



cutting through complexity

The marketability discount of controlling blocks of shares

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Foreword

KPMG's Global Valuation Institute (GVI) is pleased to introduce its sixth management paper since the launch of our research agenda. Authored by Rui Albuquerque from Boston University and Enrique Schroth from the Cass Business School, this paper sheds new light on the important issue of controlling block valuation and the estimation of marketability discounts.

As an independent think tank, we recognize that valuation is a constantly evolving discipline that has been shaped by practical and theoretical advances. Many high quality research papers on valuation subjects never find their way to influencing the evolution of standards and practice due to a lack of exposure to practitioners.

Our goal is to act as a catalyst for the adoption of breakthrough valuation research. To this end, KPMG's GVI benefits from the expertise of an Academic Advisory Board comprised

of professors from Beijing University in China, Northwestern University in the US and Oxford University in the UK. This Board designs a research agenda and selects and reviews the research we sponsor.

We work closely with researchers to present their managerial papers in a format that is understandable to a broad range of business professionals. This includes illustrative papers with applications and/or case studies. Through this process, we keep KPMG's global network of 1,200 valuation professionals informed of emerging valuation issues.

The use of marketability discounts in valuations is frequently subject to judgment and the experience of the analyst. Empirical analysis conducted at a sector level frequently provides guidance in this process. The authors have provided an approach to marketability in the valuation of a controlling block of shares.

The authors have embraced the complex effect of illiquidity for corporate control to come up with more accurate valuations of controlling blocks of shares.

This paper provides a framework to analyze and rank the determinants of liquidity based on economic and statistical significance. It also provides an approach to estimate the marketability discount on block of shares based on an analysis of liquidity determinants.

This paper is the sixth in a series sponsored by KPMG's Global Valuation Institute. As practitioners, we trust that you will find these of interest.

The views and opinions expressed herein are those of the authors and do not necessarily represent the views and opinions of KPMG International and/or any KPMG member firm.

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Abstract

This article draws on our paper, Albuquerque and Schroth (*forthcoming*), where we study block valuation and estimate the associated marketability discount. Here, we discuss the determinants of the marketability discount and quantify their relative importance. We conclude with an illustration of the applicability of our results to the valuation of a block in a privately held firm by revisiting the case *Mandelbaum et al. v. Commissioner of Internal Revenue* (1995).

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This paper borrows heavily from our paper "The Value of Control and the Costs of Illiquidity," sometimes using complete sentences without acknowledgment. We thank Gonçalo Pacheco Pereira for help with gathering information about the block deals and two anonymous reviewers from KPMG. Albuquerque gratefully acknowledges the financial support from the European Union Seventh Framework Programme (FP7/2007-2013) under grant agreement PCOFUND-GA-2009-246542 and from the Foundation for Science and Technology of Portugal. This paper has been awarded a research grant by the KPMG Global Valuation Institute. The usual disclaimer applies.



1. Introduction

To value a controlling block of shares on the equity of a corporation requires measurement of three main components.¹ First, a controlling block holder may add value by monitoring management or effectively managing the firm. We call this component the *shared security benefits* because they accrue simultaneously to all the shareholders of the corporation. Second, a controlling shareholder may extract *private benefits of control*. We think of these private benefits primarily as the social status and social network access for individual block holders, or as the economic synergies for corporate block holders that come with owning a large equity stake on a corporation. Shared and private benefits give rise to what is called a control or significant influence premium. Third, a large shareholder faces an illiquidity cost associated with the difficulty of selling a large equity stake. Such large stakes are traded through private negotiations and often at a discount with respect to prices at which dispersed shareholders trade shares of the same company in more liquid, public stock markets. These illiquidity costs give rise to what is called a *marketability discount*.

To illustrate the illiquidity costs consider Panasonic's acquisition of 70 percent of Sanyo's equity, on a fully diluted basis, from Mitsui-Sumitomo Bank, Daiwa SMBC and Goldman Sachs

(see Kruse and Suzuki, 2009, for details). On 7 November 2008, Panasonic announced that it was in talks to acquire a majority stake in Sanyo. On that day, Sanyo's share price jumped from JPY145 to JPY204 (the price had been JPY114 on 27 October 2008). Later, on 24 November 2008, Panasonic made an offer of JPY120 per share. The three banks rejected the offer, stating that the price was too low. Goldman Sachs added that the price per share should be at least JPY200 in order to reflect the control premium. Panasonic raised its bid price on 3 December 2008, to JPY130 per share, and the two Japanese banks accepted the offer. Goldman Sachs still rejected it and even suggested that it might purchase the Japanese banks' stakes. But on 16 December 2008, and following the demise of Lehman Brothers, Goldman Sachs announced its first-ever quarterly loss since going public in 1999. Shortly after, on 19 December 2008, Goldman Sachs accepted a price of JPY131, i.e., a discount of almost 10 percent to the pre-announcement price, while Sanyo's shares in the market dropped to JPY136. This example illustrates several important features about illiquidity costs: (i) that finding a buyer that can increase shareholder value following a take over is difficult; (ii) that unexpected events may occur, forcing a block holder to sell immediately, even when the block holder is a large financial institution; and,

(iii) that the block may have to be sold at a discount relative to the share price in the public market following such events.

To the best of our knowledge, there are no estimates of the marketability discount that consider these illiquidity costs.² This glaring lack of estimates is surprising for two reasons. First, the need to price these dimensions of illiquidity is not new. In the famous case of *Mandelbaum et al. v. Commissioner of Internal Revenue* (1995), the court points to the limited evidence on the proper size of the discount on the value of large blocks relative to the value of exchange traded shares. In a job aid for IRS valuation professionals dated 2009, the Internal Revenue Service acknowledges the difficulty in assessing the lack of marketability: "[...] the establishment of a Discount for Lack of Marketability is a factually intensive endeavor that is heavily dependent upon the experience and capability of the valuator." Second, the predominance of high ownership concentration as a form of corporate governance is by now well established. High ownership concentration is a pervasive phenomenon in public corporations in many countries, including the United States;³ it is also, an integral part of privately held corporations where lack of marketability may be particularly severe.

1 The results in this article are based on a sample of sales of blocks of 35 percent or more of the total equity of a corporation on a fully diluted basis. Several studies in the literature use weaker definitions of controlling blocks, requiring as little as 5 percent to 10 percent. Our use of a conservative definition of control follows Albuquerque and Schroth (2010), who show that the pricing of small controlling blocks is complicated by the availability of more alternative trading strategies. However, our estimation method isolates the marketability discount component from the effects of block size, implying that our estimates of the marketability discount are also applicable to smaller controlling blocks.

2 In contrast there is a large literature that provides techniques to value shared security benefits, e.g. discounted cash flow or multiples analyses. Valuing shared security benefits is well understood and is not the subject of this paper.

3 Using a sample of large US corporations, Dlugosz et al. (2006) find that 75 percent of all firm-year observations have block holders that own at least 10 percent of the firms' equity. Using a sample of US firms in 1995, Holderiness (2009) provides evidence that 96 percent of US public firms have block holders and that the average (median) ownership by the largest shareholder is 26 percent (17 percent). More recent and comprehensive evidence of the concentration of ownership in corporate America is unavailable. See Morck (2007) for extensive evidence outside the United States.



The goal of this paper is to show how to obtain an estimate of the marketability discount and to apply this estimate to the valuation of a controlling block of shares. To do so, we identify some of the main cross-sectional and time-series determinants of the marketability discount for publicly traded firms. We then illustrate how to apply a marketability discount adjustment to controlling blocks of both publicly traded firms and privately held firms, using as an example the Mandelbaum valuation case.⁴

To identify its *determinants*, we use the marketability discount estimates in Albuquerque and Schroth (forthcoming, henceforth AS). AS provide a model of the trading and pricing of a controlling block of shares and are able to identify the three components of block value, i.e.,

shared security benefits, private benefits of control and marketability discount, from data on the block premium and abnormal returns in a cross-section of block trades between 1990 and 2010. The main focus of this paper is on the measurement of the marketability discount on a case-by-case basis. We do not discuss here the measurement of the other two components of block value because this step requires the use of standard equity valuation techniques, i.e., for the value of shared security benefits, or the imputation of the idiosyncratic private benefits of the block holder. However, a valuer should assess whether such factors are reflected in a valuation of a large stake in a company. These two additional valuation effects, in contrast with the marketability discount, are generally viewed to

contribute to higher valuations and they must be considered in connection with the marketability discount when determining the total value of a large or controlling interest.

Section 2 presents briefly the traditional approaches to measure the illiquidity of shares, underscoring their inapplicability to the case of controlling blocks. Section 3 reviews the pricing model in AS, providing an intuitive explanation of how the model is able to decompose observed block prices into the three components of block valuation. Section 4 presents the actual estimates of the marketability discount in AS and describes the data set used to identify their cross-sectional and time-series determinants. Section 5 presents our results and shows how to use them with a specific application to the Mandelbaum case. Section 6 concludes.

⁴ We believe that our valuation framework has several applications to pricing, e.g., for tax purposes or asset sales, among others, but we also acknowledge that it is of limited use for financial reporting. For the purpose of comparables reporting, FAS 157 and IFRS 13 prohibit the use of block discounts in financial reporting when individual shares of the company are traded in an active market. Also, fair value may require longer marketing periods that may not be consistent with the need to absorb a sudden liquidity shock.

2. Literature review

2.1 Measures of the value of control

There is a vast literature that tries to measure the value of control. One approach is to use the voting premium, measured directly as the difference in prices across shares with different voting rights (e.g., Masulis et al., 2009), or indirectly, as deviations from put-call parity (Kalay et al., 2011) and equity-loan values (Christoffersen et al., 2007). This approach features two main drawbacks. First, by studying prices *per share*, it measures the *marginal* value of control. Instead, we are interested in the *total* value of control, which is the aim of any valuation analysis of controlling blocks. Second, these measures rely on prices of exchange traded shares and,

therefore, cannot be used to estimate the marketability discount in block shares. Finally, these measures only capture the net value of control, i.e., the difference between the benefits and the costs of control, without isolating each component. The method we describe below uses a direct measure of the liquidity costs arising from the lack of marketability of a block, which are of first order importance for companies with controlling blocks.

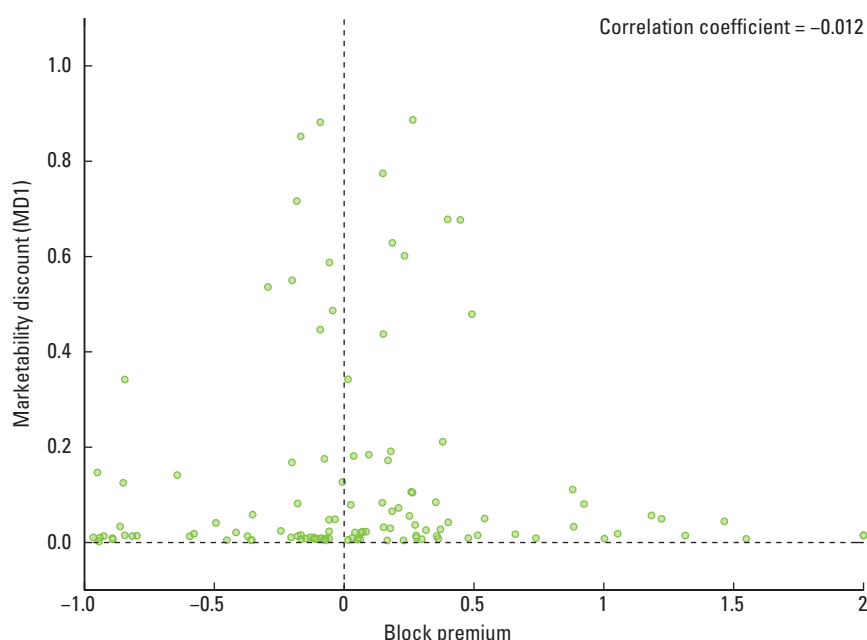
Another strand of the literature, which started with Barclay and Holderness (1989), measures the value of control via the block premium, i.e., the difference between the negotiated price per share in the block and the price of the exchange traded shares. As we argue

in our prior work (Albuquerque and Schroth, 2010), the block premium summarizes different dimensions of benefits and costs of control. Therefore, while informative, the block premium is neither a direct measure of the benefits nor of the costs of control. Not surprisingly, we find a very low unconditional correlation between the block premium and the marketability discount measure we use here (see Figure 1).

2.2 Other valuation approaches to illiquid assets

There is a related literature that studies the pricing of illiquid assets (see Longstaff, 1995, Abudy and Benninga, 2011, and the surveys by Amihud et al., 2005, and Damadoran, 2005). This literature measures the marketability discount associated with stocks with trading restrictions, such as the vesting of stock grants. Our analysis differs from these papers in two critical ways. First, we study the pricing of shares *belonging* to large blocks, which differs from the pricing of a single share because of the control premium, among other things. Second, we consider a different reason for illiquidity: the intrinsic scarcity of controlling stakes for specific firms. Indeed, we consider illiquidity costs that arise because block holders bear the risk of selling in unexpectedly low demand periods and due to unexpected personal liquidity shortages. These frictions are more relevant than trading restrictions imposed, say, on management or employees.

Figure 1: Scatter plot of the Block Premium against the Marketability Discount



3. The Albuquerque and Schroth model

In this section, we review the AS block valuation study. Instead of providing the full mathematical formulation of the model, we provide an intuitive description of how the model is used to identify, or estimate from the data the various components of block value.

The AS model specifies the block value as a function of three components. First, the model assumes that a controlling block holder affects the firm's cash flow (e.g., Holderness and Sheehan, 1988; Barclay and Holderness, 1989; and Pérez-González, 2004) and is therefore able to add value to all shareholders. Second, the model allows for pecuniary and non-pecuniary private benefits of control, as long as these are not diverted from the firm's cash flow.⁵ These are assumed to be positively correlated with the asset value generated by the controlling shareholder and do not lead to an additional trading motive. Finally, the model allows for illiquidity costs and, therefore, a marketability discount. In brief:

Value of block = Present value of cash flows under block holder + Present value of private benefits to block holder – Illiquidity costs

AS show that their model can identify these three components by using the valuations of two different types of shareholders during a block trade: the block holders' valuation implicit in the negotiated block price and the dispersed shareholders' valuation revealed in the exchange share price. This information

is summarized respectively in the *block premium* paid by the acquirer, i.e., the price paid per share in the block normalized by the exchange-traded-share price prior to the trade announcement, and in the cumulative (abnormal) *announcement return* on the exchange-traded shares, i.e., the price change on the exchange-traded shares from before to after the trade announcement.

3.1 Changes in security benefits

The model uses the announcement return on the exchange-traded shares to identify the change in security benefits from one block holder (the seller) to another (the buyer). A positive return implies that the dispersed shareholders perceive the new block holder to be better at running the firm. Vice-versa, a negative return implies that the dispersed shareholders perceive the new block holder to be worse at running the firm. There are two reasons why a trade causing a negative announcement return may occur. First, the block buyer may derive sufficient private benefits despite reducing cash flow, thus paying a high price for the block and enticing the current block holder to sell. Second, the block seller may have been hit by a liquidity shock that forces the sale. We address these two pricing effects below.

3.2 Private benefits

Trades that are accompanied by low or negative announcement returns but have a positive block premium are

likely the result of buyers with high private benefits. In addition, private benefits are important also to explain deals where the announcement return is positive, especially if the block price is significantly higher than the exchange-traded share price after the trade announcement. This is because the price response in the stock market already incorporates the gains from the increase in shared security benefits. Therefore, a block price that is even higher is likely to reflect private benefits (Barclay and Holderness (1989)).

3.3 Illiquidity costs

When both the announcement return and the block premium are negative, the model infers that those deals were caused by liquidity shocks. However, positive announcement returns may also follow trades caused by liquidity shocks. Such would be the case if the seller meets a high-value buyer, i.e., a future block-holder who provides liquidity and increases cash flow. Because the buyer pays a fire sale price to the block, the block premium is low or negative.

The model is able also to infer illiquidity costs from trades that were not caused by a liquidity shock. In the absence of a liquidity shock, the block changes hands only if the available potential block-holder can increase the cash flow. In this case, block and share prices differ partly because liquidity shocks penalize block holders more than dispersed shareholders, who are unaffected by the lower expected fire sale price in a future sale.⁶

⁵ Private benefits derived by 'tunneling' the firm's cash flow have distortionary effects on value that are not covered by AS. However, Albuquerque and Schroth (2010) show that, due to the incentive alignment effect of large stakes, this form of private benefits extraction is unlikely to have any effects on the value of blocks consisting of more than 35 percent of shares.

⁶ In reality, not all liquidity shocks lead to fire sales because the block holder's reservation value may exceed the block's fire sale value. The reservation value, which summarizes the value of all the alternatives to a fire sale, would include, for example, the use of the block as collateral on a loan or the sale of a non-controlling share of the block itself. The AS data include trades where the block holder chooses to sell the entire block over the potential alternatives. AS show that these considerations result in estimates of the marketability discount that are a lower bound to the true values. Hence, our approach, gives a conservative estimate of these discounts.

4. Data

Data for the marketability discount is from AS. Data on the determinants of the marketability discount come from three sources: the Board of Governors of the US Federal Reserve for characteristics of the aggregate economy, and CRSP and Compustat for industry and firm-level data.

4.1 Data on marketability discounts

AS estimate the marketability discount using data on block trades (114 US disclosed-value trades of blocks consisting of at least 35 percent but not more than 90 percent of the stock of a company) from Thomson One Banker Acquisitions. From their analysis, we construct two measures of the marketability discount, *MD1* and *MD2*. *MD1* and *MD2* have both a time stamp and a firm identifier. The time stamp on *MD1* and *MD2* is the date of the block trade that was used to produce the estimates. The firm identifier in *MD1* and *MD2* refers to the target firm whose shares were traded.

MD1 is the marketability discount in percentage terms that can be applied

to the value of a firm derived from a standard discounted cash flow analysis, or any similar valuation method, which are known to omit the illiquidity of the controlling block. *MD1* is the appropriate discount in the case of shares of privately held corporations where a share price is unobservable. *MD2* is the marketability discount in percentage terms that can be applied to the price of the shares traded in an active market. *MD2* is appropriate for the case of public corporations. Therefore, the total value of the block under *MD1* is:

$$\begin{aligned} \text{Value of block} = & \text{Present value of cash} \\ & \text{flows under block holder} \\ & \times (1 - MD1) \\ & + \text{Present value of private} \\ & \text{benefits to block holder} \end{aligned}$$

and under *MD2* is:

$$\begin{aligned} \text{Value of block} = & \text{Present value of cash} \\ & \text{flows under block holder} \\ & + \text{Present value of private} \\ & \text{benefits to block holder} \\ & - MD1 \times \text{Exchange share} \\ & \text{price} \end{aligned}$$

The top panel of Figure 2 contains a scatter plot of *MD1* and *MD2* over time. From the figure, we note two main patterns. First, note that *MD1* and *MD2* are very similar for almost all trades. Second, there are periods of systematically high values of the marketability discount. These periods may be captured by economy-wide variables that are associated with liquidity drying up. In fact, in the bottom panel of Figure 2, we plot the yield curve slope, measured by the difference in the 10-year rate and the 3-month Treasury bills rate, over the same time period. The periods of high marketability discount appear to correlate well with the periods where the term structure of interest rates had a negative or low slope. Below, we show that this is true for other economy-wide variables as well. Third, there appears to be a role for industry and firm-level variables as determinants of liquidity, since even during these periods there is considerable variation in *MD1* and *MD2*.



Figure 2: Marketability discounts and yield spreads over time

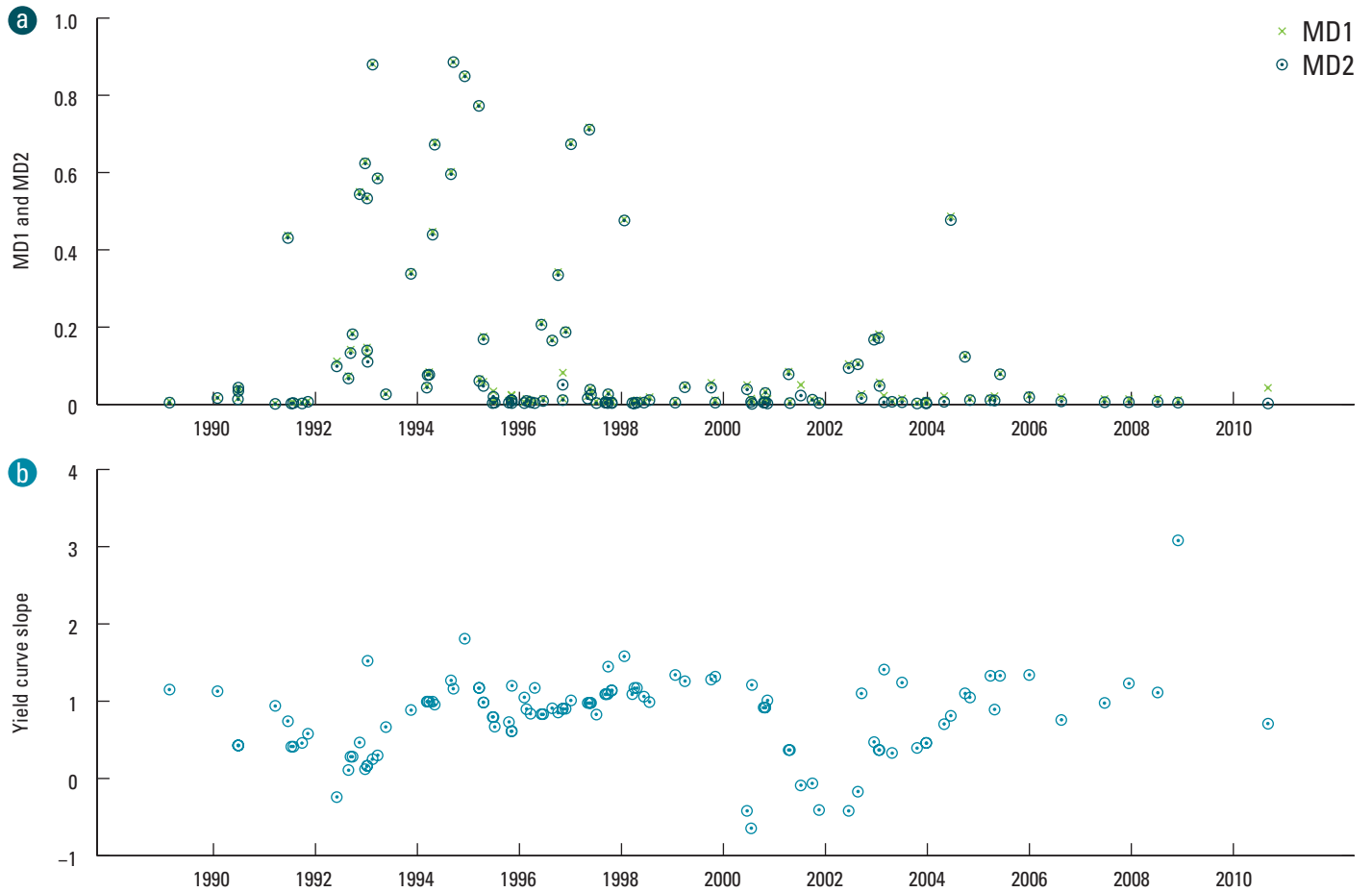


Figure 3 contains histograms of *MD1* and *MD2*. On average *MD1* is 13.15 percent and *MD2* is 12.50 percent and the respective minimum values are 0.2 percent and 0.1 percent. It is interesting to note that marketability discounts are low for a very large fraction of the observations. One way to interpret our findings is that liquidity shocks are rare; in a way they are outlier events. However, it is worth noting that Figure 3 shows the marketability discount estimates only

for the sampled firm-years. Moreover, as we discuss next, the marketability discount varies significantly depending on the external market conditions surrounding the trade. Therefore, the discount on any given firm may increase to very high levels in periods of low market liquidity. Understanding the role and significance of these external conditions, or determinants of the marketability discount, is the main focus of this article.

4.2 Determinants of the marketability discount

We classify the determinants of the marketability discount into two groups: economy-wide determinants and industry and firm-level determinants. These variables are measured with a lag relative to the respective marketability discounts. Table 1 describes these determinants in detail.

Figure 3: Distribution of marketability discounts

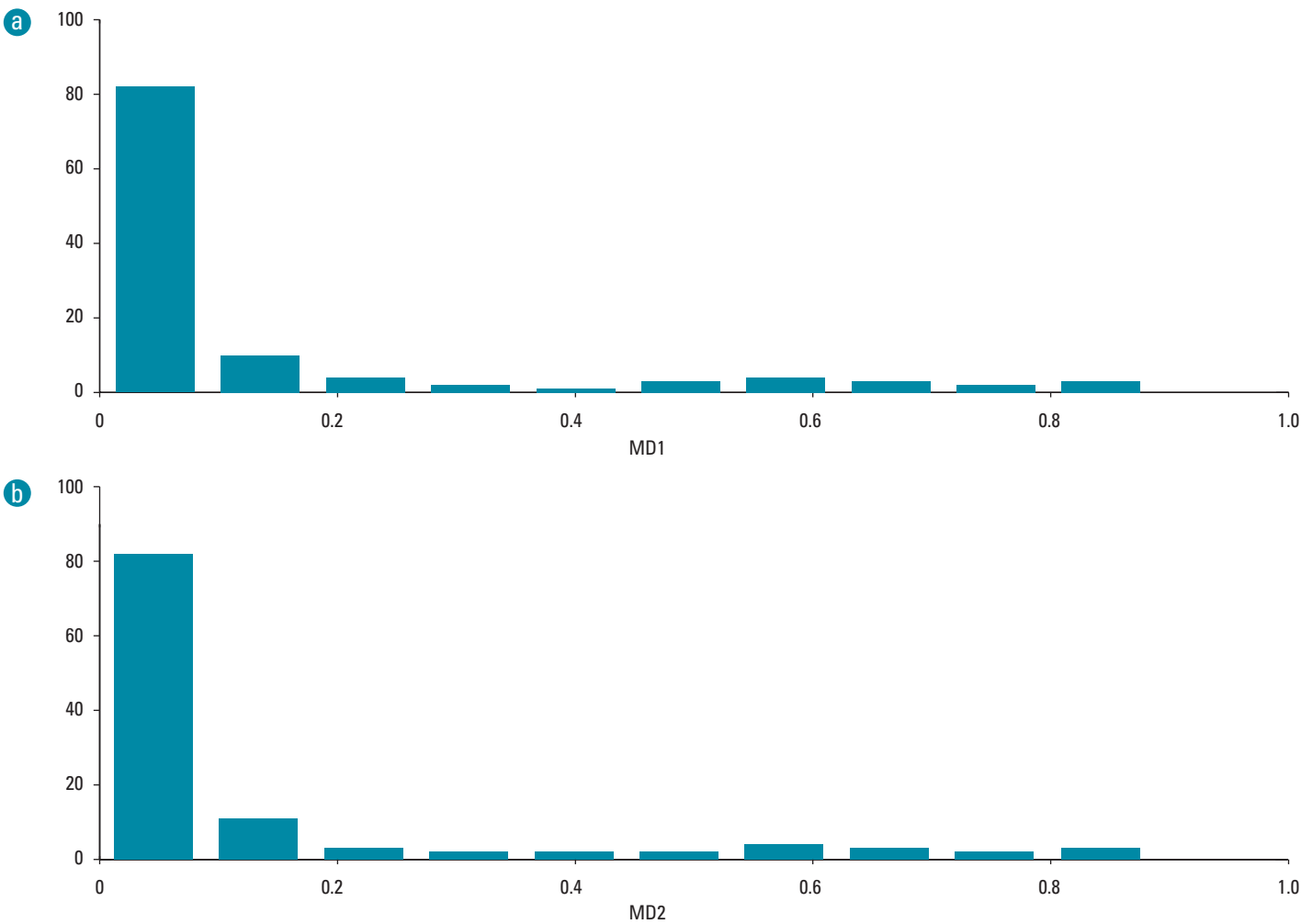


Table 1: Description of determinants of the marketability discount

Economy-wide variables	
Variable	Description
Market return	Annualized average daily returns on the equally-weighted portfolio of all NYSE, AMEX and NASDAQ stocks for the last month before the deal (%).
GDP growth	Average annual growth rate in US GDP per capita in the last quarter prior to the trade (%).
Market volatility	Standard deviation of the annualized daily returns on the equally-weighted portfolio of all NYSE, AMEX and NASDAQ stocks for the 12 month-period before the deal (%).
Yield curve slope	Difference between the interest rate on the 10 year and the three-month Treasury bill (%).
Fontaine-Garcia index	Fontaine and Garcia's (2011) monthly index of the value of funding liquidity: the higher the index, the lower the bond market liquidity.
Industry- and firm-specific variables	
Variable	Description
Industry's M&A activity	Total M&A activity during the last quarter before the deal, where the target is in the same two-digit SIC Code as the deal's target (\$ billions).
Target minus industry leverage	Difference between the proportion of total debt to total assets on the last fiscal year before the trade announcement between the target firm and the median of all firms in the same three-digit SIC code.
Target volatility	Standard deviation of the target's annualized average daily return for the 12 month-period ending two months before the trade announcement (%).
Industry's asset specificity	Median proportion of machinery and equipment to total assets of all firms in the same three-digit SIC code as the target firm, as defined by Stromberg (2001).
Industry's market-to-book ratio	Median ratio of the market value of the firm (book value of debt + market value of equity) to the book value of total assets of all firms in the same three-digit SIC code as the target firm.
Block-to-Industry Size	Ratio of the total controlling block value to the total market capitalization of all NYSE and AMEX firms in the same two-digit SIC Code as the target.

We include characteristics of the aggregate economy that may produce shocks to the block holder's liquidity and thus changes in the marketability discount. We expect these shocks to occur in times of tighter aggregate funding liquidity. Our proxy for funding liquidity is the bond liquidity premium index in Fontaine and Garcia (2012) (*Fontaine-Garcia index*). Fontaine and

Garcia (2012) identify a monthly liquidity factor from the yield spread between US Treasury bills with the same cash flows but different ages. They interpret the higher yields on otherwise identical older Treasury bills as a premium on the liquidity of on-the-run bonds. We hypothesize that *Fontaine-Garcia index* is positively associated with the marketability discount.⁷

⁷ We also considered as candidate proxies for funding liquidity the TED spread, i.e., the spread between the 3-month dollar LIBOR rate and the 3-month Treasury bill, and the Pástor and Stambaugh (2003) stock market liquidity factor. These variables produce similar results.

We include also the growth of US GDP per capita (*GDP growth*) and the average daily return on the equally-weighted portfolio of all NYSE, AMEX, and NASDAQ stocks (*Market Return*). The inclusion of these business cycle proxies is meant to capture two opposing effects: during expansions, (i) investors have stronger balance sheets and are less likely to face liquidity shocks, and, (ii) better alternative investment opportunities may generate a demand for cash.⁸ We try to separate these opposing effects by combining the business cycle variables with variables related to aggregate funding costs. We argue that having a better alternative investment opportunity would only force a block holder to sell if at the same time the cost of borrowing is high. The proxy for the cost of funding used is the slope of the yield curve (*Yield curve slope*). We expect high *GDP growth* and high *Market return* to have a negative direct effect on the marketability discount, but a positive effect via their interaction with the *Yield curve slope*.

We also include in the determinants of the marketability discount the standard deviation of the market return (*Market Volatility*). Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009) show that liquidity providers face tighter funding constraints when market returns are low and volatility is high and thereby diminish their role as liquidity providers (see also Chordia et al., 2002). We therefore predict the marketability discount to decrease with *Market Return* and to increase with *Market Volatility*.

We consider several firm and industry determinants of the marketability

discount. Williamson (1988) argues that asset liquidation values should be closely related to the asset's redeployability. Shleifer and Vishny (1992) add that, because distressed assets tend to be put to the best use by liquidating them within the same industry, redeployability is a function of the industry's capacity to absorb them. We adopt these ideas about the liquidity of physical assets to the financial asset under consideration. As a proxy for the 'financial redeployability' of the block, we use the ratio of the block value to the total market capitalization of all firms in the same two-digit SIC group (*Block-to-Industry Size*) (see Gavazza, 2010). Based on this interpretation, we expect the marketability discount to increase with the relative size of the block. However, if block holders have a preference for relatively larger blocks to exert fuller control, then the marketability discount should vary negatively with *Block-to-Industry Size*.

Another proxy for the redeployability of the block is obtained from the industry's asset specificity. We follow Stromberg (2001) and measure *Industry's asset specificity* with the median proportion of machinery and equipment to total assets of all firms in the industry (non-industry specific assets include land, commercial real estate and cash). As with *Block-to-Industry Size*, *Industry's asset specificity* measures the redeployability of the physical assets rather than of the controlling block. We view *Industry's asset specificity* as a proxy for the amount of industry-specific knowledge required by the controlling block holder and expect more potential buyers of controlling stakes in firms that use generic productive assets. Hence, we predict a positive association

between *Industry's asset specificity* and the marketability discount.

We include the total dollar volume of M&A activity involving targets in the same two-digit SIC group (*Industry's M&A activity*). High *Industry's M&A Activity* could be the result of an increased supply of industry-specific assets, which would increase the marketability discount. High *Industry's M&A Activity* could also reflect high liquidity for industry-specific assets as in Schlingemann et al. (2002) and Ortiz-Molina and Phillips (2011) and, therefore, decrease the marketability discount.

We let the marketability discount vary with the target's leverage relative to its industry's median leverage. We define *Target minus Industry Leverage* as the difference between the target's proportion of long-term debt to assets and the median proportion of long-term debt to assets of all firms in the same three-digit SIC code. We expect that block holders' price in a larger marketability discount for firms with more long-term debt as they are more constrained to borrow to fund any restructuring activities. To control for the time-series variation in investment opportunities in the same industry, we include the median ratio of the market-to-book value of assets of all firms in the same three-digit SIC code. Finally, we control for the possibility that fire sale prices are affected by the target firm's return volatility.

Ultimately, other data on the motivations for a trade may be relevant in individual cases. But a lack of such data on a systematic basis on other deals prevents us from pursuing a proper statistical analysis of their relevance.

⁸ Bates (2005) and Hovakimian and Titman (2006) show that firms that sell assets keep more of the money from the asset sale if they have more growth opportunities.

5. Cross-sectional and time-series variation in the marketability discount

5.1 Estimating elasticities

We regress the marketability discount on its determinants. We estimate two different econometric specifications, one that uses the logarithm, and another that uses the level, of each of the discount measures, *MD1* and *MD2*.

For statistical reasons, the logarithm specification is preferred since the marketability discount is truncated at zero and highly skewed.

For *MD1*, and for the logarithm specification, we obtain:

$$\begin{aligned} \ln \widehat{MD1} = & -\frac{1.98}{(0.62)} + \frac{0.08}{(0.12)} \times \text{Yield curve slope} + \frac{0.51}{(0.19)} \times \text{Fontaine-Garcia index} \\ & -\frac{7.39}{(1.48)} \times \text{Market return} - \frac{7.16}{(2.21)} \times \text{Market volatility} - \frac{17.93}{(8.06)} \times \text{GDP growth} \\ & + \frac{6.27}{(3.68)} \times \text{Yield curve slope} \times \text{GDP growth} \\ & + \frac{0.69}{(0.60)} \times \text{Yield curve slope} \times \text{Market return} \\ & + \frac{1.05}{(0.31)} \times \text{Target minus industry leverage} \\ & + \frac{0.94}{(0.47)} \times \text{Industry's asset specificity} \\ & - \frac{0.33}{(0.23)} \times \text{Industry's market-to-book ratio} - \frac{0.73}{(2.70)} \times \text{Block-to-Industry size} \\ & - \frac{0.12}{(0.03)} \times \text{Industry's M\&A activity} - \frac{0.46}{(0.21)} \times \text{Target volatility}; R^2 = 0.74 \end{aligned}$$

For *MD2*, and again for the logarithm specification, we obtain very similar estimates, both qualitatively and quantitatively:

$$\begin{aligned} \ln \widehat{MD2} = & -\frac{1.95}{(0.72)} + \frac{0.16}{(0.14)} \times \text{Yield curve slope} + \frac{0.62}{(0.23)} \times \text{Fontaine-Garcia index} \\ & -\frac{6.89}{(1.73)} \times \text{Market return} - \frac{8.59}{(2.58)} \times \text{Market volatility} - \frac{19.95}{(9.42)} \times \text{GDP growth} \\ & + \frac{8.10}{(4.30)} \times \text{Yield curve slope} \times \text{GDP growth} \\ & + \frac{0.28}{(0.70)} \times \text{Yield curve slope} \times \text{Market return} \\ & + \frac{1.21}{(0.36)} \times \text{Target minus industry leverage} \\ & + \frac{1.20}{(0.55)} \times \text{Industry's asset specificity} \\ & - \frac{0.60}{(0.27)} \times \text{Industry's market-to-book ratio} - \frac{1.25}{(3.16)} \times \text{Block-to-Industry size} \\ & - \frac{0.18}{(0.03)} \times \text{Industry's M\&A activity} - \frac{0.63}{(0.25)} \times \text{Target volatility}; R^2 = 0.76 \end{aligned}$$

Associated with each variable, there is the estimated coefficient and underneath it, in parenthesis, the standard deviation of the estimate. The high R^2 in both regressions indicate that the specifications explain well the marketability discount. The R^2 in the regressions that use the level of the discounts as the left hand side variable are in the range of 60 percent.

In Table 2, we convert the estimated coefficients from these regressions into elasticities that give the percentage change in the marketability discount induced by one percentage change in each of the liquidity determinants, keeping all others constant. By and large, the two specifications produce very similar results, with the logarithmic specification occasionally yielding slightly more statistically significant estimates.

To rank each determinant of liquidity by its explanatory power, we compute the partial correlation coefficient between the marketability discount and said determinant. The partial correlation, say between *MD1* and *GDP growth*, is calculated as the correlation between the residual of regressing *MD1* on all its determinants except for *GDP growth* and the residual of regressing *GDP growth* on all the liquidity determinants except itself. Table 3 presents the correlation coefficients between *MD1* and its determinants, as well as their squared values, which give the proportion of the residual variation in the marketability discount that is explained by each determinant. The determinants are ordered by their relevance. The results for *MD2* are almost identical and therefore omitted from the table.

Table 3 presents evidence consistent with the marketability discount being most highly correlated with economy-wide or business cycle determinants of liquidity, *Market Return* and *GDP growth*.

Periods of high *Market Return* or *GDP growth* are periods of increased liquidity and lower marketability discounts, as predicted. Quantitatively, we find that in an economic expansion, equivalent

to a one standard deviation increase in *GDP growth* relative to its mean, the average *MD1* is 1.5 percent, whereas in an economic recession, equivalent to a one standard deviation decrease in *GDP*

growth relative to its mean, the average *MD1* is 24.8 percent. The numbers for the average *MD2* are 0.6 percent and 24.4 percent, respectively.⁹

Table 2: The elasticities of the marketability discount with respect to its determinants

Econometric Specification	Log on levels		Levels on levels	
	Marketability discount 1	Marketability discount 2	Marketability discount 1	Marketability discount 2
Economy-wide variables				
Market return	-0.792*** (0.101)	-0.817*** (0.118)	-0.563*** (0.136)	-0.578*** (0.144)
GDP growth	-0.921*** (0.115)	-0.989*** (0.135)	-0.811*** (0.156)	-0.842*** (0.165)
Market volatility	-1.004*** (0.309)	-1.205*** (0.362)	-0.692 (0.418)	-0.425 (0.262)
Yield curve slope	0.621*** (0.161)	0.768*** (0.188)	0.799*** (0.217)	0.847*** (0.229)
Fontaine-Garcia index	0.412*** (0.156)	0.499*** (0.182)	0.665*** (0.211)	0.422*** (0.132)
Industry- and firm-specific variables				
Industry's M&A activity	-0.477*** (0.106)	-0.702*** (0.124)	-0.445*** (0.143)	-0.478*** (0.152)
Target minus industry leverage	0.047*** (0.014)	0.054*** (0.016)	0.039** (0.019)	0.040** (0.020)
Target volatility	-0.182** (0.084)	-0.253** (0.098)	-0.282** (0.114)	-0.303** (0.120)
Industry asset specificity	0.254** (0.126)	0.327** (0.148)	0.273*** (0.087)	0.291*** (0.097)
Industry market-to-book ratio	-0.413 (0.290)	-0.738** (0.339)	-0.45 (0.392)	-0.494 (0.414)
Block-to-Industry Size	-0.006 (0.021)	-0.01 (0.025)	0.025 (0.029)	0.027 (0.031)

This table reports the elasticities of the marketability and control discounts of majority controlling blocks with respect to their determinants. Their standard errors are shown underneath, in brackets. Estimates followed by ***, **, and * are statistically significant to the 0.01, 0.05, and 0.1 respectively. The variables are defined in Table 1. The elasticities are derived from the regression of the logarithms or the levels of the marketability discounts predicted by the search model in Albuquerque and Schroth (2013) on the determinants shown above, using a sample of 114 acquisitions of controlling blocks between 35 percent and 90 percent of the common stock, which occurred between January of 1990 and December of 2010.

9 It's possible too that fewer deals are made during recessions. For example, during the European sovereign debt crisis the affected governments sold assets in search of liquidity but sometimes backed out of intended sales possibly because the marketability discount associated would be too large. Such is the case of the main public airline company in Portugal, TAP.



Of the remaining economy-wide determinants of liquidity, the proxies for funding costs and funding liquidity, *Yield curve slope* and *Fontaine-Garcia index*, respectively, also appear quantitatively important, but their effect, as measured by the squared partial correlation, is less than one-fourth of the effect of the business cycle variables.

Of the industry and firm-level determinants of liquidity, the most significant one is *Industry's M&A Activity*. Periods with high *Industry's M&A Activity* appear to be associated with increased demand for industry assets and low marketability discounts. The effect of *Target minus Industry Leverage* is positive as expected, though smaller with a squared partial correlation of 10.4 percent. All other industry and firm-level determinants have squared partial correlations that are at or below five percent. Surprisingly, our results suggest that the variables

that proxy for the block's redeployability, *Industry's asset specificity* and *Block-to-Industry Size*, are marginally significant at best. This is not conclusive evidence that asset specificity is not relevant to determine liquidity in block trades partly because of the difficulty of finding good proxies for financial redeployability.

5.2 Predicting the marketability discount for privately held corporations: An application to the case of Mandelbaum et al v. Commissioner of Internal Revenue

Next, we use the elasticities in Table 2 to compute the change in the marketability discount given changes in its determinants. To illustrate, we use

the actual values in Mandelbaum et al. v. Commissioner of Internal Revenue (1995).

In the Court's decision regarding Mandelbaum et al. v. Commissioner of Internal Revenue, the Judge applied a 30 percent discount with respect to the standard valuation, ruling against the plaintiff's expert claim to apply a 75 percent discount on the 'virtually illiquid' block of shares of Big M, Inc., a chain of department stores specialized in sporting goods located in northern New Jersey. The Judge argued that he had no other choice but to apply the average observed discount on stocks with trading restrictions, but suggested that this discount was not ideal because the illiquidity of blocks is not the same as trading restrictions on small amounts of shares, and because discounts should vary with deal and macroeconomic effects.

Table 3: Explanatory power of the marketability discounts determinants

Determinants	Partial correlation coefficient	Squared partial correlation	Significance (p-value)	Discounts interval following a partial one sample standard deviation decrease/increase in each determinant	
				Marketability discount 1	Marketability discount 2
Economy-wide variables					
Market return	-0.652	0.425	0.000	[0.002, 0.260]	[0.000, 0.252]
GDP growth	-0.639	0.409	0.000	[0.015, 0.248]	[0.006, 0.244]
Market volatility	-0.381	0.145	0.000	[0.082, 0.181]	[0.082, 0.181]
Yield curve slope	0.304	0.092	0.002	[0.075, 0.188]	[0.059, 0.191]
Fontaine-Garcia index	0.215	0.046	0.028	[0.098, 0.165]	[0.086, 0.164]
Industry- and firm-specific variables					
Industry's M&A activity	-0.456	0.208	0.000	[0.072, 0.191]	[0.042, 0.208]
Target minus industry leverage	0.322	0.104	0.001	[0.093, 0.170]	[0.083, 0.167]
Target volatility	-0.211	0.045	0.031	[0.107, 0.156]	[0.093, 0.157]
Industry asset specificity	0.204	0.041	0.038	[0.108, 0.155]	[0.097, 0.154]
Industry market-to-book ratio	-0.073	0.005	0.462	[0.111, 0.152]	[0.090, 0.160]
Block-to-Industry Size	-0.045	0.002	0.653	[0.128, 0.135]	[0.120, 0.130]
This table reports the correlation coefficient of the marketability discount of majority controlling blocks with each of its determinants after removing the effects of all other determinants. The variables are ranked according to the square of this partial correlation. The significance level (p-value) is for the test that each determinant's partial correlation with the marketability discount is zero. The table also reports the predicted marketability discounts on the controlling block following a decrease or an increase by one sample standard deviation in each determinant, keeping the others constant.					

Table 4 shows how to use the information above to get an estimate for the marketability discount to apply in the case of Big M, Inc. stock. To conduct this exercise, we focus on the top three economy-wide determinants and the top three industry- and firm-level determinants of *MD1*. The top part of Table 4 documents the mean

and standard deviation of these determinants in the AS sample, under the heading "Sample statistics," and the actual data from Big M, Inc., its industry (three-digit SIC Code 531, Department Stores) and the US economy on December of 1990, under the heading "Big M, Inc." The last column in the top part of Table 4, shows the percentage

difference between the values for Big M, Inc. and the sample means in AS.

The resulting marketability discount is shown at the bottom part of Table 4. It is calculated by adding to the mean discount (bottom-left) the sum of the changes induced by each variable. These are estimated by

Table 4: Estimating the block discount for Big M, Inc. in December 1990

Data	Sample statistics		Big M, Inc.	
	Mean	Standard deviation	Values	% difference
Economy-wide variables				
Market return	12.74%	15.78%	-1.98%	-115.52%
GDP growth	3.23%	3.11%	0.56%	-82.80%
Market volatility	14.03%	5.25%	15.96%	13.73%
Industry- and firm-specific variables				
Industry's M&A activity	3.879	3.682	0.569	-85.34%
Target minus industry leverage	0.045	0.280	-0.139	-409.41%
Target volatility	39.83%	40.13%	24.47%	-38.56%
Discounts	Sample		Predicted	
	Mean	Standard deviation	Mean	Standard error
Marketability discount 1	13.15%	22.19%	37.13%	21.22%
Marketability discount 2	12.50%	22.27%	38.42%	20.11%

This table illustrates the estimation of the marketability and control discounts for the controlling block of Big M, Inc., a chain of sports goods retail shops located in New Jersey, using the elasticities reported in Table 2, and the determinants of discounts with an effect significant to the 0.01 level (see Table 3).

multiplying the percentage change (column 4, Table 4) times their elasticity (columns 1 and 2, Table 2). Interestingly, the Judge's estimate of 30 percent is remarkably close to the average of the relevant discount (*MD1*) in the AS sample, i.e., 37.13 percent. In our results, this high discount is explained by the facts that the US economy was growing at a pace below mean and the stock market had experienced low returns, coupled with low M&A activity in the industry. The ameliorating factor was that Big M, Inc. was significantly less levered than its industry peers as

compared with the average target in the AS sample.

This exercise underscores the need to condition the measurement of the marketability discount on firm, industry and economy-wide characteristics. One of the main results from our exercise is that the marketability discount varies dramatically across firms, industries and time. Therefore, the application of unconditional averages can lead to biased results. While the Judge for *Mandelbaum et al. v. Commissioner of Internal Revenue* (1995) raised this concern, at the time there was

no way to address it. Of course, the quantitative analysis that we propose is not to be seen as a rule of thumb to be applicable blindly. Instead, judgment is required to consider for example, any special circumstances, facts and variables ignored in the analysis, and the bargaining power of the buyer and seller in each transaction.¹⁰ In addition, one reason for exercising judgement that transpires from our exercise is the high standard error of the estimated marketability discount. In the *Mandelbaum* case, the estimated mean value of *MD1* has a standard error of 21 percent.

¹⁰ Knowing for example the prospects for a future IPO or the capacity of the firm to pay dividends may further contribute to improve these estimates.

6. Summary and conclusions

The precise valuation of controlling blocks of shares must recognize the complex effects of the illiquidity of the market for corporate control. Evidence suggests that shares in these privately negotiated blocks trade at prices that differ substantially from the share price that the same companies trade at in active markets. While it is possible and necessary to solve and estimate a model that spells out this complexity, it is also necessary to devise a way to use the model's results in a straightforward way.

We have analyzed the marketability discount and a laundry list of possible determinants of liquidity. We provided both a ranking of these determinants based on economic and statistical significance and a way of combining this information to estimate the marketability discount on blocks of shares.

Besides its simplicity, two important strengths of our results are that

(i) they allow for firm, industry and macroeconomic determinants of liquidity to affect the marketability discount, and (ii) it can be quite accurate relative to the more exact approach in AS with remarkably little data. Economy-wide variables appear to capture well periods of drying up liquidity.

The simplicity of our approach should not hide the complexity of the issue. The quantitative estimates that can be derived using the elasticities we offer have to be judged based on the statistical significance of the estimates, as well as based on information that has not been quantified in the analysis. For example, in any valuation analysis consideration may be given to behavioral factors such as the bargaining power of the buyer and seller, or non-behavioral factors that we omitted in the analysis. Also, consideration may be given to alternative ways of disposing the asset including the piecemeal sale of the block.



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