conditions of the cash and derivative instrument (floating side) match to a sufficient high degree.

FIGURE 93: The Integrated Market Model of the Cash Flow Hedge Accounting Model according to IAS 39



6.8.3 Fair Value Hedge Accounting

The economic underpinnings of cash flow and fair value hedge accounting are similar but, due to the structure of the fair value hedge accounting model, the impact of multi-curve models is different.

6.8.3.1 Compliance of Multi-Curve Models with IAS 39

According to IAS 39.AG99F the application for hedge accounting requires the following:

a) The **designated risks** and **portions** must be **separately identifiable** components of the financial instruments
b) Changes in cash flows or the fair value of the entire financial instrument arising from changes in the designated risks and portions must be **reliably measurable**.

These definitions are also included in the Exposure Draft "Hedge Accounting" (ED 2010/13, §18) for the designated "risk component", so the following analysis is assumed to hold for the new rules discussed for IFRS 9. Furthermore IAS 39.AG110 states that "the hedge must relate to a specific and designated risk (hedged risk) […] and must ultimately affect the entity's P & L."

With respect to interest rate risk the conditions above are assumed to be fulfilled for hedge accounting purposes; IAS 39 explicitly permits the designation of the LIBOR or EURIBOR component as a portion of the hedged item (e.g. bond/loan) (IAS 39.81, IAS 39.AG99C) and the utilization of a benchmark curve (e.g. "swap" curve) to determine the fair value of interest rate risk (e.g. discounting future cash flows) (IAS 39.86(a), IAS 39.78, IAS 39.AG82(a), IAS 39.AG102).[97]

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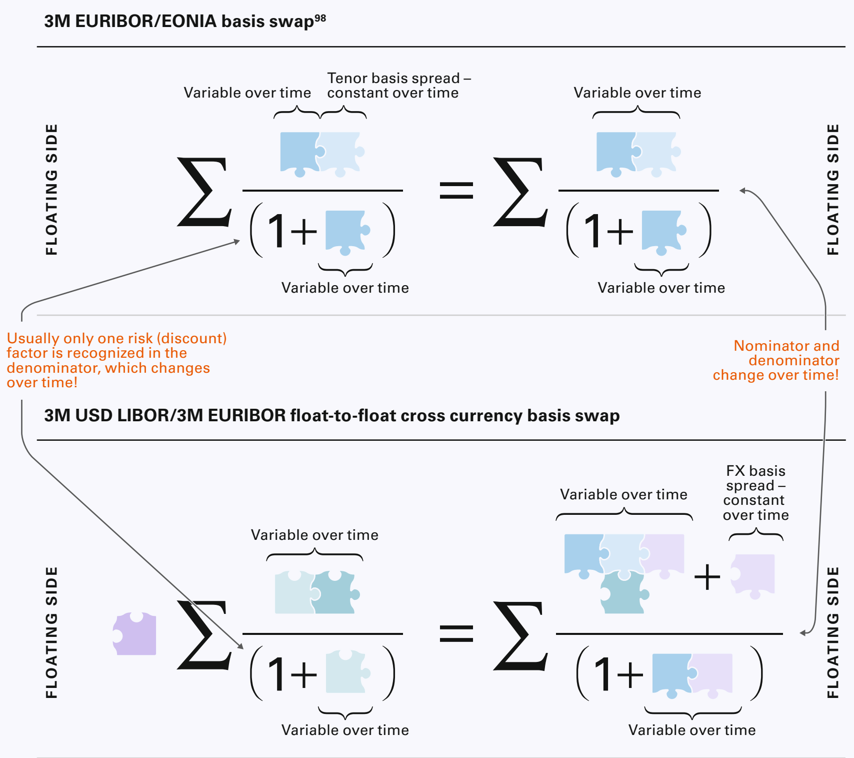
97 According to IAS 39.AG107, IAS 39.AG108, IGF.4.3 the "benchmark curve" is supposed to reflect spreads resulting from counterparty and own credit spreads. This can be achieved also by applying the absence of arbitrage principle. For the sake of simplicity we neglect the issue of "counterparty valuation adjustments".

Within the multi-curve setup EONIA and/or FED Funds serve as "benchmark curves" and are utilized to determine the fair value attributable to interest rate risk. As described above in *Section 6.6,* "interest rate risk" is an unobservable risk and requires the identification of a set of derivatives for its determination and measurement. Within the multi-curve setup EONIA and FED Fund interest rate swaps serve as the representatives of interest rate risk. Measuring fair value changes with respect to interest rate risk utilizing the EONIA or FED Fund benchmark curve ultimately affects the entity's P&L. Similarly to the single-curve model, since the benchmark curve is constructed from liquidly traded instruments (derivatives), also in the multi-curve model the conditions of being **separately identifiable** and **reliably measurable** are met for the portion of the hedged risk. As will be shown below, the **designated risks** and **portions** follow the same rationale as in the single-curve model and are represented by EONIA or FED Funds swap rates.

The role of tenor basis or cross currency basis swaps within multi-curve setups is crucial. These derivative instruments are liquidly traded instruments and, as shown above, are taken into account to measure and evaluate risks in a financial institution (e.g. VaR). Furthermore they also represent components of a performance measurement in trading and treasury departments. These traded instruments measure "differences" in benchmark curves and consequently also meet the conditions – being separately identifiable and reliably measurable – of IAS 39 above; otherwise a logical inconsistency is created. For example: liquidly traded cross currency basis swaps can be considered as the difference between EURIBOR and USD LIBOR benchmark curve. If the cross currency basis swap does not meet the above-mentioned conditions from IAS 39, this also holds true for the EURIBOR and USD LIBOR benchmark curve.

But surprisingly this does not play a role in multi-curve fair hedge accounting: According to its risk factors, a fixed-to-float hedging derivative is virtually decomposed into a fixed-to-float derivative corresponding to the discount curve and instruments incorporating different forwarding and discounting on the floating side (like tenor or cross currency basis swaps). Changes in fair value are measured with respect to the benchmark curve (denominator) and additionally with respect to changes in the cash flows ("variable cash flows") represented in the nominator of the floating sides. So even if the IAS 39 requirements of being separately identifiable and reliably measurable are met, this does not ensure effective hedging relationships due to the "variable cash flows" that are driven by separate risk factors. This is illustrated in *Figure 94*[98], the risk factors "tenor risk" and "FX basis risk" are represented

FIGURE 94: Schematic Representation of a Tenor Basis Swap and a Cross Currency Basis Swap (at t_0) in a Multi-Curve Setup



by float-to-float instruments: a 3-month EURIBOR/EONIA basis swap and a 3-month EURIBOR/3-month USD LIBOR cross currency basis swap. Both derivatives change in value with respect to changes in the nominator and in the denominator. Therefore the construction of a "risk-equivalent bond/loan" representing the hedging costs of the hedged item economically includes "variable" cash flows in order to reflect the "tenor risk" and "FX basis risk" risk factors relative to the hedged risk over the lifetime. Thus the hedged item must in a way incorporate these "variable" cash flows in order to be in line with the markets' assessment of risk and valuation factors, since at all times only one risk factor is represented by the discount curve and recognized in the denominator. All remaining risk factors have their own dynamics which are measured relatively to the discount curve (hedged risk) and have to be recognized in the cash flow of the hedged item. This is admissible as a designation of a portion of cash flow according to IAS 39.81. There is an economic rationale for this property, which is derived in the following *Section 6.8.3.2* where the "risk-equivalent bond/loan" in the multi-curve setup is determined.

6.8.3.2 Constructing the "Risk-Equivalent Bond/Loan" in the Multi-Curve Setup

Constructing the risk-equivalent bond/loan serves to determine the cash flow in the underlying bond or loan subject to economic hedging, which is a common approach in treasury departments.

Let's consider the single-curve case (forwarding = discounting). Let's assume the set of hedging instruments consists of 3-month USD LIBOR interest rate swaps and the benchmark curve is the 3-month USD LIBOR curve. We then consider a bullet bond/loan with maturity T and want to determine the cash flow (internal coupon $c^{3M}_{\text{int},t_0}(t_0,T)$) in the bond/loan subject to the 3-month USD LIBOR interest rate risk. The internal coupon $c^{3M}_{\text{int},t_0}(t_0,T)$ is determined in such a way that the discounted

98 For illustration purposes the US convention for tenor basis swaps as described in section 3.2.1 is used.

coupons including the repayment of the notional, which is assumed to be equal to 1 at time T, equals 1. Then the following equation can be derived:

EQUATION 25: Derivation of the Risk-Equivalent Bond / Loan in a Single-Curve Setup

$$c_{\text{int},t_0}^{3M}\left(t_0,T\right)\cdot A_{\$}^{3M}\left(t_0,T\right)+B_{\$}^{3M}\left(t_0,T\right)\overset{!}{=}1$$

then

$$c_{\text{int},t_0}^{3M}\left(t_0,T\right)=c_{\$}^{3M}\left(t_0,T\right)$$

since

$$c_{\$}^{3M}\left(t_0,T\right)\cdot A_{\$}^{3M}\left(t_0,T\right)=\Lambda_{\$}^{3M}\left(t_0,T\right)=1-B_{\$}^{3M}\left(t_0,T\right)$$

$$\Rightarrow\ c_{\$}^{3M}\left(t_0,T\right)\cdot A_{\$}^{3M}\left(t_0,T\right)+B_{\$}^{3M}\left(t_0,T\right)=1$$

In the single-curve setup the coupon of the risk-equivalent bond/loan is equal to the 3-month USD LIBOR swap rate for maturity T. Similar calculations hold for 3-month EURIBOR hedges, as shown in *Figure 75.* Therefore the 3-month USD LIBOR swap rate represents the **designated risk** and **portion** as well as the **separately identifiable** component of the bond/loan (financial instruments) according to IAS 39.AG99F.[99] Please note that the **designated risk** and **portion** in the single-curve setup is dependent on the discount curve derived from the set of hedging instruments. Consequently the **designated risk** and **portion** for a 3-month USD LIBOR and a 6-month USD LIBOR are different since they are derived from a different set of interest rate swaps, but before the financial crises these differences were considered as insignificant $c_{\$}^{3M}\left(t_0,T\right)\approx c_{\$}^{6M}\left(t_0,T\right)$.

In a multi-curve setup it is more complicated to derive the risk-equivalent bond/loan. Continuing the example from above, assuming a 3-month USD LIBOR interest rate swap (hedging instrument) with FED Funds as a discount curve, the following equation can be derived:

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99 For more details refer to Schubert, D. (2011).

EQUATION 26: Derivation of the Risk-Equivalent Bond / Loan in a Multi-Curve Setup

$$c_\$^{3M}(t_0,T)\cdot A_\$^{FED}(t_0,T)-\Lambda_\$^{3M/FED}(t_0,T)$$

$$=c_\$^{3M}(t_0,T)\cdot A_\$^{FED}(t_0,T)-\underbrace{c_\$^{FED}(t_0,T)\cdot A_\$^{FED}(t_0,T)+c_\$^{FED}(t_0,T)\cdot A_\$^{FED}(t_0,T)}_{=0}$$
$$+\underbrace{\Lambda_\$^{FED}(t_0,T)-\Lambda_\$^{FED}(t_0,T)}_{=0}-\Lambda_\$^{3M/FED}(t_0,T)$$

$$=c_\$^{FED}(t_0,T)\cdot A_\$^{FED}(t_0,T)+c_\$^{3M}(t_0,T)\cdot A_\$^{FED}(t_0,T)-c_\$^{FED}(t_0,T)\cdot A_\$^{FED}(t_0,T)$$
$$-\Lambda_\$^{3M/FED}(t_0,T)+\Lambda_\$^{FED}(t_0,T)-\Lambda_\$^{FED}(t_0,T)$$

$$=c_\$^{FED}(t_0,T)\cdot A_\$^{FED}(t_0,T)$$
$$+\left[\left(c_\$^{3M}(t_0,T)-c_\$^{FED}(t_0,T)\right)\cdot A_\$^{FED}(t_0,T)-\left(\Lambda_\$^{3M/FED}(t_0,T)-\Lambda_\$^{FED}(t_0,T)\right)\right]$$
$$-\left(1-B_\$^{FED}(t_0,T)\right)$$

$$\Rightarrow c_\$^{FED}(t_0,T)\cdot A_\$^{FED}(t_0,T)$$
$$+\underbrace{\left[\left(c_\$^{3M}(t_0,T)-c_\$^{FED}(t_0,T)\right)\cdot A_\$^{FED}(t_0,T)-\left(\Lambda_\$^{3M/FED}(t_0,T)-\Lambda_\$^{FED}(t_0,T)\right)\right]}_{\text{Tenor basis swap}}$$
$$+B_\$^{FED}(t_0,T)=1$$

$$c_\$^{FED}(t_0,T)\cdot A_\$^{FED}(t_0,T)$$
$$-\underbrace{\left[\left(c_\$^{3M}(t_0,T)-c_\$^{FED}(t_0,T)\right)\cdot A_\$^{FED}(t_0,T)-\left(c_\$^{3M}(t_0,T)-c_\$^{FED}(t_0,T)\right)\cdot A_\$^{FED}(t_0,T)\right]}_{\text{Tenor basis swap}}$$
$$+B_\$^{FED}(t_0,T)=1$$

$$c_\$^{FED}(t_0,T)\cdot A_\$^{FED}(t_0,T)$$
$$-\underbrace{\left[\left(c_\$^{3M}(t_0,T)-c_\$^{FED}(t_0,T)\right)-\left(c_\$^{3M}(t_0,T)-c_\$^{FED}(t_0,T)\right)\right]\cdot A_\$^{FED}(t_0,T)}_{=0 \text{ at } t_0}$$
$$+B_\$^{FED}(t_0,T)=1$$

$\Rightarrow$ but time dependent at $t>t_0$

$$\left[\begin{array}{l}c_\$^{FED}(t_0,T)-\\ \underbrace{\left[\left(c_\$^{3M}(t,T)-c_\$^{FED}(t,T)\right)-\left(c_\$^{3M}(t_0,T)-c_\$^{FED}(t_0,T)\right)\right]}_{\text{Current adjustment}}\end{array}\right]\cdot A_\$^{FED}(t,T)+B_\$^{FED}(t,T)$$

According to the equation above the following properties can be derived:

- If the hedging instrument is a 3-month USD LIBOR interest rate swap and therefore forwarding =/ discounting, the **designated risk** and **portion** which is a **separately identifiable** component of a bond/loan is equal to $c_{\$}^{\text{FED}}(t_0,T)$, but not equal to the swap rate $c_{\$}^{\text{3M}}(t_0,T)$ of the hedging instrument!
- In the single-curve model the internal coupon $c_{\$}^{\text{3M}}(t_0,T)$ is constant over time, but in the multi-curve model the internal coupon is time dependent:

$$\left[\begin{array}{l} c_{\$}^{\text{FED}}(t_0,T) \\ -\underbrace{\left[\left(c_{\$}^{\text{3M}}(t,T)-c_{\$}^{\text{FED}}(t,T)\right)-\left(c_{\$}^{\text{3M}}(t_0,T)-c_{\$}^{\text{FED}}(t_0,T)\right)\right]}_{\text{Current adjustment}} \end{array}\right].$$

- The time dependent adjustment is represented by the change in the 3-month USD LIBOR/FED Fund tenor basis swap.
- In the single-curve model the designation of a portion of cash flows (internal coupon – refer to the *Figure 75*) is permitted according to IAS 39.81, IAS 39.AG99C (refer also to KPMG, Insights into IFRS, 8th Edition 2011/12, 7.7.180.10 and 7.7.180.20), provided that the internal coupon does not exceed the contractual coupon (counterexample: sub-EURIBOR bond/loan). In the single-curve model the designated portion of cash flows and the **designated risk** and **portion,** which is a **separately identifiable** component of a bond/loan, coincide – independently of time and equal to $c_{\$}^{\text{3M}}(t_0,T)$.

- In the multi-curve model this is no longer the case: the **designated risk** and **portion (of the designated risk)** which is a **separately identifiable** component of a bond/loan is equal to $c_{\$}^{\text{FED}}(t_0,T)$, but the **portion of cash flows** (according to IAS 39.81) designated at each t is time dependent and equal to:

$$\left[\begin{array}{l} c_{\$}^{\text{FED}}(t_0,T) \\ -\underbrace{\left[\left(c_{\$}^{3\text{M}}(t,T)-c_{\$}^{\text{FED}}(t,T)\right)-\left(c_{\$}^{3\text{M}}(t_0,T)-c_{\$}^{\text{FED}}(t_0,T)\right)\right]}_{\text{Current adjustment}} \end{array}\right].$$

- In the multi-curve setup the 3-month USD LIBOR interest rate swaps are not the only hedging instruments. If a FED Fund interest rate swap is used, similar to the single-curve model, the internal coupon is determined as follows:

$$c_{\text{int},t_0}^{\text{FED}}(t_0,T)\cdot A_{\$}^{\text{FED}}(t_0,T)+B_{\$}^{\text{FED}}(t_0,T)\overset{!}{=}1$$

then

$$c_{\text{int},t_0}^{\text{FED}}(t_0,T)=c_{\$}^{\text{FED}}(t_0,T)$$

since

$$c_{\$}^{\text{FED}}(t_0,T)\cdot A_{\$}^{\text{FED}}(t_0,T)=\Lambda_{\$}^{\text{FED}}(t_0,T)=1-B_{\$}^{\text{FED}}(t_0,T)$$

$$\Rightarrow c_{\$}^{\text{FED}}(t_0,T)\cdot A_{\$}^{\text{FED}}(t_0,T)+B_{\$}^{\text{FED}}(t_0,T)=1.$$

- If the hedging instrument is a FED Funds interest rate swap, then the designated risk and portion of the designated risk, which is a separately identifiable component of a bond/loan, is equal to $c_{\$}^{\text{FED}}(t_0,T)$ and coincides with the portion of designated cash flows.

TABLE 54: **Construction of Risk-Equivalent Bonds / Loans and IAS 39 Requirements**

Single-curve model: forwarding = discounting

Hedging instrument	**3M USD LIBOR IRS**	**6M USD LIBOR IRS**	**USD FED Funds IRS**
Discount curve (benchmark curve)	3M USD LIBOR IRS	6M USD LIBOR IRS	USD FED Funds IRS
Designated risk and portion = separately identifiable component in the hedged item	$c_{\$}^{3M}(t_0,T)$	$c_{\$}^{6M}(t_0,T)$	$c_{\$}^{FED}(t_0,T)$
Designated portion of cash flows of the hedged item = risk-equivalent bond / loan	$c_{\$}^{3M}(t_0,T)$	$c_{\$}^{6M}(t_0,T)$	$c_{\$}^{FED}(t_0,T)$
Reliably measurable (portion of the designated risk)	Traded 3M USD LIBOR IRS	Traded 6M USD LIBOR IRS	Traded FED Funds IRS
Funding model 3M USD LIBOR = floating side of hedging instrument	Yes	No	No

Multi-curve model: forwarding ≠ discounting

Hedging instrument	**USD FED Funds IRS**	**3M USD LIBOR IRS**	**6M USD LIBOR IRS**
Discount curve (benchmark curve)	USD FED Funds IRS	USD FED Funds IRS	USD FED Funds IRS
Designated risk and portion = separately identifiable component in the hedged item	$c_{\$}^{FED}(t_0,T)$	$c_{\$}^{FED}(t_0,T)$	$c_{\$}^{FED}(t_0,T)$
Designated portion of cash flows of the hedged item = risk-equivalent bond / loan	$c_{\$}^{FED}(t_0,T)$	$\left[\begin{array}{l} c_{\$}^{FED}(t_0,T) \\ -\left[\begin{array}{l} \left(\begin{array}{l} c_{\$}^{3M}(t,T) \\ -c_{\$}^{FED}(t,T) \end{array}\right) \\ -\left(\begin{array}{l} c_{\$}^{3M}(t_0,T) \\ -c_{\$}^{FED}(t_0,T) \end{array}\right) \end{array}\right] \end{array}\right]$	$\left[\begin{array}{l} c_{\$}^{FED}(t_0,T) \\ -\left[\begin{array}{l} \left(\begin{array}{l} c_{\$}^{6M}(t,T) \\ -c_{\$}^{FED}(t,T) \end{array}\right) \\ -\left(\begin{array}{l} c_{\$}^{6M}(t_0,T) \\ -c_{\$}^{FED}(t_0,T) \end{array}\right) \end{array}\right] \end{array}\right]$
Reliably measurable (portion of the designated risk)	Traded FED Funds IRS	Traded FED Funds IRS	Traded FED Funds IRS
Funding model 3M USD LIBOR = floating side of hedging instrument	No	Yes	No

- The issue for hedge accounting in a multi-curve setup is the following: if the hedging instrument does not coincide with the discount curve (benchmark curve), then the risk-equivalent bond/loan becomes time dependent.
- The economic perception behind time dependency is that on a fair value basis the internal coupon needs to be adjusted dynamically in order to hedge the FED Funds risk component in the bond/loan with a 3-month USD LIBOR interest rate swap! It is important to note that it is not the contractual cash flows which change in a multi curve model, but the fair value considerations.
- In order to determine the fair value resulting from the hedged risk, the repayment of the bond/loan (entire notional) always enters into the fair value evaluation.
- The risk equivalent bond determined by the hedging costs approach is independent of the funding model.
- As a net result of the hedging relationship an OIS floating rate note corresponding to the discount curve (hedged risk) is obtained representing the corresponding interest rate risk-free instrument.
- Whether single-curve or multi-curve models are applied, the floating side of the interest rate swap not necessarily coincides with the funding model of the financial institutions.
- The analysis above also holds for EURIBOR or EONIA interest rate swaps used for economic hedging, so the results can be derived similarly.

Table 54 summarizes the results from above – for illustration purposes only also 6-month USD LIBOR interest rate swaps are given.

6.8.3.3 Fair Value Interest Rate Hedge Accounting in a Multi-Curve Setup

Continuing the previous example, the impact of multi-curve model can be illustrated by *Figure 95.* The figure shows the impact of the markets' assessment of risk and valuation factors, in the upper part the economic hedging strategy (fair value based) is portrayed. In the lower part the transition from cash flow to risk factor and valuation perspective is shown. Note that the markets' assessment of risk and valuation factors applies to all cash flows involved in the economic hedging relationship.

The determination of the risk-equivalent bond/loan also determines the economic hedging strategy. As discussed above in *Sections 6.5* or *6.6,* the economic hedging strategy is dynamic and consists of a fixed and a variable part (see also *Figure 95*). Like in the single-curve model and in the cash flow hedge accounting model the reliance on the derivative market with respect to risk and valuation factors introduces the "Law of One Price" in the fair value hedge accounting model and therefore resolves the segmentation of cash and derivative markets.

In terms of fair value hedge accounting the (hedge) fair value of the bond/loan (hedged item) can be derived from *Equation 26:*

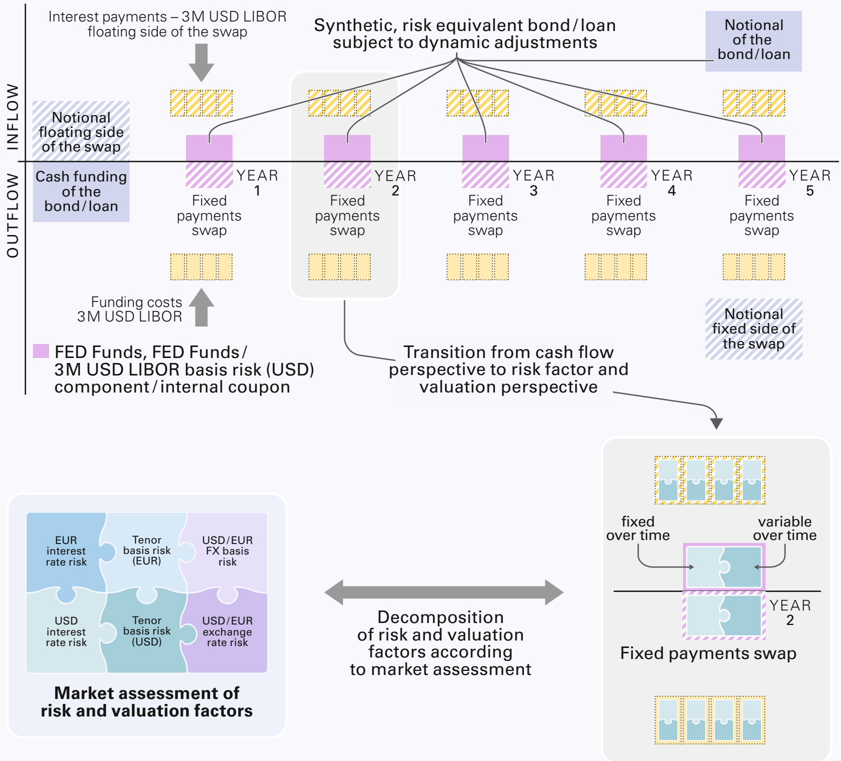
At t_0 the designated portion of cash flow of the bond/loan ($c_{\text{int},t_0}^{\text{3M,OIS}}(t_0,T)$) is:

$$c_{\text{int},t_0}^{\text{3M,OIS}}(t_0,T) = c_{\$}^{\text{FED}}(t_0,T).$$

Please note that at time t_0 the designated portion cash flow of the bond/loan equals the designated portion of risk (FED Funds interest rate risk). Provided the notional of the bond/loan is equal to 1, the hedge fair value $HFV^R(t_1,T)$ (using an analogous notation as in *Section 4.3.3* or *Section 5.3.5*) at time t_1 is determined as follows:

$$HFV^R(t_1,T) = c_{\$}^{\text{FED}}(t_0,T)\,A_{\$}^{\text{FED}}(t_1,T) + B_{\$}^{\text{FED}}(t_1,T).$$

FIGURE 95: Economic Hedging Using a 3M USD LIBOR Interest Rate Swap – the Transition from the Cash Flow Perspective to the Risk Factor and Valuation Perspective in a Multi-Curve Model



At time t_1 designated portion of cash flow in the bond/loan is:

$$c_{\text{int},t_0}^{3M,OIS}(t_1,T)$$
$$= \left[c_{\$}^{FED}(t_0,T) - \underbrace{\left[\left(c_{\$}^{3M}(t_1,T) - c_{\$}^{FED}(t_1,T)\right) - \left(c_{\$}^{3M}(t_0,T) - c_{\$}^{FED}(t_0,T)\right)\right]}_{\text{Current adjustment}} \right].$$

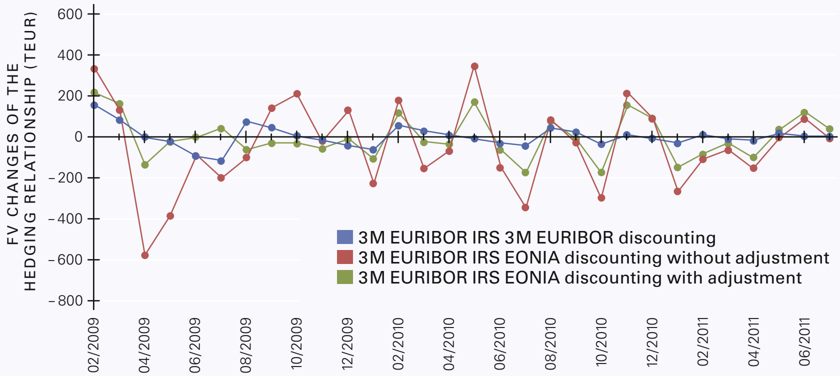
Accordingly at time t_2 the hedge fair value $HFV^R(t_2,T)$ is determined by:

$$HFV^R(t_2,T)$$
$$= \left[c_{\$}^{FED}(t_0,T) - \underbrace{\begin{bmatrix} \left(c_{\$}^{3M}(t_2,T) - c_{\$}^{FED}(t_2,T)\right) \\ -\left(c_{\$}^{3M}(t_0,T) - c_{\$}^{FED}(t_0,T)\right) \end{bmatrix}}_{\text{Current adjustment}} \right] \cdot A_{\$}^{FED}(t_2,T) + B_{\$}^{FED}(t_2,T)$$
$$= c_{\text{int},t_0}^{3M,OIS}(t_2,T) \cdot A_{\$}^{FED}(t_2,T) + B_{\$}^{FED}(t_2,T).$$

For the remaining lifetime of the hedging relationships the hedge fair values are determined similarly to the procedure outlined in *Section 4.3.3* or *Section 5.3.5.* Unfortunately the dynamic adjustment of the designated portion of cash flow in the bond/loan requires re-designating of the hedging relationship according to IAS 39 as well as the amortization of the fair value adjustment in interest income (P&L).

The variable parts subject to the dynamic adjustment of the hedged item can be considered as a dynamic hedging strategy, since in these cases the hedging instrument and the discount curve ("benchmark curve") do not coincide. An important feature of the dynamic hedging strategy is that the strategy is known at inception for the entire lifetime of the hedging relationship. The strategy is defined by changes in the fair value of tenor basis swaps – only the amount is not known. For effectiveness testing this property can be used to simulate future fair value changes of tenor basis swaps in order to prove effectiveness.

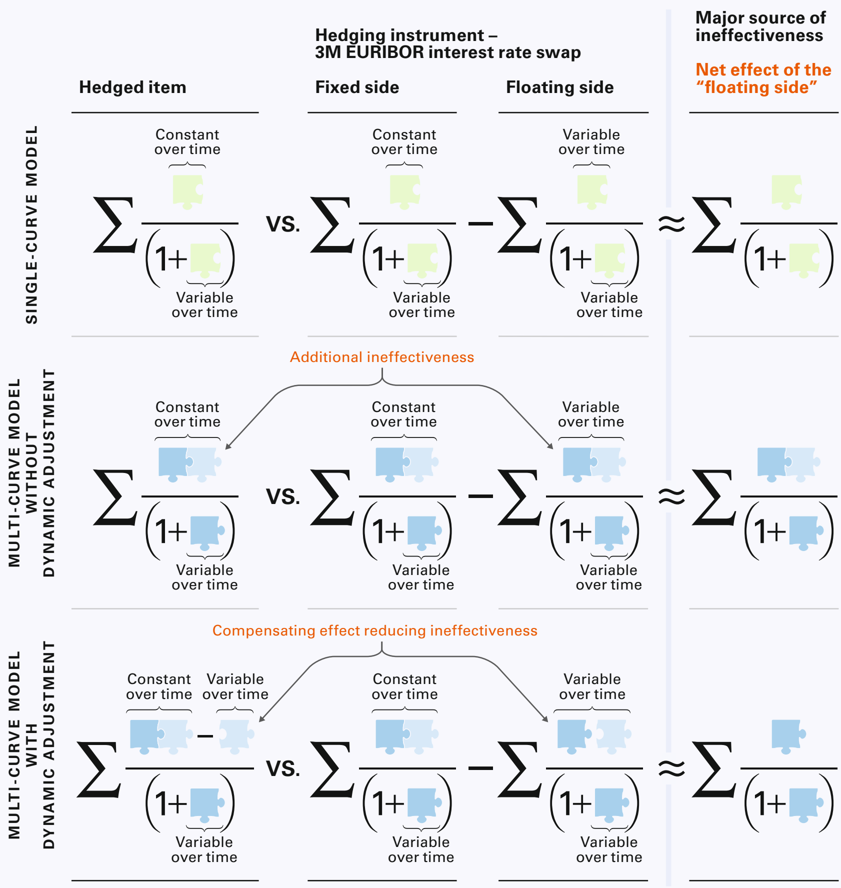
FIGURE 96: **Comparison of Different "Fair Value Hedge Accounting Strategies"**



In *Figure 96* an example of different "hedge accounting" strategies is provided. The figure shows the "net result", i.e. the difference between changes in fair value of the hedged item and the 3-month EURIBOR interest rate swap, of the hedge accounting strategies. *Figure 96* reveals: If the discount curve is changed from 3-month EURIBOR to EONIA without "dynamic adjustment", the ineffectiveness increases (red line). The "dynamic adjustment" approach (green line) reduces the ineffectiveness and the results are closer to the single-curve hedge accounting approach (blue line).

Using the metaphorical language introduced above, the impact of the different hedge accounting strategies can be illustrated as shown in *Figure 97*. In particular it reveals that the net result of the hedging relationship in the single-curve model as well as in the multi-curve model with dynamic adjustment equals a floating rate note corresponding to the discount curve (hedged risk).

FIGURE 97: Comparison of Different "Fair Value Hedge Accounting Strategies" and the Major Source of Ineffectiveness



6.8.3.4 Fair Value Hedge Accounting Involving the FX Basis

The incorporation of the FX basis in case of FX hedge accounting with a fixed-to-float cross currency swap follows a similar rationale as above. The determination of the risk-equivalent bond / loan can be omitted by analyzing directly the reset behavior of the corresponding cross currency basis swap. Using the notation above, the derivation of the hedge accounting model involving the cross currency basis swaps follows from:

$$S_{€}^{\$}(T_1)\cdot N_{\$}\left[c_{\$}^{3M}(t_0,T)\cdot A_{\$}^{FED}(T_1,T)+B_{\$}^{FED}(T_1,T)\right]$$
$$-N_{€}\left[\begin{array}{l}\sum_{j=5}^{4N}\delta(t_{j-1},t_j)\cdot\begin{pmatrix}f_{€}^{3M/EONIA}(t_4,t_{j-1},t_j)\\+b(t_0,T)\end{pmatrix}\cdot B_{€}^{FX/EONIA}(t_4,t_j)\\+B_{€}^{FX/EONIA}(t_4,T)\end{array}\right]$$

$$=S_{€}^{\$}(T_1)\cdot N_{\$}\left[c_{\$}^{3M}(t_0,T)\cdot A_{\$}^{FED}(T_1,T)+B_{\$}^{FED}(T_1,T)\right]$$
$$-N_{€}\left[\begin{array}{l}\sum_{j=5}^{4N}\delta(t_{j-1},t_j)\cdot\begin{pmatrix}f_{€}^{3M/EONIA}(t_4,t_{j-1},t_j)\\+b(t_0,T)\underbrace{-b(t_4,T)+b(t_4,T)}_{=0}\end{pmatrix}\cdot B_{€}^{FX/EONIA}(t_4,t_j)\\+B_{€}^{FX/EONIA}(t_4,T)\end{array}\right]$$

$$=S_{€}^{\$}(t_1)\cdot N_{\$}\left[c_{\$}^{3M}(t_0,T)\cdot A_{\$}^{FED}(T_1,T)+B_{\$}^{FED}(T_1,T)\right]$$
$$-N_{€}\left[\begin{array}{l}\sum_{j=5}^{4N}\delta(t_{j-1},t_j)\cdot\left(f_{€}^{3M/EONIA}(t_4,t_{j-1},t_j)+b(t_4,T)\right)\cdot B_{€}^{FX/EONIA}(t_4,t_j)\\+B_{€}^{FX/EONIA}(t_4,T)\\+\left(b(t_0,T)-b(t_4,T)\right)\cdot\sum_{j=5}^{4N}\delta(t_{j-1},t_j)\cdot B_{€}^{FX/EONIA}(t_4,t_j)\end{array}\right]$$

$$=S_{€}^{\$}(t_1)\cdot N_{\$}\left[c_{\$}^{3M}(t_0,T)\cdot A_{\$}^{FED}(T_1,T)+B_{\$}^{FED}(T_1,T)\right]$$
$$-N_{€}\left[\begin{array}{l}\sum_{j=5}^{4N}\delta(t_{j-1},t_j)\cdot f_{\$}^{3M/FED}(t_4,t_{j-1},t_j)\cdot B_{\$}^{FED}(t_4,t_j)+B_{\$}^{FED}(t_4,T)\\+\left(b(t_0,T)-b(t_4,T)\right)\cdot\sum_{j=5}^{4N}\delta(t_{j-1},t_j)\cdot B_{€}^{FX/EONIA}(t_4,t_j)\end{array}\right]$$

$$=S_{€}^{\$}(t_1)\cdot N_{\$}\left[c_{\$}^{3M}(t_0,T)\cdot A_{\$}^{FED}(T_1,T)+B_{\$}^{FED}(T_1,T)\right]$$

$$-N_{€}\left[\begin{array}{l}\underbrace{\Lambda_{\$}^{FED}(t_4,T)-\Lambda_{\$}^{FED}(t_4,T)}_{=0}+\Lambda_{\$}^{3M/FED}(t_4,T)+B_{\$}^{FED}(t_4,T)\\ +\left(b(t_0,T)-b(t_4,T)\right)\cdot\sum_{j=5}^{4N}\delta(t_{j-1},t_j)\cdot B_{€}^{FX/EONIA}(t_4,t_j)\end{array}\right]$$

$$=S_{€}^{\$}(t_1)\cdot N_{\$}\left[c_{\$}^{3M}(t_0,T)\cdot A_{\$}^{FED}(T_1,T)+B_{\$}^{FED}(T_1,T)\right]$$

$$-N_{€}\left[\begin{array}{l}\left(\Lambda_{\$}^{3M/FED}(t_4,T)-\Lambda_{\$}^{FED}(t_4,T)\right)+\left(1-B_{\$}^{FED}(t_4,T)\right)+B_{\$}^{FED}(t_4,T)\\ +\left(b(t_0,T)-b(t_4,T)\right)\cdot\sum_{j=5}^{4N}\delta(t_{j-1},t_j)\cdot B_{€}^{FX/EONIA}(t_4,t_j)\end{array}\right]$$

$$=S_{€}^{\$}(t_1)\cdot N_{\$}\left[c_{\$}^{3M}(t_0,T)\cdot A_{\$}^{FED}(T_1,T)+B_{\$}^{FED}(T_1,T)\right]-N_{€}$$

$$-N_{€}\left[\begin{array}{l}\left(\Lambda_{\$}^{3M/FED}(t_4,T)-\Lambda_{\$}^{FED}(t_4,T)\right)\\ +\left(b(t_0,T)-b(t_4,T)\right)\cdot\sum_{j=5}^{4N}\delta(t_{j-1},t_j)\cdot B_{€}^{FX/EONIA}(t_4,t_j)\end{array}\right],$$

using $\left(\Lambda_{\$}^{3M/FED}(t,T)-\Lambda_{\$}^{FED}(t,T)\right)=\left(c_{\$}^{3M}(t,T)-c_{\$}^{FED}(t,T)\right)\cdot A_{\$}^{FED}(t,T)$

and $N_{€}=\left[\frac{S_{€}^{\$}(t_1)}{S_{€}^{\$}(t_1)}\right]S_{€}^{\$}(t_0)\cdot N_{\$}$ gives

$$S_{€}^{\$}(t_1)\cdot N_{\$}\left[c_{\$}^{3M}(t_0,T)\cdot A_{\$}^{FED}(t_1,T)+B_{\$}^{FED}(t_1,T)\right]$$

$$-\left[\frac{S_{€}^{\$}(t_1)}{S_{€}^{\$}(t_1)}\right]S_{€}^{\$}(t_0)\cdot N_{\$}\left[\begin{array}{l}\left(c_{\$}^{3M}(T_1,T)-c_{\$}^{FED}(T_1,T)\right)\cdot A_{\$}^{FED}(T_1,T)\\ +\begin{pmatrix}b(t_0,T)\\ -b(t_4,T)\end{pmatrix}\cdot\sum_{j=5}^{4N}\delta(t_{j-1},t_j)\cdot B_{€}^{FX/EONIA}(t_4,t_j)\end{array}\right]-N_{€}$$

$$=S_{€}^{\$}(t_1)\cdot N_{\$}\left[\begin{array}{l}\left[c_{\$}^{3M}(t_0,T)\cdot A_{\$}^{FED}(T_1,T)+B_{\$}^{FED}(T_1,T)\right]\\ -\left[\frac{S_{€}^{\$}(t_0)}{S_{€}^{\$}(t_1)}\right]\left[\begin{array}{l}\left(c_{\$}^{3M}(T_1,T)-c_{\$}^{FED}(T_1,T)\right)\cdot A_{\$}^{FED}(T_1,T)\\ +\begin{pmatrix}b(t_0,T)\\ -b(t_4,T)\end{pmatrix}\cdot\sum_{j=5}^{4N}\delta(t_{j-1},t_j)\cdot B_{€}^{FX/EONIA}(t_4,t_j)\end{array}\right]\end{array}\right]-N_{€}.$$

Accordingly the dynamic hedging strategy with respect to FX fair value hedge accounting can be derived. In case of fair value hedges, the hedged risk includes a simultaneous hedge of FX risk and interest rate risk. In the example the hedged risk due to USD interest rate risk is represented by the USD FED Funds benchmark curve.

At t_0 the cash flow of the hedged item with internal coupon $c_{\text{int},t_0}^{\text{FX,OIS}}(t_0,T)$ subject to hedge accounting is determined as follows, since this is similar to interest rate hedge accounting in USD:

$$c_{\text{int},t_0}^{\text{FX,OIS}}(t_0,T) = c_{\$}^{\text{FED}}(t_0,T).$$

At T_1, when combining the results for USD interest rate hedge accounting and the analysis above, the internal coupon $c_{\text{int},t_0}^{\text{FX,OIS}}(T_1,T)$ is evaluated in the following. Using the result for t_0, the reset value of the floating leg at T_1 above and the identity:

$$c_{\$}^{3M}(t_0,T) = \left[c_{\$}^{3M}(t_0,T) - \underbrace{\left[\left(c_{\$}^{\text{FED}}(t_0,T) - c_{\$}^{\text{FED}}(t_0,T)\right)\right]}_{=0} \right]$$

and inserting this result into the equation above yields

$$c_{\text{int},t_0}^{\text{FX,OIS}}(T_1,T)$$
$$= \left[\begin{array}{l} c_{\$}^{\text{FED}}(t_0,T) - \left[\begin{array}{l} \left[\dfrac{S_{€}^{\$}(t_0)}{S_{€}^{\$}(T_1)}\right]\left(c_{\$}^{3M}(T_1,T) - c_{\$}^{\text{FED}}(T_1,T)\right) \\ -\left(c_{\$}^{3M}(t_0,T) - c_{\$}^{\text{FED}}(t_0,T)\right) \end{array} \right] \\ -\left[\dfrac{S_{€}^{\$}(t_0)}{S_{€}^{\$}(T_1)}\right]\left[\left(b(t_0,T) - b(t_4,T)\right) \dfrac{\sum_{j=5}^{4N} \delta(t_{j-1},t_j) \cdot B_{€}^{\text{FX/EONIA}}(t_4,t_j)}{A_{\$}^{\text{FED}}(T_1,T)} \right] \end{array} \right].$$

For general *t* this equation will take the following form:

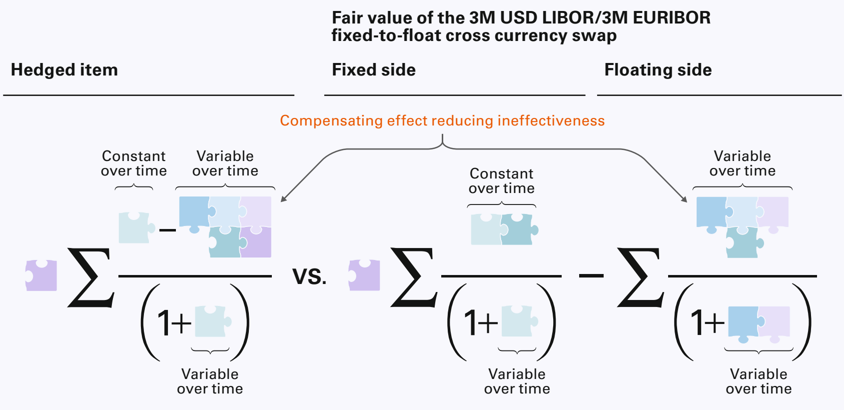
EQUATION 27: Dynamically Adjusted Internal Coupon for the Second Cross Currency Swap Representation (FX Basis as Constant Spread on the Floating Side)

$$c_{int,t_0}^{FX,OIS}(t,T) = \left[\begin{array}{l} c_{\$}^{FED}(t_0,T) - \left[\left[\frac{S_{€}^{\$}(t_0)}{S_{€}^{\$}(t)}\right]\left(c_{\$}^{3M}(t,T) - c_{\$}^{FED}(t,T)\right) - \left(c_{\$}^{3M}(t_0,T) - c_{\$}^{FED}(t_0,T)\right)\right] \\ -\left[\frac{S_{€}^{\$}(t_0)}{S_{€}^{\$}(t)}\right]\left[\left(b(t_0,T) - b(t,T)\right)\frac{\sum_{t_j > t}^{T} \delta(t_{j-1},t_j) \cdot B_{€}^{FX/EONIA}(t,t_j)}{A_{\$}^{FED}(t,T)}\right] \end{array} \right]$$

Please note that in the special case with no FX basis $b(t_0,T) \equiv b(t,T) = 0$ and no FX risk $S_{€}^{\$}(t_0) = S_{€}^{\$}(t) \equiv 1$, the cash flow subject to hedge accounting equals the cash flow in case of USD interest rate hedge accounting.

As in case of interest rate hedge accounting, the dynamic adjustment of the designated portion of cash flow in the bond/loan requires the

FIGURE 98: Illustration of the Fixed-to-Float Cross Currency Swap Hedge Accounting Strategy



re-designation of the hedging relationship according to IAS 39 as well as the amortization of the fair value adjustment in interest income (P&L).

Using the illustration metaphoric above, the hedge accounting model can be stated as shown in *Figure 98.*

When assuming the first representation variant for the CCS as defined in *Section 5.3.4,* the FX basis is taken into account in the fixed coupon of the USD leg (i.e. no fixed spread on the floating EUR leg). Accordingly the internal coupon with dynamic adjustment (portion of cash flows of the hedged item) takes the following form:

EQUATION 28: Dynamically Adjusted Internal Coupon for the First Cross Currency Swap Representation (FX Basis Incorporated in Fixed Rate)

$$c_{\text{int},t_0}^{\text{FX,OIS}}(t,T)$$

$$= \left[\begin{array}{l} c_{\$}^{\text{FED}}(t_0,T) - \left[\left[\dfrac{S_{€}^{\$}(t_0)}{S_{€}^{\$}(t)} \right] \left(c_{\$}^{3\text{M}}(t,T) - c_{\$}^{\text{FED}}(t,T) \right) - \left(c_{\$}^{3\text{M}}(t_0,T) - c_{\$}^{\text{FED}}(t_0,T) \right) \right] \\ - \left[\begin{array}{l} b(t_0,T) \dfrac{\sum_{t_j>t}^{T} \delta(t_{j-1},t_j) \cdot B_{€}^{\text{FX/EONIA}}(t_0,t_j)}{A_{\$}^{\text{FED}}(t_0,T)} \\ -b(t,T) \left[\dfrac{S_{€}^{\$}(t_0)}{S_{€}^{\$}(t)} \right] \dfrac{\sum_{t_j>t}^{T} \delta(t_{j-1},t_j) \cdot B_{€}^{\text{FX/EONIA}}(t,t_j)}{A_{\$}^{\text{FED}}(t,T)} \end{array} \right] \end{array} \right]$$

$$= c_{\$}^{\text{CCIRS}}(t_0,T) - \left[\dfrac{S_{€}^{\$}(t_0)}{S_{€}^{\$}(t)} \right] \left[\left(c_{\$}^{3\text{M}}(t,T) - c_{\$}^{\text{FED}}(t,T) \right) - b(t,T) \dfrac{\sum_{t_j>t}^{T} \delta(t_{j-1},t_j) \cdot B_{€}^{\text{FX/EONIA}}(t,t_j)}{A_{\$}^{\text{FED}}(t,T)} \right]$$

In the case of no tenor, i.e. $c_{\$}^{3\text{M}}(t,T) = c_{\$}^{\text{FED}}(t,T)$ for all t, this coincides with the definition of the internal coupon for the "pure" EURIBOR/LIBOR case as given in *Equation 20,* and in case of no FX basis and a constant exchange rate $S_{€}^{\$}(t) \equiv 1$ it coincides with the USD interest rate hedge.

FIGURE 99: **Effectiveness Test Results (Periodic Dollar Offset) for Cross Currency Swap (First Representation) Hedges without and with Dynamic Adjustment**

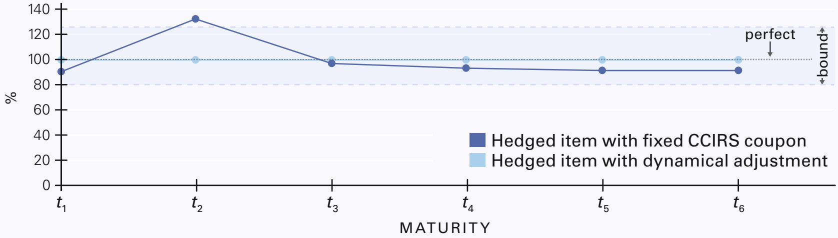


FIGURE 100: **Effectiveness Results (Periodic Dollar Offset) for Cross Currency Swap (Second Representation) Hedges without and with Dynamic Adjustment**

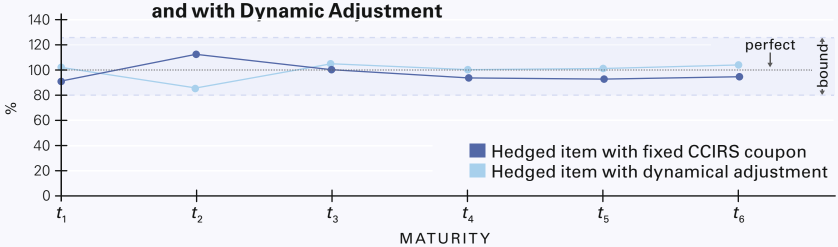
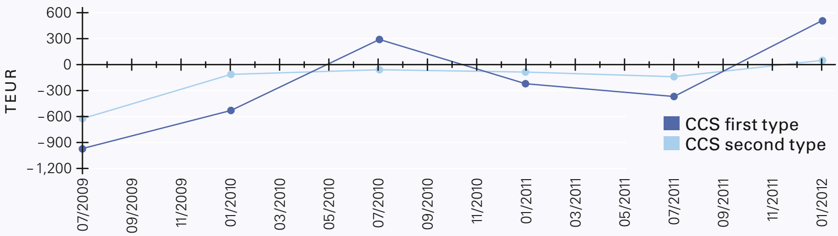


FIGURE 101: **Fair Value Changes of the Floating EUR Leg of Cross Currency Swap of First and Second Type**



The subsequent example follows the hedge relationship described in *Section 5.3.5.1* and illustrated in *Table 40,* applying the multi-curve setup as described in *Section 6.7.* Considering effectiveness tests w.r.t. a semi-annual period (on resets dates), the results (in terms of the periodic dollar offset method) shown in *Figure 99* can be obtained.

As in the "pure" EURIBOR/LIBOR example of *Section 5.3.5.1,* the dynamic adjustment leads to almost perfect effectiveness results despite the high value in t_2 which results from a "small number" effect. Without dynamic adjustment the ineffectiveness increases. For the general case it has to be kept in mind that additional ineffectiveness arises if the effectiveness measurement is not performed on reset days. The determination of booking entries follows similarly as for the examples in section 4 and 5.

Similar improvements of the effectiveness test due to the dynamic adjustment approach can be observed for the second type of CCS definition. In this representation variant the FX basis is taken into account by a constant spread on the floating EUR side (cf. *Section 5.3.4*) and the internal coupon as defined in *Equation 27.*

As before, the increased ineffectiveness in t_2 is mainly driven by a "small number" effect. The improvement in the effectiveness test due to the dynamic adjustment approach has less impact than in the case of the first CCS representation. This may be explained by the consideration of the movements of the floating EUR leg, which are the main factors of the ineffectiveness. In this case the floating EUR leg is less volatile than in the previous case of CCS first representation as shown in *Figure 101.*

For an overview the results of the example are summarized in *Table 55.*

The values in brackets in *Table 55* relate to the range including small numbers effects, the others excluding the effectiveness results of these points in time.

TABLE 55: Summary of Effectiveness Test Results in Case of Different Representations of Cross Currency Swap

	Results of effectiveness testing (periodic dollar offset method) range of results	
Definition of CCS (FX basis recognized on floating side following market convention)	**Without dynamic adjustment**	**With dynamic adjustment**
First representation: fixed coupon of the CCS adjusted to achieve PV = 0 at inception	[91 %, 97 % (133 %)]	[100 %]
Second representation: fixed spread on floating side of the CCS to achieve PV = 0 at inception	[92 %, (114 %) 101 %]	[(87 %) 101 %,105 %]

An analog analysis like the one above using the OIS rate as internal coupon instead of the fixed CCS rate for the unadjusted case leads to similar improvements in effectiveness tests as those shown in *Table 55*. For reasons of brevity the results are not portrayed.

Following the same argumentation as at the end of *Section 5.3.5.1,* a first step to take an MtM feature into account would be reflected in the alteration of the initial spot rate to that of the recent adjustment $S_{€}^{\$}\left(t^{\text{adj}}\right)$, so that the internal coupon takes this form:

$$c_{\text{int},t_0}^{\text{FX,OIS}}(t,T) = \left[\begin{array}{l} c_{\$}^{\text{FED}}(t_0,T) - \left[\begin{array}{l}\left[\dfrac{S_{€}^{\$}\left(t^{\text{adj}}\right)}{S_{€}^{\$}(t)}\right]\left(c_{\$}^{3M}(t,T)-c_{\$}^{\text{FED}}(t,T)\right) \\ -\left(c_{\$}^{3M}(t_0,T)-c_{\$}^{\text{FED}}(t_0,T)\right)\end{array}\right] \\ -\left[\dfrac{S_{€}^{\$}\left(t^{\text{adj}}\right)}{S_{€}^{\$}(t)}\right]\left[\left(b(t_0,T)-b(t,T)\right)\dfrac{\sum_{t_j>t}^{T}\delta\left(t_{j-1},t_j\right)\cdot B_{€}^{\text{FX/EONIA}}\left(t,t_j\right)}{A_{\$}^{\text{FED}}(t,T)}\right]\end{array}\right].$$

As also mentioned in *Section 5.3.5.1,* for a sophisticated derivation the valuation of the MtM feature would have to be taken into account.

Table 56 summarizes the hedge accounting model according to IAS 39 in the multi-curve setup. Similar results can be derived for the EUR case (EONIA).

TABLE 56: Summary of Single- and Multi-Curve Models of Fair Value Hedge Accounting according to IAS 39

Hedged risk (bench-mark curve)	Discount curve	Portion of cash flow designated in the hedged item: Cash flow subject to the hedged risk	Portion of cash flow designated in the hedged item: Dynamic adjustment	Hedging instrument	Type of hedge	Sources of ineffectiveness
			Model setup: single-curve			
3M LIBOR	3M LIBOR swap curve	3M LIBOR interest rate swap rate	None	3M LIBOR interest rate swap	Static	– Floating leg, – maturity mismatches, – incongruities in payment frequencies, – counterparty risk
3M LIBOR USD/EUR FX risk (spot rate)	3M LIBOR swap curve	3M LIBOR interest rate swap rate	None	USD/EUR fixed-to-float cross currency interest rate swap (no FX basis)	Static	Similar to the case above
			Model setup: multi-curve			
FED Funds rate	FED Funds interest rate swap curve	FED Funds interest rate swap rate	None	FED Funds interest rate swap	Static	– Floating leg, – maturity mismatches, – incongruities in payment frequencies, – counterparty risk
FED Funds rate	FED Funds interest rate swap curve	FED Funds interest rate swap rate	Minus changes in 3M LIBOR/FED Funds basis swap	3M LIBOR interest rate swap	Dynamic	Additionally to above: amortizations of the recognized fair value adjustment in interest result due to regular designation/de-designation
FED Funds rate USD/EUR FX risk (spot rate)	FED Funds interest rate swap curve	FED Funds interest rate swap rate	Minus changes in 3M LIBOR/FED Funds basis swap and FX basis	USD/EUR fixed-to-float cross currency interest rate swap (with FX basis)	Dynamic	Similar to the case above

7 Appendix: Details Bootstrapping

Calculation of the next step for the bootstrapping algorithm to calculate "mixed" forward rates:

$$
\begin{aligned}
&c^{6M}(t_0,T_1+6M)\cdot\Delta(t_0,t_0+6M)\cdot B^{3M}(t_0,t_0+6M)\\
&+c^{6M}(t_0,T_1+6M)\cdot\Delta(t_0+6M,T_1+6M)\cdot B^{3M}(t_0,T_1+6M)
\end{aligned}
$$

$$
\left.
\begin{aligned}
&\overset{!}{=}\delta(t_0,t_1)\cdot f^{6M/3M}(t_0,t_0,t_1)\\
&\quad\cdot B^{3M}(t_0,t_1)+\delta(t_1,t_2)\\
&\quad\cdot f^{6M/3M}(t_0,t_1,t_2)\cdot B^{3M}(t_0,t_2)
\end{aligned}
\right\} c^{6M}(t_0,t_2)\cdot\Delta(t_0,t_2)\cdot B^{3M}(t_0,t_2)
$$

$$
\quad+\delta(t_2,t_3)\cdot f^{6M/3M}(t_0,t_2,t_3)\cdot B^{3M}(t_0,t_3)
$$

$$
\begin{aligned}
\Leftrightarrow\ &c^{6M}(t_0,t_3)\cdot\left[\Delta(t_0,t_1)\cdot B^{3M}(t_0,t_1)+\Delta(t_1,t_3)\cdot B^{3M}(t_0,t_3)\right]\\
&\overset{!}{=}c^{6M}(t_0,t_2)\cdot\Delta(t_0,t_2)\cdot B^{3M}(t_0,t_2)\\
&\quad+\delta(t_2,t_3)\cdot f^{6M/3M}(t_0,t_2,t_3)\cdot B^{3M}(t_0,t_3)
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow\ &\delta(t_2,t_3)\cdot f^{6M/3M}(t_0,t_2,t_3)\\
&\overset{!}{=}c^{6M}(t_0,t_3)\cdot\left[\Delta(t_0,t_1)\cdot\frac{B^{3M}(t_0,t_1)}{B^{3M}(t_0,t_3)}+\Delta(t_1,t_3)\right]\\
&\quad-c^{6M}(t_0,t_2)\cdot\Delta(t_0,t_2)\cdot\frac{B^{3M}(t_0,t_2)}{B^{3M}(t_0,t_3)}\\
&=c^{6M}(t_0,t_3)\cdot\left[\Delta(t_0,t_1)\cdot\left(1+\delta(t_1,t_3)\cdot f^{3M}(t_0,t_1,t_3)\right)+\Delta(t_1,t_3)\right]\\
&\quad-c^{6M}(t_0,t_2)\cdot\Delta(t_0,t_2)\cdot\left(1+\delta(t_2,t_3)\cdot f^{3M}(t_0,t_2,t_3)\right).
\end{aligned}
$$

On the other hand the quoted rates can be expressed by the forward rates:

$$c^{6M}(t_0,t_3)\cdot\left[\Delta(t_0,t_1)\cdot\left(1+\delta(t_1,t_3)\cdot f^{3M}(t_0,t_1,t_3)\right)+\Delta(t_1,t_3)\right]$$

$$\stackrel{!}{=}c^{6M}(t_0,t_2)\cdot\Delta(t_0,t_2)\cdot\left(1+\delta(t_2,t_3)\cdot f^{3M}(t_0,t_1,t_3)\right)$$
$$+\delta(t_2,t_3)\cdot f^{6M/3M}(t_0,t_2,t_3)$$

$$\stackrel{!}{=}\delta(t_0,t_1)\cdot r^{6M}(t_0)\frac{B^{3M}(t_0,t_1)}{B^{3M}(t_0,t_3)}$$
$$+\delta(t_1,t_2)\cdot f^{6M/3M}(t_0,t_1,t_2)\cdot\frac{B^{3M}(t_0,t_2)}{B^{3M}(t_0,t_3)}$$
$$+\delta(t_2,t_3)\cdot f^{6M/3M}(t_0,t_2,t_3)$$

$$=\delta(t_0,t_1)\cdot r^{6M}(t_0)\left(1+\delta(t_1,t_3)\cdot f^{3M}(t_0,t_1,t_3)\right)$$
$$+\delta(t_1,t_2)\cdot f^{6M/3M}(t_0,t_1,t_2)\cdot\left(1+\delta(t_2,t_3)\cdot f^{3M}(t_0,t_2,t_3)\right)$$
$$+\delta(t_2,t_3)\cdot f^{6M/3M}(t_0,t_2,t_3).$$

Using the abbreviations introduced in *Section 4.2.6* it can be written:

$$c^{6M}(t_0,t_3)\cdot A^{3M}(t_0,t_3)$$
$$\stackrel{!}{=}c^{6M}(t_0,t_2)\cdot A^{3M}(t_0,t_2)+\delta(t_2,t_3)\cdot f^{6M/3M}(t_0,t_2,t_3)\cdot B^{3M}(t_0,t_3).$$

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